Graduate Texts in Mathematics

Serge Lang

Fundamentals of Differential Geometry

微分几何基础

Springer

世界图书出版公司
www.wpcbj.com.cn

Serge Lang

Fundamentals of Differential Geometry

With 22 Illustrations

Springer

图书在版编目（CIP）数据

微分几何基础：英文/（美）朗（Lang，S.）著.
—修订本.—北京：世界图书出版公司北京公司，
2010.2
书名原文：Fundamentals of Differential Geometry
ISBN 978-7-5100-0540-4

Ⅰ.①微… Ⅱ.①朗… Ⅲ.①微分几何—英文
Ⅳ.0186.1

中国版本图书馆 CIP 数据核字（2010）第 010689 号

书　　名：	Fundamentals of Differential Geometry
作　　者：	Serge Lang
中 译 名：	微分几何基础
责任编辑：	高蓉　刘慧
出 版 者：	世界图书出版公司北京公司
印 刷 者：	三河国英印务有限公司
发　　行：	世界图书出版公司北京公司（北京朝内大街 137 号　100010）
联系电话：	010-64021602，010-64015659
电子信箱：	kjb@ wpcbj. com. cn
开　　本：	24 开
印　　张：	23.5
版　　次：	2010 年 01 月
版权登记：	图字：01-2009-1083
书　　号：	978-7-5100-0540-4/O · 756　　**定　　价：** 65.00 元

Serge Lang
Department of Mathematics
Yale University
New Haven, CT 06520
USA

Editorial Board

S. Axler
Department of Mathematics
San Francisco State University
San Francisco, CA 94132
USA

F.W. Gehring
Mathematics Department
East Hall
University of Michigan
Ann Arbor, MI 48109
USA

K.A. Ribet
Department of Mathematics
University of California at
Berkeley
Berkeley, CA 94720-3840
USA

Mathematics Subject Classification (1991): 58-01

Library of Congress Cataloging-in-Publication Data

Lang, Serge, 1927–
Fundamentals of differential geometry / Serge Lang.
p. cm.—(Graduate texts in mathematics: 191)
Includes bibliographical references and index.
ISBN 0-387-98593-X (alk. paper)
1. Geometry, Differential. I. Title. II. Series.
QA641.L33 1999
516.3'6—dc21 98-29993

©1999 Springer-Verlag New York, Inc
All rights reserved. This work may not be translated or copied in whole or in part without the written permission of the publisher (Springer-Verlag New York, Inc., 175 Fifth Avenue, New York, NY 10010, USA), except for brief excerpts in connection with reviews or scholarly analysis. Use in connection with any form of information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed is forbidden. The use of general descriptive names, trade names, trademarks, etc., in this publication, even if the former are not epecially identified, is not to be taken as a sign that such names, as understood by the Trade Marks and Merchandise Marks Act, may be accordingly used freely by anyone.

This reprint has been authorized by Springer-Verlag (Berlin/Heidelberg/New York) for sale in the Mainland China only and not for export therefrom.

9 8 7 6 5 4 3 2 (Corrected second printing, 2001)

ISBN 0-387-98593-X

Springer-Verlag New York Berlin Heidelberg
A member of BertelsmannSpringer Science+Business Media GmbH

Graduate Texts in Mathematics 191

Editorial Board
S. Axler F.W. Gehring K.A. Ribet

Springer
New York
Berlin
Heidelberg
Barcelona
Hong Kong
London
Milan
Paris
Singapore
Tokyo

试读结束：需要全本请在线购买： www.ertongbook.com

Books of Related Interest by Serge Lang

Linear Algebra, Third Edition
1987, ISBN 0-387-96412-6

Undergraduate Algebra, Second Edition
1990, ISBN 0-387-97279-X

Undergraduate Analysis, Second Edition
1997, ISBN 0-387-94841-4

Complex Analysis, Third Edition
1993, ISBN 0-387-97886

Real and Functional Analysis, Third Edition
1993, ISBN 0-387-94001-4

Algebraic Number Theory, Second Edition
1994, ISBN 0-387-94225-4

Introduction to Complex Hyperbolic Spaces
1987, ISBN 0-387-96447-9

Other Books by Lang Published by Springer-Verlag

Introduction to Arakelov Theory • Riemann–Roch Algebra (with William Fulton) • Complex Multiplication • Introduction to Modular Forms • Modular Units (with Daniel Kubert) • Fundamentals of Diophantine Geometry • Elliptic Functions • Number Theory III • Survey of Diophantine Geometry • Cyclotomic Fields I and II • $SL_2(\mathbf{R})$ • Abelian Varieties • Introduction to Algebraic and Abelian Functions • Introduction to Diophantine Approximations • Elliptic Curves: Diophantine Analysis • Introduction to Linear Algebra • Calculus of Several Variables • First Course in Calculus • Basic Mathematics • Geometry: A High School Course (with Gene Murrow) • Math! Encounters with High School Students • The Beauty of Doing Mathematics • THE FILE • CHALLENGES

Foreword

The present book aims to give a fairly comprehensive account of the fundamentals of differential manifolds and differential geometry. The size of the book influenced where to stop, and there would be enough material for a second volume (this is not a threat).

At the most basic level, the book gives an introduction to the basic concepts which are used in differential topology, differential geometry, and differential equations. In differential topology, one studies for instance homotopy classes of maps and the possibility of finding suitable differentiable maps in them (immersions, embeddings, isomorphisms, etc.). One may also use differentiable structures on topological manifolds to determine the topological structure of the manifold (for example, à la Smale [Sm 67]). In differential geometry, one puts an additional structure on the differentiable manifold (a vector field, a spray, a 2-form, a Riemannian metric, ad lib.) and studies properties connected especially with these objects. Formally, one may say that one studies properties invariant under the group of differentiable automorphisms which preserve the additional structure. In differential equations, one studies vector fields and their integral curves, singular points, stable and unstable manifolds, etc. A certain number of concepts are essential for all three, and are so basic and elementary that it is worthwhile to collect them together so that more advanced expositions can be given without having to start from the very beginnings.

Those interested in a brief introduction could run through Chapters II, III, IV, V, VII, and most of Part III on volume forms, Stokes' theorem, and integration. They may also assume all manifolds finite dimensional.

Charts and local coordinates. A chart on a manifold is classically a representation of an open set of the manifold in some euclidean space.

Using a chart does not necessarily imply using coordinates. Charts will be used systematically. It will be observed equally systematically that finite dimensionality is hereby not used.

It is possible to lay down *at no extra cost* the foundations (and much more beyond) for manifolds modeled on Banach or Hilbert spaces rather than finite dimensional spaces. In fact, it turns out that the exposition gains considerably from the systematic elimination of the indiscriminate use of local coordinates $x_1, \ldots, x_n$ and $dx_1, \ldots, dx_n$. These are replaced by what they stand for, namely isomorphisms of open subsets of the manifold on open subsets of Banach spaces (local charts), and a local analysis of the situation which is more powerful and equally easy to use formally. In most cases, the finite dimensional proof extends at once to an invariant infinite dimensional proof. Furthermore, in studying differential forms, one needs to know only the definition of multilinear continuous maps. An abuse of multilinear algebra in standard treatises arises from an unnecessary double dualization and an abusive use of the tensor product.

I don't propose, of course, to do away with local coordinates. They are useful for computations, and are also especially useful when integrating differential forms, because the $dx_1 \wedge \cdots \wedge dx_n$. corresponds to the $dx_1 \cdots dx_n$ of Lebesgue measure, in oriented charts. Thus we often give the local coordinate formulation for such applications. Much of the literature is still covered by local coordinates, and I therefore hope that the neophyte will thus be helped in getting acquainted with the literature. I also hope to convince the expert that nothing is lost, and much is gained, by expressing one's geometric thoughts without hiding them under an irrelevant formalism.

I am aware of a widespread apprehensive reaction the moment some geometers or students see the words "Banach space" or "Hilbert manifold". As a possible palliative, I suggest reading the material assuming from the start that Banach space means finite dimensional space over the reals, and Hilbert manifold or Riemannian manifold means a finite dimensional manifold with a metric, with the local constant model being ordinary euclidean space. These assumptions will not make any proof shorter.

One major function of finding proofs valid in the infinite dimensional case is to provide proofs which are especially natural and simple in the finite dimensional case. Even for those who want to deal only with finite dimensional manifolds, I urge them to consider the proofs given in this book. In many cases, proofs based on coordinate free local representations in charts are clearer than proofs which are replete with the claws of a rather unpleasant prying insect such as Γ^i_{jkl}. Indeed, the bilinear map associated with a spray (which is the quadratic map corresponding to a symmetric connection) satisfies quite a nice local formalism in charts. I think the local representation of the curvature tensor as in Proposition 1.2 of Chapter IX shows the efficiency of this formalism and its superiority over

local coordinates. Readers may also find it instructive to compare the proof of Proposition 2.6 of Chapter IX concerning the rate of growth of Jacobi fields with more classical ones involving coordinates as in [He 78], pp. 71–73.

Applications in Infinite Dimension

It is profitable to deal with infinite dimensional manifolds, modeled on a Banach space in general, a self-dual Banach space for pseudo Riemannian geometry, and a Hilbert space for Riemannian geometry. In the standard pseudo Riemannian and Riemannian theory, readers will note that the differential theory works in these infinite dimensional cases, with the Hopf–Rinow theorem as the single exception, but not the Cartan–Hadamard theorem and its corollaries. Only when one comes to dealing with volumes and integration does finite dimensionality play a major role. Even if via the physicists with their Feynman integration one eventually develops a coherent analogous theory in the infinite dimensional case, there will still be something special about the finite dimensional case.

The failure of Hopf–Rinow in the infinite dimensional case is due to a phenomenon of positive curvature. The validity of Cartan–Hadamard in the case of negative curvature is a very significant fact, and it is only recently being realized as providing a setting for major applications. It is a general phenomenon that spaces parametrizing certain structures are actually infinite dimensional Cartan–Hadamard spaces, in many contexts, e.g. Teichmuller spaces, spaces of Riemannian metrics, spaces of Kähler metrics, spaces of connections, spaces associated with certain partial differential equations, ad lib. Cf. for instance the application to the KdV equation in [ScTZ 96], and the comments at the end of Chapter XI, §3 concerning other applications.

Actually, the use of infinite dimensional manifolds in connection with Teichmuller spaces dates back some time, because as shown by Bers, these spaces can be embedded as submanifolds of a complex Banach space. Cf. [Ga 87], [Vi 73]. Viewing these as Cartan–Hadamard manifolds comes from newer insights.

For further comments on some recent aspects of the use of infinite dimension, including references to Klingenberg's book [Kl 83/95], see the introduction to Chapter XIII.

Of course, there are other older applications of the infinite dimensional case. Some of them are to the calculus of variations and to physics, for instance as in Abraham–Marsden [AbM 78]. It may also happen that one does not need formally the infinite dimensional setting, but that it is useful to keep in mind to motivate the methods and approach taken in various directions. For instance, by the device of using curves, one can reduce what is a priori an infinite dimensional question to ordinary calculus in finite dimensional space, as in the standard variation formulas given in Chapter XI, §1.

Similarly, the proper domain for the geodesic part of Morse theory is the loop space (or the space of certain paths), viewed as an infinite dimensional manifold, but a substantial part of the theory can be developed without formally introducing this manifold. The reduction to the finite dimensional case is of course a very interesting aspect of the situation, from which one can deduce deep results concerning the finite dimensional manifold itself, but it stops short of a complete analysis of the loop space. (Cf. Boot [Bo 60], Milnor [Mi 63].) See also the papers of Palais [Pa 63] and Smale [Sm 64].

In addition, given two finite dimensional manifolds X, Y it is fruitful to give the set of differentiable maps from X to Y an infinite dimensional manifold structure, as was started by Eells [Ee 58], [Ee 59], [Ee 61], [EeS 64], and [Ee 66]. By so doing, one transcends the purely formal translation of finite dimensional results getting essentially new ones, which would in turn affect the finite dimensional case. For other connections with differential geometry, see [El 67].

Foundations for the geometry of manifolds of mappings are given in Abraham's notes of Smale's lectures [Ab 60] and Palais's monograph [Pa 68].

For more recent applications to critical point theory and submanifold geometry, see [PaT 88].

In the direction of differential equations, the extension of the stable and unstable manifold theorem to the Banach case, already mentioned as a possibility in earlier versions of *Differential Manifolds*, was proved quite elegantly by Irwin [Ir 70], following the idea of Pugh and Robbin for dealing with local flows using the implicit mapping theorem in Banach spaces. I have included the Pugh–Robbin proof, but refer to Irwin's paper for the stable manifold theorem which belongs at the very beginning of the theory of ordinary differential equations. The Pugh–Robbin proof can also be adjusted to hold for vector fields of class H^p (Sobolev spaces), of importance in partial differential equations, as shown by Ebin and Marsden [EbM 70].

It is a standard remark that the C^∞-functions on an open subset of a euclidean space do not form a Banach space. They form a Fréchet space (denumerably many norms instead of one). On the other hand, the implicit function theorem and the local existence theorem for differential equations are not true in the more general case. In order to recover similar results, a much more sophisticated theory is needed, which is only beginning to be developed. (Cf. Nash's paper on Riemannian metrics [Na 56], and subsequent contributions of Schwartz [Sc 60] and Moser [Mo 61].) In particular, some additional structure must be added (smoothing operators). Cf. also my Bourbaki seminar talk on the subject [La 61]. This goes beyond the scope of this book, and presents an active topic for research.

On the other hand, for some applications, one may complete the C^∞-space under a suitable Hilbert space norm, deal with the resulting Hilbert

manifold, and then use an appropriate regularity theorem to show that solutions of the equation under study actually are C^∞.

I have emphasized differential aspects of differential manifolds rather than topological ones. I am especially interested in laying down basic material which may lead to various types of applications which have arisen since the sixties, vastly expanding the perspective on differential geometry and analysis. For instance, I expect the books [BGV 92] and [Gi 95] to be only the first of many to present the accumulated vision from the seventies and eighties, after the work of Atiyah, Bismut, Bott, Gilkey, McKean, Patodi, Singer, and many others.

Negative Curvature

Most texts emphasize positive curvature at the expense of negative curvature. I have tried to redress this imbalance. In algebraic geometry, it is well recognized that negative curvature amounts more or less to "general type". For instance, curves of genus 0 are special, curves of genus 1 are semispecial, and curves of genus $\geqq 2$ are of general type. Thus I have devoted an entire chapter to the fundamental example of a space of negative curvature. Actually, I prefer to work with the Riemann tensor. I use "curvature" simply as a code word which is easily recognizable by people in the field. Furthermore, I include a complete account of the equivalence between seminegative curvature, the metric increasing property of the exponential map, and the Bruhat–Tits semiparallelogram law. Third, I emphasize the Cartan–Hadamard further by giving a version for the normal bundle of a totally geodesic submanifold. I am indebted to Wu for valuable mathematical and historical comments on this topic.

There are several current directions whereby spaces of negative curvature are the fundamental building blocks of some theories. They are quotients of Cartan–Hadamard spaces. I myself got interested in differential geometry because of the joint work with Jorgenson, which naturally led us to such spaces for the construction and theory of certain zeta functions. Quite generally, we were led to consider spaces which admit a stratification such that each stratum is a quotient of a Cartan–Hadamard space (especially a symmetric space) by a discrete group. That such stratifications exist very widely is a fact not generally taken into account. For instance, it is a theorem of Griffiths that given an algebraic variety over the complex numbers, there exists a proper Zariski closed subset whose complement is a quotient of a complex bounded domain, so in this way, every algebraic variety admits a stratification as above, even with constant negative curvature. Thurston's approach to 3-manifolds could be viewed from our perspective also. The general problem then arises how zeta functions, spectral invariants, homotopy and homology invariants, ad

试读结束：需要全本请在线购买： www.ertongbook.com

lib. behave with respect to stratifications, whether additively or otherwise. In the Jorgenson–Lang program, we associate a zeta function to each stratum, and the zeta functions of lower strata are the principal fudge factors in the functional equation of the zeta function associated to the main stratum. The spectral expansion of the heat kernel amounts to a theta relation, and we get the zeta function by taking the Gauss transform of the theta relation.

From a quite different perspective, certain natural "moduli" spaces for structures on finite dimensional manifolds have a very strong tendency to be Cartan–Hadamard spaces, for instance the space of Riemannian metrics, spaces of Kahler metrics, spaces of connections, etc. which deserve to be incorporated in a general theory.

In any case, I find the exclusive historical emphasis at the foundational level on positive curvature, spheres, projective spaces, grassmanians, at the expense of quotients of Cartan–Hadamard spaces, to be misleading as to the way manifolds are built up. Time will tell, but I don't think we'll have to wait very long before a radical change of view point becomes prevalent.

New Haven, 1998 SERGE LANG

Acknowledgments

I have greatly profited from several sources in writing this book. These sources include some from the 1960s, and some more recent ones.

First, I originally profited from Dieudonné's *Foundations of Modern Analysis*, which started to emphasize the Banach point of view.

Second, I originally profited from Bourbaki's *Fascicule de résultats* [Bou 69] for the foundations of differentiable manifolds. This provides a good guide as to what should be included. I have not followed it entirely, as I have omitted some topics and added others, but on the whole, I found it quite useful. I have put the emphasis on the differentiable point of view, as distinguished from the analytic. However, to offset this a little, I included two analytic applications of Stokes' formula, the Cauchy theorem in several variables, and the residue theorem.

Third, Milnor's notes [Mi 58], [Mi 59], [Mi 61] proved invaluable. They were of course directed toward differential topology, but of necessity had to cover ad hoc the foundations of differentiable manifolds (or, at least, part of them). In particular, I have used his treatment of the operations on vector bundles (Chapter III, §4) and his elegant exposition of the uniqueness of tubular neighborhoods (Chapter IV, §6, and Chapter VII, §4).

Fourth, I am very much indebted to Palais for collaborating on Chapter IV, and giving me his exposition of sprays (Chapter IV, §3). As he showed me, these can be used to construct tubular neighborhoods. Palais also showed me how one can recover sprays and geodesics on a Riemannian manifold by making direct use of the canonical 2-form and the metric (Chapter VII, §7). This is a considerable improvement on past expositions.

In the direction of differential geometry, I found Berger–Gauduchon–Mazet [BGM 71] extremely valuable, especially in the way they lead to the study of the Laplacian and the heat equation. This book has been

very influential, for instance for [GHL 87/93], which I have also found useful.

I also found useful Klingenberg's book [Kl 83/95], see especially chapter XIII. I am very thankful to Karcher and Wu for instructing me on several matters, including estimates for Jacobi fields. I have also benefited from Helgason's book [Hel 84], which contains some material of interest independently of Lie groups, concerning the Laplacian. I am especially indebted to Wu's invaluable guidance in dealing with the trace of the second fundamental form and its application to the Laplacian, giving rise to a new exposition of some theorems of Helgason in Chapters XIV and XV.

Graduate Texts in Mathematics

1 TAKEUTI/ZARING. Introduction to Axiomatic Set Theory. 2nd ed.
2 OXTOBY. Measure and Category. 2nd ed.
3 SCHAEFER. Topological Vector Spaces. 2nd ed.
4 HILTON/STAMMBACH. A Course in Homological Algebra. 2nd ed.
5 MAC LANE. Categories for the Working Mathematician. 2nd ed.
6 HUGHES/PIPER. Projective Planes.
7 SERRE. A Course in Arithmetic.
8 TAKEUTI/ZARING. Axiomatic Set Theory.
9 HUMPHREYS. Introduction to Lie Algebras and Representation Theory.
10 COHEN. A Course in Simple Homotopy Theory.
11 CONWAY. Functions of One Complex Variable I. 2nd ed.
12 BEALS. Advanced Mathematical Analysis.
13 ANDERSON/FULLER. Rings and Categories of Modules. 2nd ed.
14 GOLUBITSKY/GUILLEMIN. Stable Mappings and Their Singularities.
15 BERBERIAN. Lectures in Functional Analysis and Operator Theory.
16 WINTER. The Structure of Fields.
17 ROSENBLATT. Random Processes. 2nd ed.
18 HALMOS. Measure Theory.
19 HALMOS. A Hilbert Space Problem Book. 2nd ed.
20 HUSEMOLLER. Fibre Bundles. 3rd ed.
21 HUMPHREYS. Linear Algebraic Groups.
22 BARNES/MACK. An Algebraic Introduction to Mathematical Logic.
23 GREUB. Linear Algebra. 4th ed.
24 HOLMES. Geometric Functional Analysis and Its Applications.
25 HEWITT/STROMBERG. Real and Abstract Analysis.
26 MANES. Algebraic Theories.
27 KELLEY. General Topology.
28 ZARISKI/SAMUEL. Commutative Algebra. Vol.I.
29 ZARISKI/SAMUEL. Commutative Algebra. Vol.II.
30 JACOBSON. Lectures in Abstract Algebra I. Basic Concepts.
31 JACOBSON. Lectures in Abstract Algebra II. Linear Algebra.
32 JACOBSON. Lectures in Abstract Algebra III. Theory of Fields and Galois Theory.
33 HIRSCH. Differential Topology.
34 SPITZER. Principles of Random Walk. 2nd ed.
35 ALEXANDER/WERMER. Several Complex Variables and Banach Algebras. 3rd ed.
36 KELLEY/NAMIOKA et al. Linear Topological Spaces.
37 MONK. Mathematical Logic.
38 GRAUERT/FRITZSCHE. Several Complex Variables.
39 ARVESON. An Invitation to C^*-Algebras.
40 KEMENY/SNELL/KNAPP. Denumerable Markov Chains. 2nd ed.
41 APOSTOL. Modular Functions and Dirichlet Series in Number Theory. 2nd ed.
42 SERRE. Linear Representations of Finite Groups.
43 GILLMAN/JERISON. Rings of Continuous Functions.
44 KENDIG. Elementary Algebraic Geometry.
45 LOÈVE. Probability Theory I. 4th ed.
46 LOÈVE. Probability Theory II. 4th ed.
47 MOISE. Geometric Topology in Dimensions 2 and 3.
48 SACHS/WU. General Relativity for Mathematicians.
49 GRUENBERG/WEIR. Linear Geometry. 2nd ed.
50 EDWARDS. Fermat's Last Theorem.
51 KLINGENBERG. A Course in Differential Geometry.
52 HARTSHORNE. Algebraic Geometry.
53 MANIN. A Course in Mathematical Logic.
54 GRAVER/WATKINS. Combinatorics with Emphasis on the Theory of Graphs.
55 BROWN/PEARCY. Introduction to Operator Theory I: Elements of Functional Analysis.
56 MASSEY. Algebraic Topology: An Introduction.
57 CROWELL/FOX. Introduction to Knot Theory.
58 KOBLITZ. p-adic Numbers, p-adic Analysis, and Zeta-Functions. 2nd ed.
59 LANG. Cyclotomic Fields.
60 ARNOLD. Mathematical Methods in Classical Mechanics. 2nd ed.
61 WHITEHEAD. Elements of Homotopy Theory.
62 KARGAPOLOV/MERLZJAKOV. Fundamentals of the Theory of Groups.
63 BOLLOBAS. Graph Theory.
64 EDWARDS. Fourier Series. Vol. I. 2nd ed.
65 WELLS. Differential Analysis on Complex Manifolds. 2nd ed.

66 WATERHOUSE. Introduction to Affine Group Schemes.
67 SERRE. Local Fields.
68 WEIDMANN. Linear Operators in Hilbert Spaces.
69 LANG. Cyclotomic Fields II.
70 MASSEY. Singular Homology Theory.
71 FARKAS/KRA. Riemann Surfaces. 2nd ed.
72 STILLWELL. Classical Topology and Combinatorial Group Theory. 2nd ed.
73 HUNGERFORD. Algebra.
74 DAVENPORT. Multiplicative Number Theory. 3rd ed.
75 HOCHSCHILD. Basic Theory of Algebraic Groups and Lie Algebras.
76 IITAKA. Algebraic Geometry.
77 HECKE. Lectures on the Theory of Algebraic Numbers.
78 BURRIS/SANKAPPANAVAR. A Course in Universal Algebra.
79 WALTERS. An Introduction to Ergodic Theory.
80 ROBINSON. A Course in the Theory of Groups. 2nd ed.
81 FORSTER. Lectures on Riemann Surfaces.
82 BOTT/TU. Differential Forms in Algebraic Topology.
83 WASHINGTON. Introduction to Cyclotomic Fields. 2nd ed.
84 IRELAND/ROSEN. A Classical Introduction to Modern Number Theory. 2nd ed.
85 EDWARDS. Fourier Series. Vol. II. 2nd ed.
86 VAN LINT. Introduction to Coding Theory. 2nd ed.
87 BROWN. Cohomology of Groups.
88 PIERCE. Associative Algebras.
89 LANG. Introduction to Algebraic and Abelian Functions. 2nd ed.
90 BRONDSTED. An Introduction to Convex Polytopes.
91 BEARDON. On the Geometry of Discrete Groups.
92 DIESTEL. Sequences and Series in Banach Spaces.
93 DUBROVIN/FOMENKO/NOVIKOV. Modern Geometry—Methods and Applications. Part I. 2nd ed.
94 WARNER. Foundations of Differentiable Manifolds and Lie Groups.
95 SHIRYAEV. Probability. 2nd ed.
96 CONWAY. A Course in Functional Analysis. 2nd ed.
97 KOBLITZ. Introduction to Elliptic Curves and Modular Forms. 2nd ed.
98 BRÖCKER/TOM DIECK. Representations of Compact Lie Groups.
99 GROVE/BENSON. Finite Reflection Groups. 2nd ed.
100 BERG/CHRISTENSEN/RESSEL. Harmonic Analysis on Semigroups: Theory of Positive Definite and Related Functions.
101 EDWARDS. Galois Theory.
102 VARADARAJAN. Lie Groups, Lie Algebras and Their Representations.
103 LANG. Complex Analysis. 3rd ed.
104 DUBROVIN/FOMENKO/NOVIKOV. Modern Geometry—Methods and Applications. Part II.
105 LANG. $SL_2(\mathbf{R})$.
106 SILVERMAN. The Arithmetic of Elliptic Curves.
107 OLVER. Applications of Lie Groups to Differential Equations. 2nd ed.
108 RANGE. Holomorphic Functions and Integral Representations in Several Complex Variables.
109 LEHTO. Univalent Functions and Teichmüller Spaces.
110 LANG. Algebraic Number Theory.
111 HUSEMÖLLER. Elliptic Curves.
112 LANG. Elliptic Functions.
113 KARATZAS/SHREVE. Brownian Motion and Stochastic Calculus. 2nd ed.
114 KOBLITZ. A Course in Number Theory and Cryptography. 2nd ed.
115 BERGER/GOSTIAUX. Differential Geometry: Manifolds, Curves, and Surfaces.
116 KELLEY/SRINIVASAN. Measure and Integral. Vol. I.
117 SERRE. Algebraic Groups and Class Fields.
118 PEDERSEN. Analysis Now.
119 ROTMAN. An Introduction to Algebraic Topology.
120 ZIEMER. Weakly Differentiable Functions: Sobolev Spaces and Functions of Bounded Variation.
121 LANG. Cyclotomic Fields I and II. Combined 2nd ed.
122 REMMERT. Theory of Complex Functions. *Readings in Mathematics*
123 EBBINGHAUS/HERMES et al. Numbers. *Readings in Mathematics*
124 DUBROVIN/FOMENKO/NOVIKOV. Modern Geometry—Methods and Applications. Part III.
125 BERENSTEIN/GAY. Complex Variables: An Introduction.
126 BOREL. Linear Algebraic Groups. 2nd ed.
127 MASSEY. A Basic Course in Algebraic Topology.
128 RAUCH. Partial Differential Equations.
129 FULTON/HARRIS. Representation Theory: A First Course. *Readings in Mathematics*
130 DODSON/POSTON. Tensor Geometry.
131 LAM. A First Course in Noncommutative Rings. 2nd ed.

132 BEARDON. Iteration of Rational Functions.
133 HARRIS. Algebraic Geometry: A First Course.
134 ROMAN. Coding and Information Theory.
135 ROMAN. Advanced Linear Algebra.
136 ADKINS/WEINTRAUB. Algebra: An Approach via Module Theory.
137 AXLER/BOURDON/RAMEY. Harmonic Function Theory. 2nd ed.
138 COHEN. A Course in Computational Algebraic Number Theory.
139 BREDON. Topology and Geometry.
140 AUBIN. Optima and Equilibria. An Introduction to Nonlinear Analysis.
141 BECKER/WEISPFENNING/KREDEL. Gröbner Bases. A Computational Approach to Commutative Algebra.
142 LANG. Real and Functional Analysis. 3rd ed.
143 DOOB. Measure Theory.
144 DENNIS/FARB. Noncommutative Algebra.
145 VICK. Homology Theory. An Introduction to Algebraic Topology. 2nd ed.
146 BRIDGES. Computability: A Mathematical Sketchbook.
147 ROSENBERG. Algebraic K-Theory and Its Applications.
148 ROTMAN. An Introduction to the Theory of Groups. 4th ed.
149 RATCLIFFE. Foundations of Hyperbolic Manifolds.
150 EISENBUD. Commutative Algebra with a View Toward Algebraic Geometry.
151 SILVERMAN. Advanced Topics in the Arithmetic of Elliptic Curves.
152 ZIEGLER. Lectures on Polytopes.
153 FULTON. Algebraic Topology: A First Course.
154 BROWN/PEARCY. An Introduction to Analysis.
155 KASSEL. Quantum Groups.
156 KECHRIS. Classical Descriptive Set Theory.
157 MALLIAVIN. Integration and Probability.
158 ROMAN. Field Theory.
159 CONWAY. Functions of One Complex Variable II.
160 LANG. Differential and Riemannian Manifolds.
161 BORWEIN/ERDELYI. Polynomials and Polynomial Inequalities.
162 ALPERIN/BELL. Groups and Representations.
163 DIXON/MORTIMER. Permutation Groups.
164 NATHANSON. Additive Number Theory: The Classical Bases.
165 NATHANSON. Additive Number Theory: Inverse Problems and the Geometry of Sumsets.
166 SHARPE. Differential Geometry: Cartan's Generalization of Klein's Erlangen Program.
167 MORANDI. Field and Galois Theory.
168 EWALD. Combinatorial Convexity and Algebraic Geometry.
169 BHATIA. Matrix Analysis.
170 BREDON. Sheaf Theory. 2nd ed.
171 PETERSEN. Riemannian Geometry.
172 REMMERT. Classical Topics in Complex Function Theory.
173 DIESTEL. Graph Theory. 2nd ed.
174 BRIDGES. Foundations of Real and Abstract Analysis.
175 LICKORISH. An Introduction to Knot Theory.
176 LEE. Riemannian Manifolds.
177 NEWMAN. Analytic Number Theory.
178 CLARKE/LEDYAEV/STERN/WOLENSKI. Nonsmooth Analysis and Control Theory.
179 DOUGLAS. Banach Algebra Techniques in Operator Theory. 2nd ed.
180 SRIVASTAVA. A Course on Borel Sets.
181 KRESS. Numerical Analysis.
182 WALTER. Ordinary Differential Equations.
183 MEGGINSON. An Introduction to Banach Space Theory.
184 BOLLOBAS. Modern Graph Theory.
185 COX/LITTLE/O'SHEA. Using Algebraic Geometry.
186 RAMAKRISHNAN/VALENZA. Fourier Analysis on Number Fields.
187 HARRIS/MORRISON. Moduli of Curves.
188 GOLDBLATT. Lectures on the Hyperreals: An Introduction to Nonstandard Analysis.
189 LAM. Lectures on Modules and Rings.
190 ESMONDE/MURTY. Problems in Algebraic Number Theory.
191 LANG. Fundamentals of Differential Geometry.
192 HIRSCH/LACOMBE. Elements of Functional Analysis.
193 COHEN. Advanced Topics in Computational Number Theory.
194 ENGEL/NAGEL. One-Parameter Semigroups for Linear Evolution Equations.
195 NATHANSON. Elementary Methods in Number Theory.
196 OSBORNE. Basic Homological Algebra.
197 EISENBUD/HARRIS. The Geometry of Schemes.
198 ROBERT. A Course in p-adic Analysis.
199 HEDENMALM/KORENBLUM/ZHU. Theory of Bergman Spaces.
200 BAO/CHERN/SHEN. An Introduction to Riemann–Finsler Geometry.
201 HINDRY/SILVERMAN. Diophantine Geometry: An Introduction.
202 LEE. Introduction to Topological Manifolds.

试读结束：需要全本请在线购买： www.ertongbook.com