Pierre Meystre
Murray Sargent III

Elements of Quantum Optics

量子光学基础

第4版

4th Edition

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Preface

This book grew out of a 2-semester graduate course in laser physics and quantum optics. It requires a solid understanding of elementary electromagnetism as well as at least one, but preferably two, semesters of quantum mechanics. Its present form resulted from many years of teaching and research at the University of Arizona, the Max-Planck-Institut für Quantenoptik, and the University of Munich. The contents have evolved significantly over the years, due to the fact that quantum optics is a rapidly changing field. Because the amount of material that can be covered in two semesters is finite, a number of topics had to be left out or shortened when new material was added. Important omissions include the manipulation of atomic trajectories by light, superradiance, and descriptions of experiments.

Rather than treating any given topic in great depth, this book aims to give a broad coverage of the basic elements that we consider necessary to carry out research in quantum optics. We have attempted to present a variety of theoretical tools, so that after completion of the course students should be able to understand specialized research literature and to produce original research of their own. In doing so, we have always sacrificed rigor to physical insight and have used the concept of "simplest nontrivial example" to illustrate techniques or results that can be generalized to more complicated situations. In the same spirit, we have not attempted to give exhaustive lists of references, but rather have limited ourselves to those papers and books that we found particularly useful.

The book is divided into three parts. Chapters 1–3 review various aspects of electromagnetic theory and of quantum mechanics. The material of these chapters, especially Chaps. 1–3, represents the minimum knowledge required to follow the rest of the course. Chapter 2 introduces many nonlinear optics phenomena by using a classical nonlinear oscillator model, and is usefully referred to in later chapters. Depending on the level at which the course is taught, one can skip Chaps. 1–3 totally or at the other extreme, give them considerable emphasis.

Chapters 4–12 treat semiclassical light-matter interactions. They contain more material than we have typically been able to teach in a one-semester course. Especially if much time is spent on the Chaps. 1–3, some of Chaps. 4–12 must be skipped. However, Chap. 4 on the density matrix, Chap. 5 on the interaction between matter and cw fields, Chap. 7 on semi-classical laser theory, and to some extent Chap. 9 on nonlinear spectroscopy are central to the book and cannot be ignored. In contrast one could omit Chap. 8 on optical bistability, Chap. 10 on phase conjugation, Chap. 11 on optical instabilities, or Chap. 12 on coherent transients.

Chapters 13–19 discuss aspects of light-matter interaction that require the quantization of the electromagnetic field. They are tightly knit together and it is difficult to imagine skipping one of them in a one-semester course. Chapter 13 draws an analogy between electromagnetic field modes and harmonic oscillators to quantize the field in a simple way. Chapter 14 discusses simple aspects of the interaction between a single mode of the field and a two-level atom. Chapter 15 on reservoir theory in essential for the discussion of resonance fluorescence (Chap. 16) and squeezing (Chap. 17). These chapters are strongly connected to the nonlinear spectroscopy discussion of Chap. 9. In resonance fluorescence and in squeezing the quantum nature of the field appears mostly in the form of noise. We conclude in Chap. 19 by giving elements of the quantum theory of the laser, which requires a proper treatment of quantum fields to all orders.

In addition to being a textbook, this book contains many important formulas in quantum optics that are not found elsewhere except in the original literature or in specialized monographs. As such, and certainly for our own research, this book is a very valuable reference. One particularly gratifying feature of the book is that it reveals the close connection between many seemingly unrelated or only distantly related topics, such as probe absorption, four-wave mixing, optical instabilities, resonance fluorescence, and squeezing.

We are indebted to the many people who have made important contributions to this book: they include first of all our students, who had to suffer through several not-so-debugged versions of the book and have helped with their corrections and suggestions. Special thanks to S. An, B. Capron, T. Carty, P. Dobiasch, J. Grantham, A. Guzman, D. Holm, J. Lehan, R. Morgan, M. Pereira, G. Reiner, E. Schumacher, J. Watanabe, and M. Watson. We are also very grateful to many colleagues for their encouragements and suggestions. Herbert Walther deserves more thanks than anybody else: this book would not have been started or completed without his constant encouragement and support. Thanks are due especially to the late Fred Hopf as well as to J.H. Eberly, H.M. Gibbs, J. Javanainen, S.W. Koch, W.E. Lamb, Jr., H. Pilloff, C.M. Savage, M.O. Scully, D.F. Walls, K. Wodkiewicz, and E.M. Wright. We are also indebted to the Max-Planck-Institut fur Quantenoptik and to the U.S. Office of Naval Research for direct or indirect financial support of this work.

Tucson, August 1989

Pierre Meystre
Murray Sargent III

Preface to the Second Edition

This edition contains a significant number of changes designed to improve clarity. We have also added a new section on the theory of resonant light pressure and the manipulation of atomic trajectories by light. This topic is of considerable interest presently and has applications both in high resolution spectroscopy and in the emerging field of atom optics. Smaller changes include a reformulation of the photon-echo problem in a way that reveals its relationship to four-wave mixing, as well as a discussion of the quantization of standing-waves versus running-waves of the electromagnetic field. Finally, we have also improved a number of figures and have added some new ones.

We thank the readers who have taken the time to point out to us a number of misprints. Special tanks are due to Z. Bialynicka-Birula. S. Haroche, K. Just, S. LaRochelle, E. Schumacher, and M. Wilkens.

Tucson, February 1991 *P.M. M.S. III*

Preface to the Third Edition

Important developments have taken place in quantum optics in the last few years. Particularly noteworthy are cavity quantum electrodynamics, which is already moving toward device applications, atom optics and laser cooling, which are now quite mature subjects, and the recent experimental demonstration of Bose-Einstein condensation in low density alkali vapors. A number of theoretical tools have been either developed or introduced to quantum optics to handle the new situations at hand.

The third edition of Elements of Quantum Optics attempts to include many of these developments, without changing the goal of the book, which remains to give a broad description of the basic tools necessary to carry out research in quantum optics. We have therefore maintained the general structure of the text, but added topics called for by the developments we mentioned. The discussion of light forces and atomic motion has been promoted to a whole chapter, which includes in addition a simple analysis of Doppler cooling. A new chapter on cavity QED has also been included. We have extended the discussion of quasi-probability distributions of the electromagnetic field, and added a section on the quantization of the Schrödinger field, *aka* second quantization. This topic has become quite important in connection with atom optics and Bose condensation, and is now a necessary part of quantum optics education. We have expanded the chapter on system-reservoir interactions to include an introduction to the Monte Carlo wave functions technique. This method is proving exceedingly powerful in numerical simulations as well as in its intuitive appeal in shedding new light on old problems. Finally, at a more elementary level we have expanded the discussion of quantum mechanics to include a more complete discussion of the coordinate and momentum representations. We have also fixed whatever misprints have been brought to our attention in the previous edition.

Because Murray Sargent moved from the sunny Southwest to the rainy Northwest to pursue his interests in computer science, it rested on my shoulders to include these changes in the book. Fans of Murray's style and physical understanding will no doubt regret this, as I missed his input, comments and enthusiasm. I hope that the final product will nonetheless meet his and your approval.

As always, I have benefited enormously from the input of my students and colleagues. Special thanks are due this time to J.D. Berger, H. Giessen, E.V. Goldstein, G. Lenz and M.G. Moore.

Tucson, November 1997 *P.M.*

Preface to the Fourth Edition

It has been 10 years since the publication of the third edition of this text, and quantum optics continues to be a vibrant field with exciting and oftentimes unexpected new developments. This is the motivation behind the addition of a new chapter on quantum entanglement and quantum information, two areas of considerable current interest. A section on the quantum theory of the beam splitter has been included in that chapter, as this simple, yet rather subtle device is central to much of the work on that topic. Spectacular progress also continues in the study of quantum-degenerate atoms and molecules, and quantum optics plays a leading role in that research, too. While it is well beyond the scope of this book to cover this fast moving area in any kind of depth, we have included a section on the Gross-Pitaevskii equation, which is a good entry point to that exciting field. New sections on atom interferometry, electromagnetically induced transparency (EIT), and slow light have also been added. There is now a more detailed discussion of the electric dipole approximation in Chap. 3, complemented by three problems that discuss details of the minimum coupling Hamiltonian, and an introduction to the input-output formalism in Chap. 18. More minor changes have been included at various places, and hopefully all remaining misprints have been fixed. Many of the figures have been redrawn and replace originals that dated in many cases from the stone-age of word processing. I am particularly thankful to Kiel Howe for his talent and dedication in carrying out this task.

Many thanks are also due to M. Bhattacharya, W. Chen, O. Dutta, R. Kanamoto, V. S. Lethokov, D. Meiser, T. Miyakawa, C. P. Search, and H. Uys. The final touches to this edition were performed at the Kavli Institute for Theoretical Physics, University of California, Santa Barbara. It is a pleasure to thank Dr. David Gross and the KITP staff for their perfect hospitality.

Tucson, June 2007 *P.M.*

Contents

1 Classical Electromagnetic Fields

In this book we present the basic ideas needed to understand how laser light interacts with various forms of matter. Among the important consequences is an understanding of the laser itself. The present chapter summarizes classical electromagnetic fields, which describe laser light remarkably well. The chapter also discusses the interaction of these fields with a medium consisting of classical simple harmonic oscillators. It is surprising how well this simple model describes linear absorption, a point discussed from a quantum mechanical point of view in Sect. 3.3. The rest of the book is concerned with *nonlinear* interactions of radiation with matter. Chapter 2 generalizes the classical oscillator to treat simple kinds of nonlinear mechanisms, and shows us a number of phenomena in a relatively simple context. Starting with Chap. 3, we treat the medium quantum mechanically. The combination of a classical description of light and a quantum mechanical description of matter is called the *semiclassical* approximation. This approximation is not always justified (Chaps. 13–19), but there are remarkably few cases in quantum optics where we need to quantize the field.

In the present chapter, we limit ourselves both to *classical* electromagnetic fields and to *classical* media. Section 1.1 briefly reviews Maxwell's equations in a vacuum. We derive the wave equation, and introduce the slowly-varying amplitude and phase approximation for the electromagnetic field. Section 1.2 recalls Maxwell's equations in a medium. We then show the roles of the in-phase and in-quadrature parts of the polarization of the medium through which the light propagates, and give a brief discussion of Beer's law of light absorption. Section 1.3 discusses the classical dipole oscillator. We introduce the concept of the self-field and show how it leads to radiative damping. Then we consider the classical Rabi problem, which allows us to introduce the classical analog of the optical Bloch equations. The derivations in Sects. 1.1–1.3 are not necessarily the simplest ones, but they correspond as closely as possible to their quantum mechanical counterparts that appear later in the book.

Section 1.4 is concerned with the coherence of the electromagnetic field. We review the Young and Hanbury Brown-Twiss experiments. We introduce the notion of nth order coherence. We conclude this section by a brief comment on antibunching, which provides us with a powerful test of the quantum nature of light.

With knowledge of Sects. 1.1–1.4, we have all the elements needed to understand an elementary treatment of the Free-Electron Laser (FEL), which is presented in Sect. 1.5. The FEL is in some way the simplest laser to understand, since it can largely be described classically, i.e., there is no need to quantize the matter.

1.1 Maxwell's Equations in a Vacuum

In the absence of charges and currents, Maxwell's equations are given by

$$\nabla \cdot \mathbf{B} = 0\,, \tag{1.1}$$

$$\nabla \cdot \mathbf{E} = 0\,, \tag{1.2}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}\,, \tag{1.3}$$

$$\nabla \times \mathbf{B} = \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}\,, \tag{1.4}$$

where $\mathbf{E}$ is the electric field, $\mathbf{B}$ is the magnetic field, μ_0 is the permeability of the free space, and ε_0 is the permittivity of free space (in this book we use MKS units throughout). Alternatively it is useful to write c^2 for $1/\mu_0\varepsilon_0$, where c is the speed of light in the vacuum. Taking the curl of (1.3) and substituting the rate of change of (1.4) we find

$$\nabla \times \nabla \times \mathbf{E} = -\frac{1}{c^2}\frac{\partial^2 \mathbf{E}}{\partial t^2}\,. \tag{1.5}$$

This equation can be simplified by noting that $\nabla \times \nabla = \nabla(\nabla \cdot) - \nabla^2$ and using (1.2). We find the wave equation

$$\nabla^2 \mathbf{E} - \frac{1}{c^2}\frac{\partial^2 \mathbf{E}}{\partial t^2} = 0\,. \tag{1.6}$$

This tells us how an electromagnetic wave propagates in a vacuum. By direct substitution, we can show that

$$\mathbf{E}(\mathbf{r}, t) = \mathbf{E}_0 f(\mathbf{K} \cdot \mathbf{r} - \nu t) \tag{1.7}$$

is a solution of (1.6) where f is an arbitrary function, $\mathbf{E}_0$ is a constant, ν is an oscillation frequency in radians/second ($2\pi \times Hz$), $\mathbf{K}$ is a constant vector in the direction of propagation of the field, and having the magnitude $K \equiv |\mathbf{K}| = \nu/c$. This solution represents a transverse plane wave propagating along the direction of $\mathbf{K}$ with speed $c = \nu/K$.

A property of the wave equation (1.6) is that if $\mathbf{E}_1(\mathbf{r}, t)$ and $\mathbf{E}_2(\mathbf{r}, t)$ are solutions, then the superposition $a_1\mathbf{E}_1(\mathbf{r}, t) + a_2\mathbf{E}_2(\mathbf{r}, t)$ is also a solution, where a_1 and a_2 are any two constants. This is called the principle of superposition. It is a direct consequence of the fact that differentiation is a linear operation. In particular, the superposition

$$\mathbf{E}(\mathbf{r},t) = \sum_k \mathbf{E}_k f(\mathbf{K}_k\cdot\mathbf{r} - \nu t) \tag{1.8}$$

is also a solution. This shows us that nonplane waves are also solutions of the wave equation (1.6).

Quantum opticians like to decompose electric fields into "positive" and "negative" frequency parts

$$\mathbf{E}(\mathbf{r},t) = \mathbf{E}^+(\mathbf{r},t) + \mathbf{E}^-(\mathbf{r},t)\ , \tag{1.9}$$

where $\mathbf{E}^+(\mathbf{r},t)$ has the form

$$\mathbf{E}^+(\mathbf{r},t) = \frac{1}{2}\sum_n \mathcal{E}_n(\mathbf{r})\mathrm{e}^{-\mathrm{i}\nu_n t}\ , \tag{1.10}$$

where $\mathcal{E}_n(\mathbf{r})$ is a complex function of $\mathbf{r}, \nu_n$ is the corresponding frequency, and in general

$$\mathbf{E}^-(\mathbf{r},t) = [\mathbf{E}^+(\mathbf{r},t)]^*\ . \tag{1.11}$$

In itself this decomposition is just that of the analytic signal used in classical coherence theory [see Born and Wolf (1970)], but as we see in Chap. 13, it has deep foundations in the quantum theory of light detection. For now we consider this to be a convenient mathematical trick that allows us to work with exponentials rather than with sines and cosines. It is easy to see that since the wave equation (1.6) is real, if $\mathbf{E}^+(\mathbf{r},t)$ is a solution, then so is $\mathbf{E}^-(\mathbf{r},t)$, and the linearity of (1.6) guarantees that the sum (1.9) is also a solution.

In this book, we are concerned mostly with the interaction of monochromatic (or quasi-monochromatic) laser light with matter. In particular, consider a linearly-polarized plane wave propagating in the z-direction. Its electric field can be described by

$$\mathbf{E}^+(z,t) = \frac{1}{2}\hat{\mathbf{x}}E_0(z,t)\mathrm{e}^{\mathrm{i}[Kz-\nu t-\phi(z,t)]}\ , \tag{1.12}$$

where $\hat{\mathbf{x}}$ is the direction of polarization, $E_0(z,t)$ is a real amplitude, ν is the central frequency of the field, and the wave number $K = \nu/c$. If $E(z,t)$ is truly monochromatic, E_0 and ϕ are constants in time and space. More generally, we suppose they vary sufficiently slowly in time and space that the following inequalities are valid:

$$\left|\frac{\partial E_0}{\partial t}\right| \ll \nu E_0 \,, \tag{1.13}$$

$$\left|\frac{\partial E_0}{\partial z}\right| \ll K E_0 \,, \tag{1.14}$$

$$\left|\frac{\partial \phi}{\partial t}\right| \ll \nu \,, \tag{1.15}$$

$$\left|\frac{\partial \phi}{\partial z}\right| \ll K \,. \tag{1.16}$$

These equations define the so-called ***slowly-varying amplitude and phase approximation (SVAP)***, which plays a central role in laser physics and pulse propagation problems. Physically it means that we consider light waves whose amplitudes and phases vary little within an optical period and an optical wavelength. Sometimes this approximation is called the SVEA, for ***slowly-varying envelope approximation.***

The SVAP leads to major mathematical simplifications as can be seen by substituting the field (1.12) into the wave equation (1.6) and using (1.13–1.16) to eliminate the small contributions $\ddot{E}_0, \ddot{\phi}, E_0'', \phi''$, and $\dot{E}\dot{\phi}$. We find

$$\frac{\partial E_0}{\partial z} + \frac{1}{c}\frac{\partial E_0}{\partial t} = 0 \,, \tag{1.17}$$

$$\frac{\partial \phi}{\partial z} + \frac{1}{c}\frac{\partial \phi}{\partial t} = 0 \,, \tag{1.18}$$

where (1.17) results from equating the sum of the imaginary parts to zero and (1.18) from the real parts. Thus the SVAP allows us to transform the second-order wave equation (1.6) into first-order equations. Although this does not seem like much of an achievement right now, since we can solve (1.6) exactly anyway, it is a tremendous help when we consider Maxwell's equations in a medium. The SVAP is not always a good approximation. For example, plasma physicists who shine light on targets typically must use the second-order equations. In addition, the SVAP approximation also neglects the backward propagation of light.

1.2 Maxwell's Equations in a Medium

Inside a macroscopic medium, Maxwell's equations (1.1–1.4) become

$$\nabla\cdot\mathbf{B} = 0 \,, \tag{1.19}$$

$$\nabla\cdot\mathbf{D} = \rho_{\text{free}} \,, \tag{1.20}$$

$$\nabla\times\mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \,, \tag{1.21}$$

$$\nabla\times\mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \,. \tag{1.22}$$

These equations are often called the *macroscopic* Maxwell's equations, since they relate vectors that are averaged over volumes containing many atoms but which have linear dimensions small compared to significant variations in the applied electric field. General derivations of (1.19–1.22) can be very complicated, but the discussion by Jackson (1999) is quite readable. In (1.20, 1.22), the displacement electric field $\mathbf{D}$ is given for our purpose by

$$\mathbf{D} = \varepsilon\mathbf{E} + \mathbf{P} \,, \tag{1.23}$$

where the permittivity ε includes the contributions of the host lattice and $\mathbf{P}$ is the induced polarization of the resonant or nearly resonant medium we wish to treat explicitly. For example, in ruby the Al_2O_3 lattice has an index of refraction of 1.76, which is included in ε. The ruby color is given by Cr ions which are responsible for laser action. We describe their interaction with light by the polarization $\mathbf{P}$. Indeed much of this book deals with the calculation of $\mathbf{P}$ for various situations. The free charge density ρ_{free} in (1.20) consists of all charges other than the bound charges inside atoms and molecules, whose effects are provided for by $\mathbf{P}$. We don't need ρ_{free} in this book. In (1.22), the magnetic field $\mathbf{H}$ is given by

$$\mathbf{H} = \frac{\mathbf{B}}{\mu} - \mathbf{M} \,, \tag{1.24}$$

where μ is the permeability of the host medium and $\mathbf{M}$ is the magnetization of the medium. For the media we consider, $\mathbf{M} = 0$ and $\mu = \mu_0$. The current density $\mathbf{J}$ is often related to the applied electric field $\mathbf{E}$ by the constitutive relation $\mathbf{J} = \sigma\mathbf{E}$, where σ is the conductivity of the medium.

The macroscopic wave equation corresponding to (1.6) is given by combining the curl of (1.21) with (1.23, 1.24). In the process we find $\nabla\times\nabla\times\mathbf{E} = \nabla(\nabla\cdot\mathbf{E}) - \nabla^2\mathbf{E} \simeq -\nabla^2\mathbf{E}$. In optics $\nabla\cdot\mathbf{E} \simeq 0$, since most light field vectors vary little along the directions in which they point. For example, a plane-wave field is constant along the direction it points, causing its $\nabla\cdot\mathbf{E}$ to vanish identically. We find

$$-\nabla^2\mathbf{E} + \mu\frac{\partial\mathbf{J}}{\partial t} + \frac{1}{c^2}\frac{\partial^2\mathbf{E}}{\partial t^2} = -\mu\frac{\partial^2\mathbf{P}}{\partial t^2} \,, \tag{1.25}$$

where $c = 1/\sqrt{\varepsilon\mu}$ is now the speed of light in the host medium. In Chap. 7 we use the $\partial\mathbf{J}/\partial t$ term to simulate losses in a Fabry–Perot resonator. We drop this term in our present discussion.

For a quasi-monochromatic field, the polarization induced in the medium is also quasi-monochromatic, but generally has a different phase from the field. Thus as for the field (1.9) we decompose the polarization into positive and negative frequency parts

$$\mathbf{P}(z,t) = \mathbf{P}^+(z,t) + \mathbf{P}^-(z,t) \,,$$

but we include the complex amplitude $\mathcal{P}(z,t) = N\lceil\, X(z,t)$, that is,

$$\mathbf{P}^+(z,t) = \frac{1}{2}\hat{\mathbf{x}}\mathcal{P}(z,t)\mathrm{e}^{\mathrm{i}[Kz-\nu t-\phi(z,t)]}$$
$$= \frac{1}{2}\hat{\mathbf{x}}N(z)\lceil\, X(z,t)\mathrm{e}^{\mathrm{i}[Kz-\nu t-\phi(z,t)]} \,. \tag{1.26}$$

Here $N(z)$ is the number of systems per unit volume, $\lceil$ is the dipole moment constant of a single oscillator, and $X(z,t)$ is a complex dimensionless amplitude that varies little in an optical period or wavelength. In quantum mechanics, $\lceil$ is given by the electric dipole matrix element $\wp$. Since the polarization is real, we have

$$\mathbf{P}^-(z,t) = [\mathbf{P}^+(z,t)]^* \,. \tag{1.27}$$

It is sometimes convenient to write $X(z,t)$ in terms of its real and imaginary parts in the form

$$X \equiv U - \mathrm{i}V \,. \tag{1.28}$$

The classical real variables U and V have quantum mechanical counterparts that are components of the Bloch vector $U\hat{e}_1 + V\hat{e}_2 + W\hat{e}_3$, as discussed in Sect. 4.3. The slowly-varying amplitude and phase approximation for the polarization is given by

$$\left|\frac{\partial U}{\partial t}\right| \ll \nu|U| \,, \tag{1.29}$$

$$\left|\frac{\partial V}{\partial t}\right| \ll \nu|V| \,. \tag{1.30}$$

or equivalently by

$$\left|\frac{\partial X}{\partial t}\right| \ll \nu|X| \,.$$

We generalize the slowly-varying Maxwell equations (1.17, 1.18) to include the polarization by treating the left-hand side of the wave equation (1.25) as before and substituting (1.26) into the right-hand side of (1.25). Using (1.29, 1.30) to eliminate the time derivatives of U and V and equating real imaginary parts separately, we find

$$\frac{\partial E_0}{\partial z} + \frac{1}{c}\frac{\partial E_0}{\partial t} = -\frac{K}{2\varepsilon}\mathrm{Im}(\mathcal{P}) = \frac{K}{2\varepsilon}N(z)\lceil\, V \tag{1.31}$$

$$E_0\left(\frac{\partial \phi}{\partial z} + \frac{1}{c}\frac{\partial \phi}{\partial t}\right) = -\frac{K}{2\varepsilon}\mathrm{Re}\mathcal{P} = -\frac{K}{2\varepsilon}N(z)\lceil\, U \tag{1.32}$$

These two equations play a central role in optical physics and quantum optics. They tell us how light propagates through a medium and specifically how the real and imaginary parts of the polarization act. Equation (1.31) shows that the field amplitude is driven by the *imaginary* part of the polarization. This *in-quadrature* component gives rise to absorption and emission.