Springer 大学数学图书——影印版

Second Year Calculus

高等微积分

David M. Bressoud 著

TSINGHUA UNIVERSITY PRESS

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内 容 提 要

本书是本科生的微积分教学用书，主要内容为：牛顿运动学基本定律（开篇），向量代数，天体力学简介，线性变换，微分形式和微分演算，隐函数反函数定理，重积分演算，曲线曲面积分，微积分基本定理，经典场论基本定理，爱因斯坦狭义相对论简介。本书特别注意数学与物理、力学等自然科学的内在联系和应用。作者在理念导引、内容选择、程度深浅、适用范围等方面都有相当周密的考虑。从我们国内重点大学的教学角度看，本书的难易程度与物理、力学和电类专业数学课的微积分相当，而思想内容则要深刻和生动些，因此适于用作这些专业本科生的教科书或学习参考书。

David M. Bressoud
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序　言

在学校教书多年，当学生（特别是本科生）问有什么好的参考书时，我们所能推荐的似乎除了教材还是教材，而且不同教材之间的差别并不明显、特色也不鲜明。所以多年前我们就开始酝酿，希望为本科学生引进一些好的参考书，为此清华大学数学科学系的许多教授与清华大学出版社共同付出了很多心血。

这里首批推出的十余本图书，是从 Springer 出版社的多个系列丛书中精心挑选出来的。在丛书的筹划过程中，我们挑选图书最重要的标准并不是完美，而是有特色并包容各个学派（有些书甚至有争议，比如从数学上看也许不够严格），其出发点是希望我们的学生能够吸纳百家之长；同时，在价格方面，我们也做了很多工作，以使得本系列丛书的价格能让更多学校和学生接受，使得更多学生能够从中受益。

本系列图书按其定位，大体有如下四种类型（一本书可以属于多类，但这里限于篇幅不能一一介绍）。

一、适用面比较广、有特色并可以用作教材或参考书的图书。例如：

- Lovász et al.: Discrete Mathematics. 2003

  该书是离散数学的入门类型教材。与现有的教材（包括国外的教材）相比，它涵盖了离散数学新颖而又前沿的研究课题，同时还涉及信息科学方面既基本又有趣的应用；在着力打好数学基础的同时，也强调了数学与信息科学的关联。姚期智先生倡导和主持的清华大学计算机科学试验班，已经选择该书作为离散数学课程的教材。

二、在目前国内的数学教育中，课程主要以学科的纵向发展为主线，而对数学不同学科之间的联系讨论很少。学生缺乏把不同学科视为一个数学整体的训练，这方面的读物尤其欠缺。这是本丛书一个重要的着力点。最典型的是：

- Fine/Rosenberger: The Fundamental Theorem of Algebra. 1997

  该书对数学中最重要的定理——代数基本定理给出了六种证明，方法涉及到分析、代数与拓扑；附录中还给出了 Gauss 的证明和 Cauchy 的证明。全书以一个数学问题为主线展开，纵横数学的核心领域；结构严谨、文笔流畅、浅显易懂、引人入胜，是一本少见的能够让读者入迷的好读物，用它来引导学生欣赏和领会“数学的美”绝对不会落于空谈。该书适于自学、讨论，也是极好的短学期课程教材。

- Baker: Matrix Groups. 2001

  就内容而言，本书并不超出我国大学线性代数、抽象代数和一般拓扑学课程的内容，但是本书所讲的是李群和李代数的基础理论——这是现代数学和物理学非常重要的工具。各种矩阵群和矩阵代数是李群和李代数最典型和

最重要的例子，同时也能帮助学生建立数学不同学科之间的联系。从矩阵出发，既能把握李群和李代数的实质，又能学会计算和运用，所以这是一本不可多得的好书。

三、科学与技术的发展不断为数学提出新的研究课题，因此在数学学科的发展过程中，来自其他学科的推动力是不能忽视的。本系列中第三种类型的读物就是强调数学与其他学科的联系。例如：

- Woodhouse: Special Relativity. 2003

  该书将物理与数学有机结合，体现了物理学家伽利略的名言："大自然是一部用数学语言写成的巨著。" 不仅如此，本书作者还通过对线性代数、微积分、场论等数学的运用进一步强调并贯穿这样的观点：数学的真谛和发展存在并产生于物理或自然规律及其发现中。精读此书有助于理解物理学和数学的整体关系。

- Britton: Essential Mathematical Biology. 2003

  生命科学在本世纪一定会有很大发展，其对数学的需求和推动是可以预见的。因此生物数学在应用数学中占有日益重要的地位，数学系培养的学生至少一部分人应当对这个领域有所了解。随着生命科学的迅速发展，生物数学也发展很快。本书由浅入深，从经典的问题入手，最后走向学科前沿和近年的热点问题。该书至少可以消除学生对生物学的神秘感。

四、最后一类是适合本科学生的课外读物。这类图书对激发和引导学生学习数学的兴趣会非常有帮助，而且目前国内也急需这样的图书。例如：

- Daepp/Gorkin: Reading, Writing and Proving. 2003

  该书对初学高等数学的读者来说特别有意义。它的基本出发点是引导读者以研究的心态去学习，让读者养成独立思考的习惯，并进而成为研究型的学习者。该书将一个学习数学的过程在某种意义下程序化，努力让学习者养成一个好的学习习惯，以及学会如何应对问题。该书特色鲜明，类似的图书确实很少。

- Brzezniak/Zastawniak: Basic Stochastic Processes. 1998

  随机过程理论在数学、科学和工程中有越来越广泛的应用，本书适合国内的需要。其主要特点是：书中配有的习题是巩固和延伸学习内容的基本手段，而且有十分完整的解答，非常适合自学和作为教学参考书。这是一本难得的好书，它 1999 年出版，到 2000 年已经是第 3 次印刷，到 2003 年则第 6 次重印。

- Anglin/Lambek: The Heritage of Thales. 1995

  该书的基本内容是数学的历史和数学的哲学。数学历史是该书的线索，数学是内容的主体，引申到的是数学哲学。它不是一本史论型的著作，而是采用专题式编写方式，每个专题相对独立，所以比较易读、易懂，是本科生学习数学过程中非常好的课外读物。

本系列丛书中的大部分图书还将翻译为中文出版，以适应更多读者的需要。丛书筹划过程中，冯克勤、郑志勇、卢旭光、郑建华、王殿军、杨利军、叶俊、扈志明等很多清华大学的教授都投入了大量精力。他们之中很多人也将是后面中文版的译者。此外，我们今后还将不断努力丰富引进丛书的种类，同时也会将选书的范围在可能情况下进一步扩大到其他高水平的出版机构。

教育是科学和技术发展的基石，数学教育更是基石的基础。因为是基础所以它重要；也因为是基础所以它显示度不高，容易不被重视。只有将人才培养放到更高的地位上，中国成为创新型国家的目标才会成为可能。

本系列丛书的正式推出，圆了一个我们多年的梦，但这无疑仅仅是开始。

白峰杉

2006 年 6 月于清华园

David M. Bressoud

Second Year Calculus

From Celestial Mechanics to Special Relativity

With 98 Illustrations

Springer

To my wife, Jan

Preface

This is a textbook on multivariate and vector calculus, but it is also a story. It is a story that begins and ends with revolutions in our understanding of the physical world in which we live. It carries us from the birth of the mechanized view of the world in Isaac Newton's *Mathematical Principles of Natural Philosophy*, in which mathematics becomes the ultimate tool for modeling physical reality, to the dawn of a radically new and often counterintuitive age in Albert Einstein's Special Theory of Relativity, in which it is the mathematical model that suggests new aspects of that reality.

This is also a chance to have some fun with mathematics. Here is the promised reward of being able to do something interesting and useful with the calculus that you have mastered in the past year. A colleague once told me of a high school experience in which he had the opportunity to participate in a special class that promised to reveal the basic tools of mathematics. Visions of orbit calculations and other mathematical applications danced through his head in anticipation. He was disappointed when the course turned out to be set theory. Presented here is the course for which he was hoping. We shall compute orbits and rocket trajectories, see how to model flows and force fields, derive the laws of electricity and magnetism, and show how observations of mathematical symmetry lead to the conclusion that matter and energy are interchangeable.

If I stand accused of blurring the line between mathematics and physics, I enthusiastically plead "Guilty!" Mathematics is often viewed as all technique, the foundation for interesting studies of the world but dull and tedious in and of itself. I hope that this book will reveal to you some of the intimate interplay between mathematics and our understanding of the physical universe and, in the process, illuminate some of the intrinsic beauty of mathematics itself.

I have also tried to emphasize the mathematical structure underlying this subject. For this, I have taken my inspiration from two of the great texts on several variable calculus: Tom Apostol's *Calculus* and Harold Edwards' *Advanced Calculus*. The former was my own textbook as an undergraduate, and I admire its clarity and precision and, above all, its treatment of the derivative of a vector field as a linear transformation. Edwards opened the world of differential forms to me, and I am especially indebted to him for revealing the natural progression from the fundamental theorem of calculus to Maxwell's equations to special relativity. I have taken what I found most imitation as sincere flattery.

The physicist and occasional mathematician Freeman Dyson, in an address to the American Mathematical Society in 1972, spoke of the passage from the equations of electricity and magnetism to the insights of special relativity as one of the opportunities missed by mathematicians as they divorced themselves from the problems of physics. Even today, most mathematicians have a poor appreciation of Maxwell's contributions. An excerpt from Dyson's comments is included as Appendix A. I hope that many of you will be motivated to read all of his address and to participate in the reintegration of physics and mathematics that I feel has begun.

This book grew out of an honors calculus class at Penn State. It is intended to be covered in one 15-week semester of four classroom hours per week. That is a fast pace, and yet it is one that dedicated students can maintain. There are no interchangeable parts. This text was designed to be used as a whole. That does not mean that each chapter need be given the same emphasis. Chapters 1, 3, and 11, as well as Sections 8.2, 8.5, 9.5, and 10.9, can be touched lightly and rapidly with the student responsible for reading and assimilating much of the material. But, I hope that the instructor will not give these portions too scant attention for they provide the flesh of a subject that too often is reduced to the dry bones of technique.

There are many people to whom I owe a debt of gratitude for help with this book, among them Don Albers and Freeman Dyson for their early encouragement; Ray Ayoub, Allan Krall, Steve Maurer, and David Rosen, who went through much of the manuscript and suggested improvements; the National Security Agency for its support; and my editor at Springer-Verlag, who expressed consistent confidence in this project and made a number of valuable suggestions. But, most especially, I wish to thank the students who struggled through the first draft of the text in the fall of 1990 and pointed out many of my misprints, as well as places where the explanations were obscure, the examples inadequate, and the exercises impossible. I do not claim that they or I have now found all such faults, but this is a better book for their efforts. They are Jeffrey Caruso, Robert Colbert, David Druist, Mark Flood, Kathleen Galvin, Darren Gibula, Steven Gradess, Stanley Hsu, Steven Jackson, John Johnson, Timothy Keane, Brian Ledell, Kurt Ludwick, Lara Palmer, Brian Pavlakovic, Christine Penney, Julie Richards, Alexander Richman, Nicola Schussler, Andrew Shropshire, Michael Smith, Peter Stone, Xiong Sun, Christopher Tatnall, Melissa Wallner, Marc Weinstein, and Jill Wyant.

To anyone who requests it, I will send a current list of corrections for this book, and I ask your help in finding misprints and errors. Regular mail should be sent to Macalester College, St. Paul, MN 55105, USA; e-mail to bressoud@macalstr.edu.

David M. Bressoud March 6, 1991

Contents

1

$F = ma$

The heavens declare the glory of God,
and the firmament shows his handiwork.
—*Psalm 19*

Had I been present at the creation, I would have given some useful hints for the better ordering of the universe.
—attributed to King *Alfonso X* of Castile (1221–1284)

[Newton] has so clearly laid open and set before our eyes the most beautiful frame of the System of the World, that if King Alfonso were now alive, he would not complain for want of the graces either of simplicity or of harmony in it.
—*Roger Cotes*, from the Preface to the second edition of *Philosophiæ Naturalis Principia Mathematica* (1713)

1.1 Prelude to Newton's *Principia*

Popular mathematical history attributes to Isaac Newton (1642–1727) and Gottfried Wilhelm Leibniz (1646–1716) the distinction of having invented calculus. Of course, it is not nearly so simple as that. Techniques for evaluating areas and volumes as limits of computable quantities go back to the Greeks of the classical era. The rules for differentiating polynomials and the uses of these derivatives were current before Newton or Leibniz were born. Even the fundamental theorem of calculus, relating integral and differential calculus, was known to Isaac Barrow (1630–1677), Newton's teacher. Yet it is not inappropriate to date calculus from these two men for they were the first to grasp the power and universal applicability of the fundamental theorem of calculus. They were the first to see an inchoate collection of results as the body of a single unified theory.

Newton's preeminent application of calculus is his account of celestial mechanics in *Philosophiæ Naturalis Principia Mathematica* or *Mathematical Principles of Natural Philosophy*. Ironically, he makes very little specific mention of calculus in it. This may, in part, be due to the fact that calculus was still sufficiently new that he felt it would be suspect. In part, it is a reflection of an earlier age in which mathematicians jealously guarded powerful new techniques and only revealed the fruits of their labors. Newton's