普通高等教育“十一五”国家级规划教材

高等学校双语教学课程教材

MECHANICS OF MATERIALS

（材料力学）

Edited by Li Yaochen

同济大学出版社
TONGJI UNIVERSITY PRESS

普通高等教育"十一五"国家级规划教材
高等学校双语教学课程教材

MECHANICS OF MATERIALS

（材料力学）

Edited by Li Yaochen

同济大学出版社
TONGJI UNIVERSITY PRESS

内 容 提 要

本书是“十一五”国家级规划教材。内容有轴向拉伸与压缩，剪切与挤压，扭转，弯曲内力，弯曲应力，弯曲变形，应力应变变换与强度理论，组合变形，能量法，压杆稳定；在附录中有截面图形几何性质及型钢表。书中的例题较多，形式新颖；英语表述流畅。

本书主要作高等学校材料力学双语教学的教科书，也可作研究生及工程技术人员的参考书。

图书在版编目(CIP)数据

材料力学＝Mechanics of Materials：英文/李尧臣编.
--上海：同济大学出版社，2010.7
ISBN 978-7-5608-4235-6

Ⅰ.①材… Ⅱ.①李… Ⅲ.①材料力学－双语教学－高等学校－教材－英文 Ⅳ.①TB301

中国版本图书馆 CIP 数据核字(2010)第 006896 号

高等学校双语教学课程教材

Mechanics of Materials(材料力学)

Edited by Li Yaochen

责任编辑 解明芳　责任校对 徐春莲　封面设计 潘向蓁

出版发行 同济大学出版社　www.tongjipress.com.cn
(地址：上海市四平路 1239 号　邮编：200092　电话：021－65985622)
经　销 全国各地新华书店
印　刷 同济大学印刷厂
开　本 787mm×1092mm 1/16
印　张 26.5
印　数 1—1500
字　数 661000
版　次 2010 年 7 月第 1 版　2010 年 7 月第 1 次印刷
书　号 ISBN 978-7-5608-4235-6

定　价 47.00 元

前 言

在大学里开展双语教学是教育部的要求，也是许多高校的要求。这样做有两个目的：一是在学习科技知识的同时提高英语水平，二是可以促进中西方文化的交流，学习国外的一些好的教学方法。为了更好地开展材料力学课程的双语教学，必须要有合适的教材。在近几年来的双语教学中，我们从美国的原版书中选用了部分内容作为讲义使用。在实践中，我们感到，这样选用原版书作为教材还有不少问题。将原版书与中文《材料力学》教材作一比较，可以发现，它们的侧重点不同，部分内容不同，内容的安排次序和讲述方法也不同，习题风格也有所不同。当然，原版书的内容安排和讲述方法有其历史的原因和优点，习题风格也给我们耳目一新的感觉。但是，如果选用原版书，将给学生后续课程的学习和研究生入学考试带来不方便。而且，英文教材的篇幅很长，即使有选择地复印以后，其篇幅也是中文教科书的两倍多。为了使双语教学工作能够健康、有序、持续地发展，对原版书作些改造是必要的。

编者在编写本书时，参考了美国 3 本名为《Mechanics of Materials》的教科书，分别由 F. P. Beer & E. R. Johnston. Jr.，A. Pytel & J. Kiusalaas，R. C. Hibbeler 所作。还参考了由宋子康、蔡文安编写的《材料力学》，孙训方等编写的《材料力学》，也有编者的一些创新内容。原则是要与目前材料力学教学大纲中的内容和要求保持一致。在本书中仍然保留原版教材“重视基本概念，讲述详尽，推理严谨，有较强的科学性；习题有较强的工程背景，插图精美而逼真”这些优点。讲述可简则简，习题数量与中文教科书相当。本书版式紧凑，篇幅已缩减到与中文教科书相当。

本书部分插图由杨昌锦博士绘制，在此表示感谢。

我们希望本书能对材料力学课程的双语教学起到推动作用，也希望关注、使用本教材的老师和同学提出宝贵的意见。

编者感谢同济大学教育教改研究项目对本项工作的支持。

编 者

2009 年 9 月

LIST OF SYMBOLS

A, a	area
b	distance, width
C	centroid
C_1, C_2, $\cdots$	constants of integration
d	distance; diameter; depth
D	diameter
e	distance; eccentricity; dilatation
E	modulus of elasticity
f	frequency; function
$\boldsymbol{F}$	force
F_N	internal axial force
F_S	internal shear force
F_P	compressive load of columns
g	gravity acceleration
G	modulus of rigidity; shear modulus
h	distance; height
i_x, i_y, i_z	radius of gyration
I, I_x, I_y, I_z	moment of inertia
I_{xy}	product of inertia
I_P	polar moment of inertia
k	spring constant; stiffness coefficient
K	stress concentration factor; torsional spring constant
l	length, span
L	length, span
L_e	effective length
m	mass
M	couple; moment
M, M_y, M_z	bending moment
n	number; factor of safety; ratio of cross sectional areas; rotation speed
n_b	factor of safety for buckling
n_d	dynamic coefficient of impact
p	pressure
q	shearing force per unit length, shear flow; load per unit length
r, R	radius
s	length; spacing
S_x, S_y, S_z	first moment of area
t	thickness
$\boldsymbol{T}$, T	torque; temperature
u, v	rectangular coordinates
u	strain energy density
U	strain energy;
v	velocity
V	volume
w	width; distance; load per unit length
$\boldsymbol{W}$, W	weight; load; work
W, W_x, W_y, W_z	section modulus of bending
W_P	section modulus pf torsion
x, y, z	rectangular coordinates; deflections
$\bar{x}$, $\bar{y}$, $\bar{y}$	coordinates of centroid
$y_{\max}$, y_{m}	distance from N. A. to edge of the cross section
α, β, γ	angles
α	coefficient of thermal expansion
γ	shearing strain; specific weight
δ	displacement; deflection; ratio of elongation of specimen
ε	ormal strain
κ	shape factor of section for shearing
λ	slenderness ratio
μ	coefficient of effective length
ν	poisson's ratio
θ	angle; slope
ρ	radius of curvature; distance; density
σ	normal stress
τ	shearing stress
ϕ	angle; angle of twist
ψ	ratio of cross section reduction of specimen

SI Units

Selected SI units			Commonly used SI prefixes		
Quantity	Name	SI symbol	Factor	Prefix	SI Symbol
Energy	joule	J(1 J=1 N·m)	10^9	giga	G
Force	newton	N(1 N=1 kg · m/s^2)	10^6	mega	M
Length	meter[1]	m	10^3	kilo	k
Mass	kilogram[1]	kg	10^{-3}	milli	m
Moment (torque)	newton meter	N·m	10^{-6}	micro	μ
Rotational frequency	revolution per second	r/s	10^{-9}	nano	n
	hertz	Hz(1 Hz=1 r/s)			
Stress (pressure)	pascal	Pa(1 Pa=1 N/m^2)			
Time	second[1]	s			
Power	watt	W(1 W=1 J/s)			

① SI base unit.

Selected Rules and Suggestions for SI Usage

1. Be careful in the use of capital and lowercase for symbols, units, and prefixes (e. g. , m for meter or milli, M for mega).

2. In compound units formed by multiplication, use the product dot (e. g. , N·m).

3. Division may be indicated by a slash (m/s), or a negative exponent with a product dot (m · s^{-1}).

4. Avoid the use of prefixes in the denominator (e. g. , km/s is preferred over m/ms). The exception to this rule is the prefix k in the base unit kg (kilogram).

Equivalence of U. S. Customary and SI Units (Asterisks indicate exact values; others are approximations.)

	U. S. Customary to SI	SI to U. S. Customary
1. Length	1 in. =25. 4 mm=0. 025 4 m	1 mm=0. 039 370 in.
	1 ft=304. 8 mm=0. 304 8 m	1 m =39. 370 in. =3. 281ft
2. Area	1 in.^{2}=645. 16 mm^2	1 mm^2=0. 001 550 in.^{2}
	1 ft^2=0. 092 903 04 m^2	1 m^2 =1 550. 0 in.^{2} =10. 764 ft^2
3. Volume	1 in.^{3}=16 387. 064 mm^3	1 mm^3=0. 000 061 024 in.^{3}
	1 ft^3=0. 028 317 m^3	1 m^3 =61 023. 7 in.^{3} =35. 315 ft^3
4. Force	1 lb=4. 448 N	1 N=0. 224 8 lb
	1 lb/ft=14. 594 N/m	1 N/m=0. 068 522 lb/ft
5. Mass	1 lbm=0. 453 59 kg	1 kg=2. 205 lbm
	1slug=14. 593 kg	1 kg=0. 068 53 slugs
6. Moment of a force	1 lb · in. =0. 112 985 N·m	1 N·m=8. 850 75 lb · in.
	1 lb · ft=1. 355 82 N·m	1 N·m=0. 737 56 lb · ft
7. Stress (pressure)	1 lb/ft^2(psf)=47. 88 Pa	1 Pa=0. 020 886 psf
	1 lb/in.^{2}(psi)=6. 895 kPa	=0. 000 145 03 psi
8. Power	1 hp (550 lb · ft/s)=0. 745 7 kW	1 kW=1. 341 0 hp

CONTENTS

前言

LIST OF SYMBOLS

SI Units

1 Introduction .... (1)

1.1 The Task of Mechanics of Materials .... (1)

1.2 Basic Assumptions of Deformable Solid Bodies .... (2)

1.3 Geometrical Characteristics of the Members .... (3)

1.4 Analysis of Internal Forces; Stress .... (4)

1.5 Basic Forms of Rod Deformation .... (7)

2 Axially Loaded Bars .... (9)

2.1 Introduction .... (9)

2.2 Internal Forces .... (9)

2.3 Stresses on Cross-sections and Oblique-sections .... (11)

2.4 Axial Deformation; Normal Strain .... (17)

2.5 Stress-strain Diagram in Tensile Test .... (18)

2.6 Stress-strain Diagram in Compressive Test .... (24)

2.7 Deformations of Rods under Axial Loading; Hooke's Law .... (26)

2.8 Allowable Stress; Factor of Safety .... (30)

2.9 Statically Indeterminate Problems .... (34)

2.10 Problems Involving Manufacture Errors and Temperature Changes .... (39)

2.11 Stress Concentrations .... (44)

Problems .... (47)

3 Shearing and Bearing .... (56)

3.1 Introduction .... (56)

3.2 Shear Stress .... (57)

3.3 Bearing Stress .... (58)

Problems .... (66)

4 Torsion .... (71)

4.1 Introduction .... (71)

4.2 Torsion of Circular Shafts .... (72)

4.3 Design of Transmission Shafts .... (76)

4.4 Angle of Twist in the Elastic Range; Design of Transmission Shafts for Stiffness .... (81)

4.5 Statically Indeterminate Shafts .... (85)

4.6 Torsion of Noncircular Members .... (88)

4.7 Thin-walled Hollow Shafts .... (90)

Problems .... (94)

5 Shear and Moment in Beams .... (99)

5.1 Introduction .... (99)

5.2 Supports and Loads .... (100)

5.3 Shear and Bending-moment Diagrams .... (101)

5.4 Relations among Load, Shear, and Bending Moment .... (107)

5.5 Moving Loads .... (114)

5.6 Internal-Force Diagrams of Rigid Frames and Curved Bars .... (118)

Problems .... (120)

6 Stresses in Beams ······ (124)
6.1 Introduction ······ (124)
6.2 Bending Stresses in Prismatic Beams in Pure Bending ······ (124)
6.3 Economic Sections ······ (136)
6.4 Shear Stress in Beams ······ (139)
6.5 Design for Flexure and Shear ······ (148)
6.6 Design of Fasteners in Built-up Beams ······ (151)
6.7 Shear Flow in Thin-Walled Beams ······ (153)
6.8 Shear Center ······ (155)
6.9 Unsymmetrical Bending ······ (160)
6.10 Beams of Constant Strength ······ (165)
Problems ······ (168)
7 Deflection of Beams ······ (179)
7.1 Introduction ······ (179)
7.2 Equation of the Elastic Curve; Double Integration Method ······ (179)
7.3 Method of Superposition ······ (186)
7.4 Statically Indeterminate Beams ······ (193)
7.5 Deflection of Beams in Unsymmetric Bending ······ (202)
Problems ······ (205)
8 Transformations of Stress and Strain
8.1 Introduction ······ (211)
8.2 Preliminary Discussion of Transformation of Stress ······ (212)
8.3 Transformation of Plane Stress ······ (214)
8.4 Mohr's Circle for Plane Stress ······ (222)
8.5 Absolute Maximum Shear Stress ······ (228)
8.6 Theories of Failure ······ (231)
8.7 Transformation of Strain; Mohr's Circle for Strain ······ (239)
8.8 The Strain Rosette ······ (246)
8.9 Multiaxial Loading; Generalized Hooke's Law ······ (249)
Problems ······ (254)
9 Stresses due to Combined Loads ······ (260)
9.1 Introduction ······ (260)
9.2 Thin-Walled Pressure Vessels ······ (260)
9.3 Combined of Axial and Lateral Loads ······ (264)
9.4 Eccentric Axial Loading ······ (267)
9.5 Combination of Torsion and Bending; Design of Transmission Shafts ······ (278)
Problems ······ (281)
10 Energy Method ······ (288)
10.1 Introduction ······ (288)
10.2 Strain Energy and Strain Energy Density ······ (288)
10.3 Impact Loading ······ (299)
10.4 Design for Impact Loading ······ (303)
10.5 Work and Energy under Several Loads; Maxwell's Reciprocal Theorem ······ (306)
10.6 Castigliano's Theorem ······ (309)
10.7 Statically Indeterminate Structures ······ (314)
Problems ······ (317)
11 Columns ······ (324)
11.1 Introduction ······ (324)
11.2 Euler's Formula for Pin-Ended Columns ······ (326)
11.3 Extension of Euler's Formula to Columns with Other End Conditions

············ (329)

11.4 Applicability of Euler's Formula; Critical Stress Diagram ············ (334)

11.5 Discount-Coefficient Method ············ (342)

11.6 Optimization of Column Design for Stability ············ (348)

Problems ············ (354)

Appendix A Moments of Areas ······ (361)

A.1 First moment of an area; Centriod ············ (361)

A.2 Determination of the first moment and centroid of a composite area ············ (363)

A.3 Second moment, or moment of inertia, of an area; Radius of gyration ············ (366)

A.4 Parallel-axis theorem ······ (369)

A.5 Determination of the moment of interia of a composite area ············ (370)

A.6 Product of inertia for an area ············ (372)

A.7 Moments of inertia for an area about inclined axes ········ (375)

A.8 Mohr's circle for moments of inertia ············ (378)

Problems ············ (384)

Appendix B Typical Properties of Selected Materials Used in Engineering ············ (388)

Appendix C Properties of Rolled-Steel Shapes ············ (390)

ANSWERS ············ (403)

1 INTRODUCTION

1.1 THE TASK OF MECHANICS OF MATERIALS

In engineering practice, we are always dealing with structures. There are many examples of structures such as buildings, bridges, airplanes, boats, vehicles, trains and all kinds of machines. Structures are quite different in type, but they have a common character that all structures bear loads. For example, buildings bear wind load, seismic load, and load from residents. Bridges carry wind load, earthquake load and the load from vehicles. Vehicles carry the load from tourists, etc. Structures consist of several main portions called members such as rods, beams, columns, plates, shells, shafts, bolts, wheels etc. In civil engineering, these members are called structural components, while in mechanical engineering, they are called component parts.

Mechanics of materials studies members. Let us see what kind of requirements they should meet when members are working. First of all, they can not be broken; otherwise the structures including these members will collapse. Bridges can not be damaged when vehicles are running on them. When a lathe is processing parts, its shafts can not be broken. If such things happen, disastrous consequences will occur. In other words, members have to have enough strength to bear the loads safely. Second, deformation or deflection of members should be limited within some allowable ranges. For example, the deflection of a crane beam should not be too large otherwise the card on the beam will be driven with difficulty. If the principal shaft of a lathe has large deformation, the accuracy of the parts being processed will not be guaranteed. That is, the members should have enough stiffness to resist deformation. The third, members have to have stability. Right now, we are not able to give the definition of stability, but we can give an example. We take a slender column with a compressive force exerted on top of it. If the force is sufficiently large, the column will bend. That is, the equilibrium state of the column with a straight-line shape becomes unstable, or the column loses its stability. Stability is also an important requirement for members. If a column of a building loses its stability, the building will collapse, the consequence is also catastrophic.

To meet the requirements of strength, stiffness and stability for members, we may use the members with large size. For example we can use very thick plates, use rods with a very large sectional area. Or we can use high-strength materials for the members. But doing so will waste materials, raise the price of structures, as well as increase the weight of the members. Hence, we should design economic members. If the requirements of strength, stiffness and stability of members are satisfied, we say that the members are safe, otherwise they fail. Safety and economy of members are in contradiction with each other. The task of mechanics of materials is to provide a simple and practical means for analyzing the strength, stiffness and stability of members so as to give the data for engineering design, so that the requirements of both safety and economy are satisfied.

1.2 BASIC ASSUMPTIONS OF DEFORMABLE SOLID BODIES

If a solid body bears loads, its shape will change. This phenomenon is known as deformation. We say that the material of the body is a deformable solid. The microstructure of materials is quite complex. We need to make some assumptions for it to establish a simplified mode of materials so that we can study the members easily. The assumptions are:

1. Solid bodies are continuous

The solid body is stuffed with materials. The body is absolutely solid, without any gaps and cavities in it. In fact, a solid body always includes some gaps, cavities, voids and inclusions, but they are so small in size in comparison with the body that they can be neglected. Introducing this assumption, we may isolate an infinitesimal element anywhere in the body for analyzing. And the differentiation and integration calculi can be employed for our study. But it should be noted that the deformation of the body has to satisfy the geometrical compatibility condition, namely, neither separation nor penetrating will occur in materials of the body.

2. Solid bodies are homogeneous

The material property in any place of the body is the same. In other words, it is independent of the position of the material points in the member. Thus, if a piece of material is isolated in any place of the body, its mechanical property can represent the one of all the material of the body. In micro viewpoint, the material is inhomogeneous. For example, steel have inclusions of carbon and some other elements in it. But, if we isolate an element which size is much smaller than the steel body, the element is still large enough to hold sufficiently large number of steel and carbon crystals. Hence, the mechanical property of the element is in fact a stochastically mean value of steel and carbon crystals. This macro property is the same anywhere in the body. Hence, in the macro viewpoint, the material is homogeneous. The illustration is also applicable for concrete, but the size of gravel grains may not be much smaller than that of the concrete body. Thus, formulas in mechanics of materials are very accurate for metallic materials, but are approximate for concrete.

3. Materials are isotropic

Material property is the same in any direction, or it is direction insensitive in the member. We know that properties of metal crystals are different in various orientations. For examle they are different in the side orientation and in the diagonal orientation of the lettuce of the metal crystals. But, the orientations of crystal grains of metal are randomly distributed. Thus, the material can be considered isotropic. Metal is isotropic, concrete is isotropic. But some materials are anisotropic such as wood, plastic cord etc. Their mechanical properties along the fibers and across the fibers are different.

The latter two assumptions bring us convenience for study of members since the mechanics constants corresponding to the material properties are the same anywhere in the member and in any direction in the member.

Besides, we have two additional conditions in our studying:

1. Materials are elastic

The shape of a member changes under the load. But the deformation disappears if the load is removed. This nature of the material is called elasticity. Sometimes the deformation can not be completely eliminated if the load is removed. In this case, the portion of deformation, which can be eliminated, is called elastic deformation, the other portion which can not be eliminated is called plastic deformation or permanent deformation. If a plastic deformation occurs in a member, we think that the member fails already. Hence, in mechanics of materials we only study the members in elastic deformation range. Furthermore, if the deformation is proportional to the load, we say that the material is linearly elastic. For convenience, we only consider that the materials are in linearly elastic range in our study.

2. Deformation is very much smaller than the size of the member

In many cases, this assumption is true indeed. Hence, if the size of the member is employed in some basic equations, e. g. equilibrium equations, we can always use the original size before deformation of the member. This condition also brings us convenience in our study.

As a summary, we can say that in mechanics of materials, the materials are considered to be continuous, homogeneous, isotropic, and in general, linearly elastic. The deformation of the members is very much smaller than the member's size.

1.3 GEOMETRICAL CHARACTERISTICS OF THE MEMBERS

In mechanics of materials, members are abstracted as rods (Fig. 1. 1(a)). Rods are usually slender in shape. Each rod has many cross-sections. The centroids of the cross-sections of a rod form a line called the axis or the longitudinal axis of the rod. The cross-sections are perpendicular to the axis. The size of the cross-sections is much smaller than the length of the rod. If the axis of a rod is a straight line, the rod is called the straight rod, if curved, the curved rod. In mechanics of materials, we only study straight rods, and specifically, the straight rods with uniform cross-sections. But results obtained from the straight rods with uniform cross-sections can also be used for curved rods approximately if their curvature radii are very much larger than the size of the cross-sections, and for the rods with variable cross-sections if the cross-sections vary less rapidly along the axes of the rods.

Plates and shells belong to the second category of the members. The thickness of plates and shells is much smaller than the other two dimensions. The surface which bisects the thickness is defined as mid-surface. If the mid-surface is a plane, the member is called plate (Fig. 1. 1(b)), e. g. plates of buildings. If it is curved, the member is known as shell (Fig. 1. 1(b)), e. g. the roofs of buildings. The third category of the members is the solid block, which 3 dimensions are in the same order (Fig. 1. 1(c)). Examples are the foundations of buildings and machines. These members, which are more complex than rods in shape, will be studied in Theory of Elasticity.

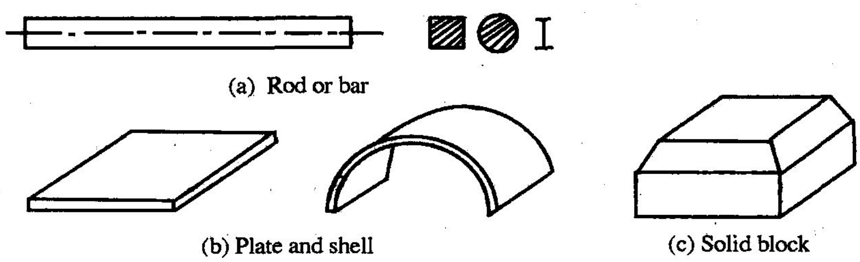


Fig. 1. 1 Different forms of members

1. 4 ANALYSIS OF INTERNAL FORCES; STRESS

There are three fundamental areas of engineering mechanics: statics, dynamics, and mechanics of materials. Statics and dynamics are devoted primarily to the study of the *external effects upon rigid bodies* — that is, bodies for which the change in shape (deformation) can be neglected. In contrast, *mechanics of materials* deals with the *internal effects and deformations* that are caused by the applied loads. Both considerations are of paramount importance in the analysis of the above-mentioned three classes of problems — strength, stiffness and stability.

The differences between rigid-body mechanics and mechanics of materials can be appreciated if we consider the bar shown in Fig. 1. 2. The force $\boldsymbol{F}$ required to support the load W in the position shown can be found easily from equilibrium analysis. After we draw the free-body diagram of the bar, summing up the moments about the pin at O, we can determine the value of $\boldsymbol{F}$. In this solution, we assume that the bar is both rigid (the deformation of the bar is neglected) and strong enough to support the load W. In mechanics of materials, the statics solution is extended to include an analysis of the forces acting inside the bar to confirm that the bar will neither break nor deform excessively.

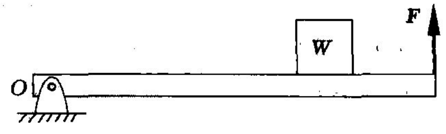


Fig. 1. 2 Equilibrium analysis of a bar to determine the force $\boldsymbol{F}$

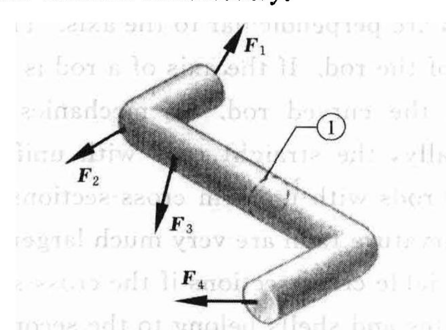
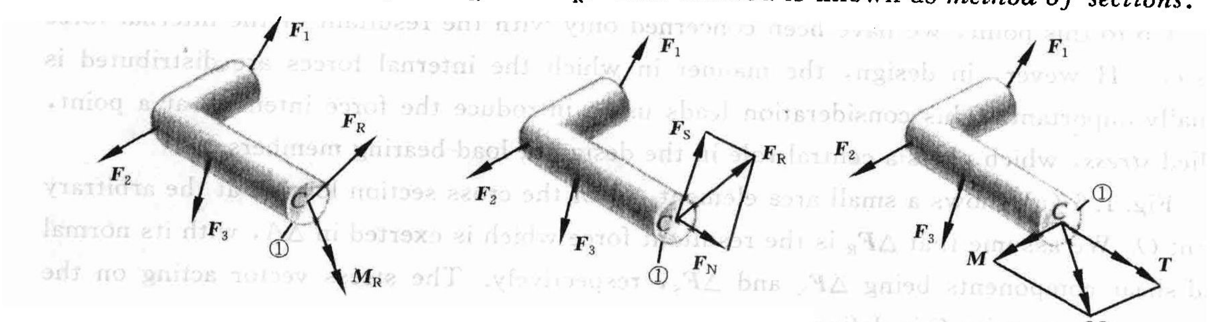


Fig. 1. 3 External forces acting on a curved bar

1. Internal Forces

The equilibrium analysis of a rigid body is concerned primarily with the calculation of external reactions and internal reactions (forces that act at internal connections, such as internal pin-connections). In mechanics of material, we must extend this analysis to determine internal forces — the forces that act on cross sections of the body itself. In addition, we must investigate the manner in which these internal forces are distributed within the body. Only after these computations have been made, can the design engineer select the proper dimensions for a member and select the material from which the member should be fabricated.

If the external forces that hold a body in equilibrium are known, we can compute the internal forces by straightforward equilibrium analysis. For example, consider the bar in Fig. 1.3 that is loaded by the external forces $\boldsymbol{F}_1$, $\boldsymbol{F}_2$, $\boldsymbol{F}_3$ and $\boldsymbol{F}_4$. To determine the internal force system acting on the cross section labeled ①, we must first isolate the segments of the bar lying on either side of section ①. The free-body diagram of the segment to the left of section ① is shown in Fig. 1.4 (a). In addition to the external forces $\boldsymbol{F}_1$, $\boldsymbol{F}_2$ and $\boldsymbol{F}_3$, this free-body diagram shows the resultant force-couple system of the internal forces that are distributed over the cross section: the resultant force $\boldsymbol{F}_{\mathrm{R}}$, acting at the centroid C of the cross section, and $\boldsymbol{M}_{\mathrm{R}}$, the resultant couple (we use double-headed arrows to represent couple-vectors). If the external forces are known, the equilibrium equation $\sum \boldsymbol{F}=0$ and $\sum \boldsymbol{M}_{\mathrm{C}}=0$ can be used to compute $\boldsymbol{F}_{\mathrm{R}}$ and $\boldsymbol{M}_{\mathrm{R}}$. This method is known as *method of sections.*



(a) Free-body diagram for determination of the internal force system acting on the section

(b) Resolving the internal force $\boldsymbol{F}_{\mathrm{R}}$ into the axial force $\boldsymbol{F}_{\mathrm{N}}$ and the shear force $\boldsymbol{F}_{\mathrm{S}}$

(c) Resolving the internal couple $\boldsymbol{M}_{\mathrm{R}}$ into the torque $\boldsymbol{T}$ and the bending moment $\boldsymbol{M}$

Fig. 1.4 Analysis of the internal forces of the curved bar

It is conventional to represent both $\boldsymbol{F}_{\mathrm{R}}$ and $\boldsymbol{M}_{\mathrm{R}}$ in terms of two components: one perpendicular to the cross section and the other lying in the cross section, as shown in Fig. 1.4(b) and Fig. 1.4 (c). These components are given the following physically meaningful names:

$\boldsymbol{F}_{\mathrm{N}}$: The force component that is perpendicular to the cross section, tending to elongate or shorten the bar, is called the *normal force.*

$\boldsymbol{F}_{\mathrm{S}}$: The force component lying in the plane of the cross section, tending to shear (slide) one segment of the bar relative to the other segment, is called the *shear force.*

$\boldsymbol{T}$: The component of the resultant couple that tends to twist (rotate) the bar is called the *twisting moment or torque.*

$\boldsymbol{M}$: The component of the resultant couple that tends to bend the bar is called the *bending moment.*

The deformations produced by these internal forces and internal couples are shown in Fig. 1.5. They are known as four basic forms of deformations of the rods: *axial tension or compression,* *transverse shear*, *torsion and bending*, they will be discussed in the next section.

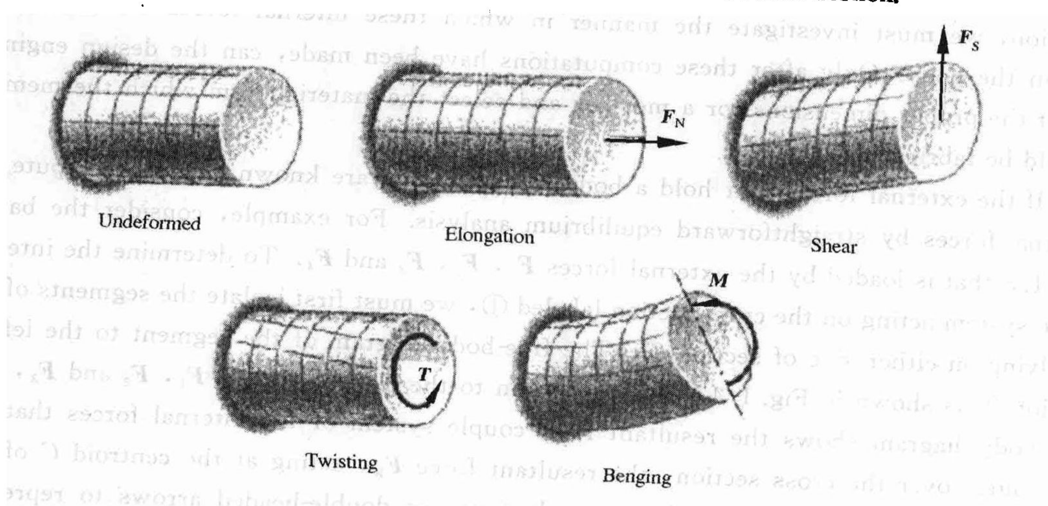


Fig. 1.5 Deformations produced by the components of internal forces and couples

2. Stresses

Up to this point, we have been concerned only with the resultant of the internal force system. However, in design, the manner in which the internal forces are distributed is equally important. This consideration leads us to introduce the force intensity at a point, called *stress*, which plays a central role in the design of load-bearing members.

Fig. 1.6 (a) shows a small area element ΔA of the cross section located at the arbitrary point O. We assume that $\Delta \boldsymbol{F}_R$ is the resultant force which is exerted in ΔA, with its normal and shear components being $\Delta \boldsymbol{F}_N$ and $\Delta \boldsymbol{F}_S$, respectively. The stress vector acting on the cross section at point O is defined as

$$\boldsymbol{p} = \lim_{\Delta A \to 0} \frac{\Delta \boldsymbol{F}_R}{\Delta A} \tag{1.1}$$

Its normal component σ (lowercase Greek *sigma*) and shear component τ (lowercase Greek *tau*), shown in Fig. 1.6 (b), are

$$\sigma = \lim_{\Delta A \to 0} \frac{\Delta F_N}{\Delta A} = \frac{dF_N}{dA}, \quad \tau = \lim_{\Delta A \to 0} \frac{\Delta F_S}{dA} = \frac{dF_S}{dA} \tag{1.2}$$

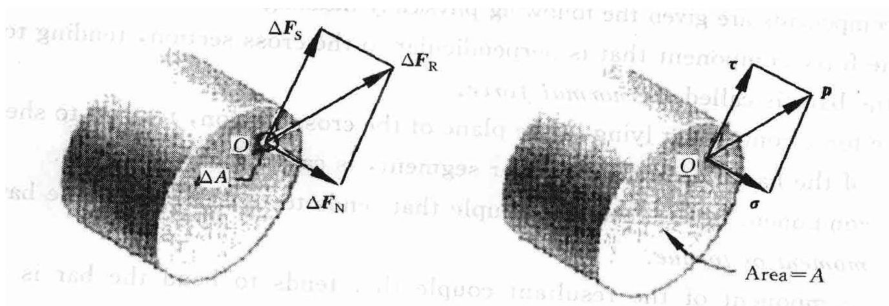


Fig. 1.6 Normal and shear stresses acting on the section at point O

The dimension of stress is $[F/L^2]$——that is, force divided by area. In SI units, force is measured in newtons (N) and area in square meters(m^2), from which the unit of stress is *newtons per square meter* (N/m^2) or, equivalently, pascals (Pa): 1.0 Pa = 1.0 N/m^2. Because 1 pascal is a very small quantity in most engineering applications, stress is usually expressed with the SI prefix M (read as "mega"), which indicates multiples of 10^6: 1.0MPa $=1.0\times10^6$ Pa. We find that it is conventient sometimes to use $1MPa=1N/mm^2$.

The commonly used *sign convention* for axial forces is to define tensile forces as positive and compressive forces as negative. This convention is carried over to normal stresses: Tensile stresses are considered to be positive, compressive stresses negative. A simple sign convention for shear stresses does not exist; a convention that depends on a coordinate system will be introduced later in the text. If the stresses are *uniformly distributed*, they can be computed from

$$\sigma=\frac{F_N}{A}, \quad \tau=\frac{F_S}{A} \tag{1.3}$$

where A is the area of the cross section. If the stress distribution is not uniform, Eqs. (1.3) should be viewed as the *average stress* acting on the cross section.

1.5 BASIC FORMS OF ROD DEFORMATION

In the previous section, four basic forms of rod deformation were given. In engineering applications, it is more perceptible to show them by a free rod with the external forces applied at its ends. According to the manner that the external forces are applied to the rod, we have:

1. Axial tension or compression

A pair of external forces in opposite directions is applied at both ends of a rod with the action lines coinciding with the axis of the rod. Deformation of this action is elongation or contraction of the rod. If the external forces at both ends point into the member, the deformation is called compression, if out of the member, tension. Examples are truss members. For the very simple truss shown in Fig. 1.7(a), rod AC is in axial tension while boom BC is in axial compression.

2. Shear

There are two types of shear: *transverse shear* and *direct shear*. The common characteristic of them is that the action lines of the external forces are perpendicular to the axis of the rod. For transverse shear, the action lines of the external forces may be far from one another. It can cause shearing as well as bending deformations in the rod. While for the direct shear, the action lines of the forces are very close. Only very little bending moment is produced in the rod. The cross section between the two adjacent and opposite forces is called shear plane since the failure or slippage occurs along the plane. Portions on the two sides of the section have a relative sliding. Examples are bolts, rivets, welding joints and some other joint members (Fig. 1.7(b)).

3. Torsion

A pair of couples in opposite sense with action planes perpendicular to the axis of the rod is applied on the rod (Fig. 1.7(c)). Relative rotation of sections around the axis of the rod takes place. Transmit shafts are the examples.

4. Bending

① Pure bending: A pair of couples in the opposite sense with the action planes coinciding with the longitudinal plane of the rod is applied on the rod (Fig. 1.7(d)). Relative rotation of sections around some axes in the sections takes place. A straight axis of the rod becomes a curved axis, called a deflection curve; ② Transverse bending: Transverse loads act on the rod. The deformation form of transverse bending is a combination of pure bending and transverse shearing. Members suffering bending deformation are called beams.

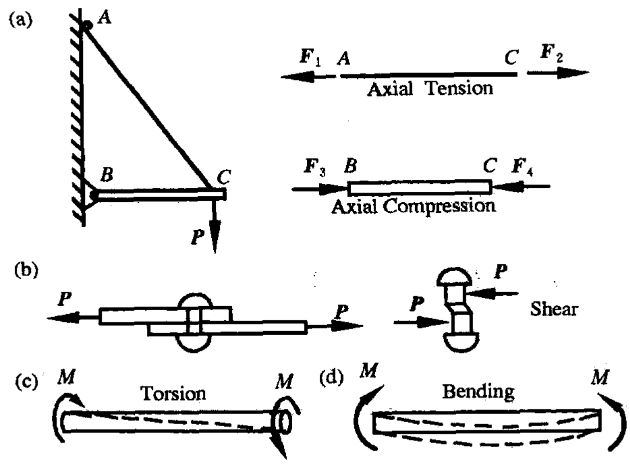


Fig. 1.7 Four basic deformations

In engineering practice, members usually suffer a combination of the basic deformation forms. For example, deformation of transmit shafts is a combination of torsion and bending. A chimney subjected to wind load and self-weight undergoes the deformation of the combination of bending and axial compression (Fig. 1.8).

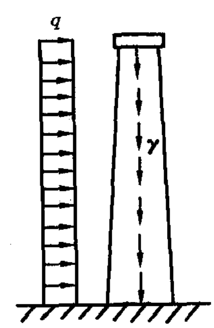


Fig. 1.8 Chimney subjected to wind load and self-weight