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Andre Weil

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Foreword

Ἀριθμόν, ἔξοχον σοφισμάτων

Αἰσχ., Προμ. Δεσμ.

The first part of this volume is based on a course taught at Princeton University in 1961–62; at that time, an excellent set of notes was prepared by David Cantor, and it was originally my intention to make these notes available to the mathematical public with only quite minor changes. Then, among some old papers of mine, I accidentally came across a long-forgotten manuscript by Chevalley, of pre-war vintage (forgotten, that is to say, both by me and by its author) which, to my taste at least, seemed to have aged very well. It contained a brief but essentially complete account of the main features of classfield theory, both local and global; and it soon became obvious that the usefulness of the intended volume would be greatly enhanced if I included such a treatment of this topic. It had to be expanded, in accordance with my own plans, but its outline could be preserved without much change. In fact, I have adhered to it rather closely at some critical points.

To improve upon Hecke, in a treatment along classical lines of the theory of algebraic numbers, would be a futile and impossible task. As will become apparent from the first pages of this book, I have rather tried to draw the conclusions from the developments of the last thirty years, whereby locally compact groups, measure and integration have been seen to play an increasingly important role in classical number-theory. In the days of Dirichlet and Hermite, and even of Minkowski, the appeal to "continuous variables" in arithmetical questions may well have seemed to come out of some magician's bag of tricks. In retrospect, we see now that the real numbers appear there as one of the infinitely many completions of the prime field, one which is neither more nor less interesting to the arithmetician than its p-adic companions, and that there is at least one language and one technique, that of the adeles, for bringing them all together under one roof and making them cooperate for a common purpose. It is needless here to go into the history of these developments; suffice it to mention such names as Hensel, Hasse, Chevalley, Artin; every one of these, and more recently Iwasawa, Tate, Tamagawa, helped to make some significant step forward along this road. Once the presence of the real field, albeit at infinite distance, ceases to be regarded as a necessary ingredient in the arithmetician's brew, it goes without saying that the function-fields over finite fields must be granted a fully simultaneous treatment with number-fields, instead of the segregated status, and at best the separate but equal facilities, which hitherto have been their lot. That, far from losing by such treatment, both races stand to gain by it, is one fact which will, I hope, clearly emerge from this book.

It will be pointed out to me that many important facts and valuable results about local fields can be proved in a fully algebraic context, without any use being made of local compacity, and can thus be shown to preserve their validity under far more general conditions. May I be allowed to suggest that I am not unaware of this circumstance, nor of the possibility of similarly extending the scope of even such global results as the theorem of Riemann-Roch? We are dealing here with mathematics, not with theology. Some mathematicians may think that they can gain full insight into God's own way of viewing their favorite topic; to me, this has always seemed a fruitless and a frivolous approach. My intentions in this book are more modest. I have tried to show that, from the point of view which I have adopted, one could give a coherent treatment, logically and aesthetically satisfying, of the topics I was dealing with. I shall be amply rewarded if I am found to have been even moderately successful in this attempt.

Some of my readers may be surprised to find no explicit mention of cohomology in my account of classfield theory. In this sense, while my approach to number-theory may be called a "modern" one in the first half of this book, it may well be described as thoroughly "unmodern" in the second part. The sophisticated reader will of course perceive that a certain amount of cohomology, and in fact no more and no less than is required for the purposes of classfield theory, hides itself in the theory of simple algebras. For anyone familiar with the language of "Galois cohomology", it will be an easy and not unprofitable exercise to translate into it some of the definitions and results of our Chapters IX, XII and XIII; in one or two places (the most conspicuous case being that of the "transfer theorem" in Chapter XII, § 5), this even makes it possible to substitute more satisfactory proofs for ours. For me to develop such an approach systematically would have meant loading a great deal of unnecessary machinery on a ship which seemed well equipped for this particular voyage; instead of making it more seaworthy, it might have sunk it.

In charting my course, I have been careful to steer clear of the arithmetical theory of algebraic groups; this is a topic of deep interest, but obviously not yet ripe for book treatment. Partly for this reason, I have refrained from discussing zeta-functions of simple algebras beyond what was needed for the sake of classfield theory. Artin's non-abelian L-functions have also been excluded; the reader of this book will find it easy to proceed to the study of Artin's beautiful papers on this subject and will find himself well prepared to enjoy them, provided he has some knowledge of the representation theory of finite groups.

It remains for me to discharge the pleasant duty of expressing my thanks to David Cantor, who prepared from my lectures at Princeton University the set of notes which reappears here as Chapters I to VII of this book (in many places with no change at all), and to Chevalley, who generously allowed me to make use of the above-mentioned manuscript and expand it into Chapters XII and XIII. My thanks are also due to Iwasawa and Lazard, who read the book in manuscript and offered many suggestions for its improvement; to H. Pogorzelski, for his assistance in proofreading; to B. Eckmann, for the interest he took in its publication; and to the staff of the Springer Verlag, and that of the Zechnersche Buchdruckerei, for their expert cooperation and their invaluable help in the process of bringing out this volume.

Princeton, May 1967. ANDRÉ WEIL

Foreword to the second edition

The text of the first edition has been left unchanged. A few corrections, references, and some brief remarks, have been added as Notes at the end of the book; the corresponding places in the text have been marked by a * in the margin. Somewhat more substantial additions will be found in the Appendices, originally prepared for the Russian edition (M. I. R., Moscow 1971). The reader's attention should be drawn to the collective volume: J. W. S. Cassels and A. Fröhlich (edd.), *Algebraic Number Theory*, Acad. Press 1967, which covers roughly the same ground as the present book, but with far greater emphasis on the cohomological aspects.

Princeton, December 1971. ANDRÉ WEIL

Chronological table

(In imitation of Hecke's "*Zeittafel*" at the end of his "*Theorie der algebraischen Zahlen*", and as a partial substitute for a historical survey, we give here a chronological list of the mathematicians who seem to have made the most significant contributions to the topics treated in this volume.)

Fermat (1601–1665)	Riemann (1826–1866)
Euler (1707–1783)	Dedekind (1831–1916)
Lagrange (1736–1813)	H. Weber (1842–1913)
Legendre (1752–1833)	Hensel (1861–1941)
Gauss (1777–1855)	Hilbert (1862–1943)
Dirichlet (1805–1859)	Takagi (1875–1960)
Kummer (1810–1893)	Hecke (1887–1947)
Hermite (1822–1901)	Artin (1898–1962)
Eisenstein (1823–1852)	Hasse (1898–)
Kronecker (1823–1891)	Chevalley (1909–)

Prerequisites and notations

No knowledge of number-theory is presupposed in this book, except for the most elementary facts about rational integers; it is useful but not necessary to have some superficial acquaintance with the p-adic valuations of the field $\mathbf{Q}$ of rational numbers and with the completions $\mathbf{Q}_p$ of $\mathbf{Q}$ defined by these valuations. On the other hand, the reader who wishes to acquire some historical perspective on the topics treated in the first part of this volume cannot do better than take up Hecke's unsurpassed *Theorie der algebraischen Zahlen*, and, if he wishes to go further back, the *Zahlentheorie* of Dirichlet-Dedekind (either in its 4th and final edition of 1894, or in the 3rd edition of 1879), with special reference to Dedekind's famous "eleventh Supplement". For similar purposes, the student of the second part of this volume may be referred to Hasse's *Klassenkörperbericht* (J. D. M. V., Part I, 1926; Part II, 1930).

The reader is expected to possess the basic vocabulary of algebra (groups, rings, fields) and of linear algebra (vector-spaces, tensor-products). Except at a few specific places, which may be skipped in a first reading, Galois theory plays no role in the first part (Chapters I to VIII). A knowledge of the main facts of Galois theory for finite and for infinite extensions is an indispensable requirement in the second part (Chapters IX to XIII).

Already in Chapter I, and throughout the book, essential use is made of the basic properties of locally compact commutative groups, including the existence and unicity of the Haar measure; the reader is expected to have acquired some familiarity with this topic before taking up the present book. The Haar measure for non-commutative locally compact groups is used in Chapters X and XI (but nowhere else). The basic facts from the duality theory of locally compact commutative groups are briefly recalled in Chapter II, § 5, and those about Fourier transforms in Chapter VII, § 2, and play an essential role thereafter.

As to our basic vocabulary and notations, they usually agree with the usage of Bourbaki. In particular, this applies to $\mathbf{N}$ (the set of the "finite cardinals" or "natural integers" $0, 1, 2, \ldots$), $\mathbf{Z}$ (the ring of rational integers), $\mathbf{Q}$ (the field of rational numbers), $\mathbf{R}$ (the field of real numbers), $\mathbf{C}$ (the field of complex numbers), $\mathbf{H}$ (the field of "classical", "ordinary" or "Hamiltonian" quaternions). If p is any rational prime, we write $\mathbf{F}_p$ for the prime field with p elements, $\mathbf{Q}_p$ for the field of p-adic numbers (the completion of $\mathbf{Q}$ with respect to the p-adic valuation; cf. Chapter I,

§ 3), $\mathbf{Z}_p$ for the ring of p-adic integers (i.e. the closure of $\mathbf{Z}$ in $\mathbf{Q}_p$). The fields $\mathbf{R}$, $\mathbf{C}$, $\mathbf{H}$, $\mathbf{Q}_p$ are always understood to be provided with their usual (or "natural") topology; so are all finite-dimensional vector-spaces over these fields. By $\mathbf{F}_q$ we understand the finite field with q elements when there is one, i.e. when q is of the form p^n, p being a rational prime and n an integer $\geqslant 1$ (cf. Chapter I, § 1). We write $\mathbf{R}_+$ for the set of all real numbers $\geqslant 0$.

All rings are assumed to have a unit. If R is a ring, its unit is written 1_R, or 1 when there is no risk of confusion; we write $R^\times$ for the multiplicative group of the invertible elements of R; in particular, when K is a field (commutative or not), $K^\times$ denotes the multiplicative group of the non-zero elements of K. We write $\mathbf{R}_+^\times$ for the multiplicative group of real numbers >0. If R is any ring, we write $M_n(R)$ for the ring of matrices with n rows and n columns whose elements belong to R, and we write 1_n for the unit in this ring, i.e. the matrix (δ_{ij}) with $\delta_{ij}=1_R$ or 0 according as $i=j$ or $i\neq j$. We write ${}^tX$ for the transpose of any matrix $X\in M_n(R)$, and $\mathrm{tr}(X)$ for its trace, i.e. the sum of its diagonal elements; if R is commutative, we write $\det(X)$ for its determinant. Occasionally we write $M_{m,n}(R)$ for the set of the matrices over R with m rows and n columns.

If R is a commutative ring, and T is an indeterminate, we write $R[T]$ for the ring of polynomials in T with coefficients in R; such a polynomial is called *monic* if its highest coefficient is 1. If S is a ring containing R, and x an element of S commuting with all elements of R, we write $R[x]$ for the subring of S generated by R and x; it consists of the elements of S of the form $F(x)$, with $F\in R[T]$. If K is a commutative field, L a field (commutative or not) containing K, and x an element of L commuting with all elements of K, we write $K(x)$ for the subfield of L generated by K and x; it is commutative. We do not speak of a field L as being an "extension" of a field K unless both are commutative; usually this word is reserved for the case when L is of finite degree over K, and then we write $[L:K]$ for this degree, i.e. for the dimension of L when L is regarded as a vector-space over K (the index of a group g' in a group g is also denoted by $[g:g']$ when it is finite; this causes no confusion).

All topologies should be understood to be Hausdorff topologies, i.e. satisfying the Hausdorff "separation" axiom ("separated" in the sense of Bourbaki). The word "homomorphism", for groups, rings, modules, vector-spaces, should be understood with the following restrictions: (a) when topologies are involved, *all homomorphisms are understood to be continuous*; (b) homomorphisms of rings are understood to be "unitary"; this means that a homomorphism of a ring R into a ring S is assumed to map 1_R onto 1_S. On the other hand, in the case of groups, homomorphisms are *not* assumed to be open mappings (i.e. to map open sets onto open sets); when necessary, one will speak of an "open homomorphism". The word "morphism" is used as a shorter synonym for "homomorphism"; the word "representation" is used occasionally, as a synonym for "homomorphism", in certain situations, e. g. when the homomorphism is one of a group into $\mathbf{C}^\times$, or for certain homomorphisms of simple algebras (cf. Chapter IX, § 2). By a *character* of a group G, commutative or not, we understand as usual a homomorphism (or "representation") of G into the subgroup of $\mathbf{C}^\times$ defined by $z\bar{z}=1$; as explained above, this should be understood to be continuous when G is given as a topological group. The words "endomorphism", "automorphism", "isomorphism" are subject to the same restrictions (a), (b) as "homomorphism"; for "automorphism" and "isomorphism", this implies, in the topological case, that the mapping in question is bijective and bicontinuous. Occasionally, when a mapping f of a set A into a set B, both with certain structures (usually fields), determines an isomorphism of A onto its image in B, we speak of it by "abuse of language" as an "isomorphism" of A into B.

In a group G, an element x is said to be of *order* n if n is the smallest integer $\geqslant 1$ such that $x^n = e$, e being the neutral element of G. If K is a field, an element of $K^\times$ of finite order is called a *root of* 1 *in* K; in accordance with a long-standing tradition, any root of 1 of order dividing n is called an n-th root of 1 in K; it is called a *primitive* n-th root of 1 if its order is n. Thus the n-th roots of 1 in K are the roots of the equation $X^n = 1$ in K.

If a, b are in $\mathbf{Z}$, (a, b) denotes their g.c.d., i.e. the element d of $\mathbf{N}$ such that $d\mathbf{Z} = a\mathbf{Z} + b\mathbf{Z}$. If R is any ring, the mapping $n \to n \cdot 1_R$ of $\mathbf{Z}$ into R maps $\mathbf{Z}$ onto the subring $\mathbf{Z} \cdot 1_R$ of R, known as "the prime ring" in R; the kernel of the morphism $n \to n \cdot 1_R$ of $\mathbf{Z}$ onto $\mathbf{Z} \cdot 1_R$ is a subgroup of $\mathbf{Z}$, hence of the form $m \cdot \mathbf{Z}$ with $m \in \mathbf{N}$; if R is not $\{0\}$ and has no zero-divisor, m is either 0 or a rational prime and is known as the *characteristic* of R. If $m = 0$, $n \to n \cdot 1_R$ is an isomorphism of $\mathbf{Z}$ onto $\mathbf{Z} \cdot 1_R$, by means of which $\mathbf{Z} \cdot 1_R$ will frequently be identified with $\mathbf{Z}$. If the characteristic of R is a prime $p > 1$, the prime ring $\mathbf{Z} \cdot 1_R$ is isomorphic to the prime field $\mathbf{F}_p$.

We shall consider left modules and right modules over non-commutative rings, and fix notations as follows. Let R be a ring; let M and N be two left modules over R. Then morphisms of M into N, for their structures as left R-modules, will be written as *right operators* on M; in other words, if α is such a morphism, we write it as $m \to m\alpha$, where $m \in M$; thus the property of being a morphism, apart from the additivity, is expressed by $r(m\alpha) = (rm)\alpha$ for all $r \in R$ and all $m \in M$. This applies in particular to endomorphisms of M. Morphisms of right R-modules are similarly written as left operators. This notation will be consistently used, in particular in Chapter IX.

As morphisms of fields into one another are assumed to be "unitary" (as explained above), such morphisms are always injective; as we have said, we sometimes refer to a morphism of a field K into a field L as an "isomorphism", or also as an *embedding*, of K into L. In part of this book, we use for such mappings the "functional" notation; beginning with Chapter VIII, § 3, where the role of Galois theory becomes essential, we shall use for them the "exponential" notation. This means that such a mapping λ is written in the former case as $x \to \lambda(x)$ and in the latter case as $x \to x^\lambda$. If L is a Galois extension of K, and λ, μ are two automorphisms of L over K, we define the law of composition $(\lambda, \mu) \to \lambda\mu$ in the Galois group $\mathfrak{g}$ of L over K as being identical with the law $(\lambda, \mu) \to \lambda \circ \mu$ in the former case, and as its opposite in the latter case; in other words, it is defined in the former case by $(\lambda\mu)x = \lambda(\mu x)$, and in the latter case by $x^{\lambda\mu} = (x^\lambda)^\mu$. For instance, if K' is a field between K and L, and $\mathfrak{h}$ is the corresponding subgroup of $\mathfrak{g}$, consisting of the automorphisms which leave fixed all the elements of K', the automorphisms of L over K which coincide on K' with a given one λ make up the right coset $\lambda\mathfrak{h}$ when the functional notation is used, and the left coset $\mathfrak{h}\lambda$ when the exponential notation is used.

When A, B, C are three additively written commutative groups (usually with some additional structures) and a "distributive" (or "bi-additive", or "bilinear") mapping $(a, b) \to ab$ of $A \times B$ into C is given, and when X, Y are respectively subgroups of A and of B, it is customary to denote by $X \cdot Y$, not the image of $X \times Y$ under that mapping, but the subgroup of C generated by that image, i.e. the group consisting of the finite sums $\sum x_i y_i$ with $x_i \in X$ and $y_i \in Y$ for all i. This notation will be used occasionally, e.g. in Chapter V.

For typographical reasons, we frequently write $\exp(z)$ instead of e^z, and $\mathbf{e}(z)$ instead of $\exp(2\pi i z) = e^{2\pi i z}$, for $z \in \mathbf{C}$; ordinarily $\mathbf{e}(z)$ occurs only for $z \in \mathbf{R}$.

Finally we must explain the method followed for cross-references; these have been inserted quite generously, with a view to helping the inexperienced reader; the reader is advised to follow them up only when the argument is not otherwise clear. Theorems have been numbered continuously throughout each chapter; the same is true for propositions, for lemmas, for definitions, for the numbered formulas. Each theorem and each proposition may be followed by one or several corollaries. Generally speaking, theorems are to be regarded as more important than propositions, but the distinction between them would hardly stand a close scrutiny. Lemmas are merely auxiliary results. Not all new concepts are the object of a numbered definition; all concepts, except those which are assumed to be known, are listed in the index at the end of the book, with proper references. Formulas are numbered only for purposes of quotation, and not as an indication of their importance. When a reference is given thus: "by prop. 2", "by corollary 1 of th. 3", etc., it refers to a result in the same §; when thus: "by prop. 2 of § 2", "by th. 3 of § 3", etc., it refers to another § of the same chapter; when thus: "by prop. 2 of Chap. IV-2", it refers to proposition 2 of Chapter IV, § 2. Numbers of Chapter and § are given at the top of every page. A table of the most frequently used notations is given below, in the order of their first appearance.

Table of notations

Chapter I.

§ 2: mod_G, mod_V, mod_K.
§ 3: $|x|_p$, $|x|_\infty$, $\mathbf{Q}_\infty=\mathbf{R}$, $|x|_v$, $\mathbf{Q}_v$ (v=rational prime or ∞).
§ 4: K (any p-field), R, P, π, q, ord_K, ord, $M^\times$, M.

Chapter II.

§ 3: $1+P^n$ (as subgroup of $K^\times$ for $n\geqslant 1$).
§ 5: $\langle g,g^*\rangle_G$, $\langle g,g^*\rangle$, G^*, H_*, V^*, L_*, V', $[v,v']_V$, $[v,v']$, χ, $\mathrm{ord}(\chi)$.

Chapter III.

§ 1: (for a place v of an A-field k) $|x|_v$, k_v, r_v, p_v (for q_v, see Chap. VII-1); ∞ (as a place of $\mathbf{Q}$), $w|v$, $E_v=E\otimes_k k_v$, ε_v, $\mathscr{A}_v$, α_v.
§ 3: End(E), $Tr_{\mathscr{A}/k}$, $N_{\mathscr{A}/k}$, $Tr_{k'/k}$, $N_{k'/k}$.

Chapter IV.

§ 1: P, P_∞, $k_{\mathbf{A}}(P)$, $k_{\mathbf{A}}$, χ, χ_v, $E_{\mathbf{A}}(P,\varepsilon)$, $E_{\mathbf{A}}$, $\mathscr{A}_{\mathbf{A}}$, $\mathscr{A}_{\mathbf{A}}(P,\alpha)$, $(k'/k)_{\mathbf{A}}$, $(E/k)_{\mathbf{A}}$.
§ 3: Aut(E), $\mathscr{A}_{\mathbf{A}}^\times$, $\mathscr{A}_{\mathbf{A}}(P,\alpha)^\times$, $|a|_{\mathbf{A}}$.
§ 4: $k_{\mathbf{A}}^1$, M, $\Omega(P)=k_{\mathbf{A}}(P)^\times$, $\Omega_1(P)$, $E(P)$.

Chapter V.

§ 2: k_∞, E_∞, $\mathfrak{r}$, L_v.
§ 3: $\mathfrak{p}_v$, $I(k)$, id(a), $P(k)$, h, $\mathfrak{N}(\mathfrak{a})$.
§ 4: $|dx\wedge d\bar{x}|$, R, c_k.

Chapter VI.

deg($\mathfrak{a}$), $\mathfrak{a}\succ\mathfrak{b}$, div($a$), $D(k)$, $P(k)$, $D_0(k)$, g, div(χ).

Chapter VII.

§ 1: q_v, ζ_k(cf. § 6).
§ 2: Φ^*, $\prod\Phi_v$, $\prod\alpha_v$.
§ 3: $\Omega(G)$, Ω_1, ω_s.
§ 4: $G_k = k_{\mathbf{A}}^\times/k^\times$, $\Omega(G_k)$, ω_1, ω_s, G_k^1, Ω_1, M, N, ω_v, $\prod\omega_v$, $Z(\omega,\Phi)$.
§ 6: $G_1(s)$, $G_2(s)$, c_k (cf. Chap. V-4), $G_w(s)$, $\zeta_k(s)$, $Z_k(s)$.
§ 7: $f(v)$, s_v, A, B, N_v, Φ_ω, $\kappa=\prod\kappa_v$, $a=(a_v)$, $b=(b_v)$, G_w, $\lambda(v)$, π_v, $L(s,\omega)$, $\mathfrak{f}$, $\Lambda(s,\omega)$.
§ 8: G_{P}, $I(P)$, $D(P)$.

Chapter VIII.

§ 1: K, K', n, q, R, P, π, q', R', P', π', f, e, Tr, N, $\mathfrak{N}$, d, $D(K'/K)$, D, ι'.
§ 2: Δ.
§ 3: $\nu(\lambda)$, $\mathfrak{g}_\nu$.
§ 4: $\mathfrak{d}$, ι, $\mathfrak{N}_{k'/k}$, $\mathfrak{N}$, $\mathfrak{D}$.

Chapter IX.

§ 1: A_L, $A\otimes B$, A^0.
§ 2: τ, ν.
§ 3: $\mathrm{Cl}(A)$, $B(K)$, $\bar{K}$, K_{sep}, $\mathfrak{G}$, $\mathfrak{H}$, K', $\bar{K}'$, K'_{sep}, $\mathfrak{G}'$, ρ, $H(K)$.
§ 4: $\{\chi,\theta\}$, $[L/K;\chi,\theta]$.
§ 5: $\chi_{n,\xi}$, $\{\xi,\theta\}_n$, $\chi_{p,\xi}$, $\{\xi,\theta\}_p$.

Chapter X.

§ 1: $\mathrm{Hom}(V,W)$, $\mathrm{Hom}(V,L;W,M)$, $\mathrm{End}(V,L)$, $\mathrm{Aut}(V,L)$.
§ 3: $\mathfrak{T}$, $\mathfrak{T}'$, $\mathfrak{T}''$; $\mathfrak{T}$, $\mathfrak{U}$.

Chapter XII.

§ 1: $\mathrm{K_{ab}}$, $\mathfrak{G}^{(1)}$, $\mathfrak{A}$, X_K, ρ, G_K, $(\chi,g)_K$, $\mathfrak{a}$, G_K^1, U_K, X_0, $\mathfrak{A}_0$, K_0.
§ 2: $h(A)$, η, $(\chi,\theta)_K$ (for $K=\mathbf{R},\mathbf{C}$); $\mathfrak{M}$, K_0, $\mathfrak{H}_0$, K_n, φ_0, X_0, φ, η, $(\chi,\theta)_K$, $\mathfrak{a}$, $h(A)$.
§ 3: U_K, $\mathfrak{A}_0$.

Chapter XIII.

§ 1: $\bar{k}$, K_v, k_{sep}, $k_{v,\mathrm{sep}}$, k_{ab}, $k_{v,\mathrm{ab}}$, $\mathfrak{G}$, $\mathfrak{A}$, $\mathfrak{G}_v$, $\mathfrak{A}_v$, ρ_v, X_k, χ_v, $(\chi_v,z)_v$, $(\chi,z)_k$, $\mathfrak{a}$, $k_{\infty+}^\times$, F, q, k_0, $\mathfrak{H}_0$, X_0, k_n, $\mathfrak{A}_0$, φ_0, φ, $\mathbf{\bar{Q}}$, ε, $\mathfrak{H}_m$, $\mathfrak{g}$, $\mathfrak{h}$.
§ 3: $h_v(A)$, U_k.
§ 5: $(x,y)_{n,K}$, $(z,z')_n$, $\Omega(P)$ (cf. Chap. IV-4), $\Omega'(P)$.
§ 7: $(x,z)_{p,K}$, Φ, $\Omega'(m,K)$, $(x,z)_p$, $\Omega'(\mathrm{m})$.
§ 9: $\mathfrak{B}(L)$, $N(L)$.
§ 10: k', $\mathfrak{g}$, $\mathfrak{h}$, U, $\mathfrak{B}$, U_v, $\mathfrak{B}_v$, γ, γ_v, $\mathfrak{f}(\omega)$, $\mathfrak{D}$.
§ 11: G_{P}, G'_{P}, L_{P}, l_{P}, pr, $\mathfrak{U}_{\mathrm{P}}$, $J(U,P)$.

Contents

PART II. CLASSFIELD THEORY