Robert S. Strichartz

The Way of Analysis

Revised Edition

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世界图书出版公司
www.wpcbj.com.cn

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JONES AND BARTLETT PUBLISHERS

Sudbury, Massachusetts

BOSTON TORONTO LONDON SINGAPORE

图书在版编目（CIP）数据

分析方法 = The Way of Analysis：英文/（美）斯特里沙兹（Strichartz，R. S.）著．—影印本．—北京：世界图书出版公司北京公司，2010.5

ISBN 978-7-5100-0556-5

Ⅰ.①分… Ⅱ.①斯… Ⅲ.①数学分析—分析方法—英文 Ⅳ.①O17

中国版本图书馆 CIP 数据核字（2010）第 036291 号

书　　名：The Way of Analysis Revised ed.

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中 译 名：分析方法 修订版

责任编辑：高蓉　刘慧

出 版 者：世界图书出版公司北京公司

印 刷 者：三河国英印务有限公司

发　　行：世界图书出版公司北京公司（北京朝内大街 137 号　100010）

联系电话：010-64021602，010-64015659

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开　　本：24 开

印　　张：32

版　　次：2010 年 04 月

版权登记：图字：01-2009-5060

书　　号：978-7-5100-0556-5/O · 771　　　定　　价：99.00 元

Preface

Do not ask permission to understand.
Do not wait for the word of authority.
Seize reason in your own hand.
With your own teeth savor the fruit.

Mathematics is more than a collection of theorems, definitions, problems and techniques; it is a way of thought. The same can be said about an individual branch of mathematics, such as analysis. Analysis has its roots in the work of Archimedes and other ancient Greek geometers, who developed techniques to find areas, volumes, centers of gravity, arc lengths, and tangents to curves. In the seventeenth century these techniques were further developed, culminating in the invention of the calculus of Newton and Leibniz. During the eighteenth century the calculus was fashioned into a tool of bold computational power and applied to diverse problems of practical and theoretical interest. At the same time the foundation of analysis—the logical justification for the success of the methods—was left in limbo. This had practical consequences: for example, Euler—the leading mathematician of the eighteenth century—developed all the techniques needed for the study of Fourier series, but he never carried out the project. On the contrary, he argued in print *against* the possibility of representing functions as Fourier series, when this proposal was put forth by Daniel Bernoulli, and his argument was based on fundamental misconceptions concerning the nature of functions and infinite series.

In the nineteenth century, the problem of the foundation of analysis was faced squarely and resolved. The theory that was developed forms most of the content of this book. We will describe it in its logical order, starting from the most basic concepts such as sets and numbers and building up to the more involved concepts of limits, continuity, derivative, and integral. The actual historical order of discovery was almost the reverse; much like peeling a cabbage, mathematicians began with the outermost layers and worked their way inward. Cauchy and Bolzano began the process in the 1820s by developing the theory of functions without defining the real numbers. The first rigorous definition of the real number system came in the work of Dedekind, Weierstrass, and Heine in the 1860s. Set theory came later in the work of Cantor, Peano, and Frege.

The consequences of the nineteenth century foundational work were enormous and are still being felt today. Perhaps the least important consequence was the establishment of a logically valid explanation of the calculus. More important, with the clearing away of the conceptual murk, new problems emerged with clarity and were developed into important theories. We will give some illustrations of these new nineteenth century discoveries in our discussions of differential equations, Fourier series, higher dimensional calculus, and manifolds. Most important of all, however, the nineteenth century foundational work paved the way for the work of the twentieth century. Analysis today is a subject of vast scope and beauty, ranging from the abstract to the concrete, characterized both by the bold computational power of the eighteenth century and the logical subtlety of the nineteenth century. Most of these developments are beyond the scope of this book or at best merely hinted at. Still, it is my hope that the reader, after having entered so deeply along the way of analysis, will be encouraged to continue the study.

My goal in writing this book is to communicate the mathematical ideas of the subject to the reader. I have tried to be generous with explanations. Perhaps there will be places where I belabor the obvious, nevertheless, I think there is enough truly challenging material here to inspire even the strongest students. On the other hand, there will inevitably be places where each reader will find difficulties in following the arguments. When this happens, I suggest that you write your questions in the margins. Later, when you go over the material, you may find that you can answer the question. If not, be sure to ask your instructor or another student; often, it is a minor misunderstanding that causes confusion and can easily be cleared up. Sometimes, the inherent difficulty of the material will demand considerable effort on your part to attain understanding. I hope you will not become frustrated in the process; it is something which all students of mathematics must confront. I believe that what you learn through a process of struggle is more likely to stick with you than what you learn without effort.

Understanding mathematics is a complex process. It involves not only following the details of an argument and verifying its correctness, but seeing the overall strategy of the argument, the role played by every hypothesis, and understanding how different theorems and definitions fit together to create the whole. It is a long-term process; in a sense, you cannot appreciate the significance of the first theorem until you have learned the last theorem. So please be sure to review old material; you may find the chapter summaries useful for this purpose. The mathematical ideas presented in this book are of fundamental importance, and you are sure to encounter them again in further studies in both pure and applied mathematics. Learn them well and they will serve you well in the future. It may not be an easy task, but it is a worthy one.

To the Instructor

This book is designed so that it may be used in several ways, including

1. a one-semester introductory real analysis course,
2. a two-semester real analysis course not including Lebesgue integration,
3. a two-semester real analysis course including an introduction to Lebesgue integration.

There are many optional sections, marked with an asterisk (*), that can be covered or omitted at your discretion. There is some flexibility in the ordering of the later chapters. Thus you can design a course in accordance with your interests and requirements. There are three chapters on applications (Chapter 11, Ordinary Differential Equations; Chapter 12, Fourier Series; and Chapter 13, Implicit Functions, Curves and Surfaces). These topics are often omitted, or treated very briefly, in a real analysis course because they are covered in other courses. However, they serve an important purpose in illustrating how the abstract theory may be applied to more concrete situations. I would urge you to try to fit as much of this material as time allows into your course.

The chapters may be divided into four groupings:

1. functions of one variable: 1, 2, 3, 4, 5, 6, 7, 8;
2. functions of several variables: 9, 10, 15;
3. applications: 11, 12, 13;
4. Lebesgue integration: 14, 15.

Note that Chapter 15, Multiple Integrals, may be used either with or without the Lebesgue integral.

The first 10 chapters are designed to be used in the given order (sections marked with an asterisk may be omitted or postponed). If you are not covering the Lebesgue integral, then selections from Chapter 15 (15.1.1, 15.2.1, and 15.2.3) can be covered any time after Chapter 10. The applications, Chapters 11, 12, and 13, can be done in any order. It is advisable to do at least some of Chapter 12, Fourier series, before doing the Lebesgue Integral, Chapter 14. In Chapter 6 (section 6.1.3) I have included a preview of some results in integration theory that are covered in detail later in the book—this is the only place I have violated the principle of presenting full proofs of all results in the order they are discussed. I think this is a reasonable compromise, in view of the facts that (a) the students will want to use these results in doing exercises, and (b) to present proofs at this point in the text would require long detours.

Here are some concrete suggestions for using this book.

1. *One-semester course*: do Chapters 1–8 in order, omitting all sections marked with an asterisk. This will cover the one variable theory. If time remains at the end, return to omitted sections.

2. *Two-semester course (without Lebesgue integrals)*

   First semester: do chapters 1–7 in order, including most sections marked with an asterisk.

Second semester: do Chapters 8–10; then 15.1.1, 15.2.1, 15.2.3, then Chapters 11–13, including some sections marked with an asterisk.

3. *Two-semester course (with Lebesgue integrals)*

First semester: do Chapters 1–7 in order, including most sections marked with an asterisk, but omit 6.2.3.

Second semester: do Chapters 8–10; then selections from chapters 11–13; then chapters 14 and 15.

This book contains a generous selection of exercises, ranging in difficulty from straightforward to challenging. The most difficult ones are marked with an asterisk.

All the main results are presented with complete proofs; indeed the emphasis is on a careful explanation of the ideas behind the proofs. One important goal is to develop the reader's mathematical maturity. For many students, a course in real analysis may be their first encounter with rigorous mathematical reasoning. This can be a daunting experience but also an inspiring one. I have tried to supply the students with the support they will need to meet the challenge.

My recommendation is that students be required to read the material before it is discussed in class. (This may be difficult to enforce in practice, but here is one suggestion: have students submit brief written answers to a question based on the reading and also a question they would like to have answered in class.) The ability to read and learn from a mathematical text is a valuable skill for students to develop. This book was written to be read—not deciphered. If I have perhaps coddled the students too much, I'm sure they won't complain about that!

The presentation of the material in this book is often informal. A lot of space is given to motivation and a discussion of proof strategies. Not every result is labeled as a theorem, and sometimes the precise statement of the result does not emerge until after the proof has been given. Formulas are not numbered, and theorems are referred to by name and not number. To compensate for the informality of the body of the text, I have included summaries at the end of each chapter (except the first) of all the main results, in standard dry mathematical format. The students should find these chapter summaries handy both for review purposes and for references.

I have tried to give some historical perspective on the material presented, but the basic organization follows logical rather than historical order. I use conventional names for theorems, even if this perpetuates injustices and errors (for example, I believe it is more important to know what the Cauchy-Schwartz inequality is than to decide whether or not Bunyakowsky deserves some/most/all of the credit for it). One important lesson from the historical record is that abstract theorems did not grow up in a vacuum: they were motivated by concrete problems and proved their worth through a variety of applications. This text gives students ample opportunity to see this interplay in action, especially in Chapters 11, 12, and 13.

In order to give the material unity, I have emphasized themes that recur. Also, many results are presented twice, first in a more concrete setting. For example, I develop the topology of the real line first, postponing the general theory of metric spaces to Chapter 9. This is perhaps not the most efficient route, but I think it makes it easier for the students. Whenever possible I give the most algorithmic proof, even if it is sometimes harder (for example, I construct a Fourier series of a continuous function that diverges at a point). I have tried to emphasize techniques that can be used again in other contexts. I construct the real number system by Cauchy completion of the rationals, since Cauchy completion is an important technique. The derivative in one variable is defined by best affine approximation, since the same definition can be used in $\mathbb{R}^n$. Chapter 7, Sequences and Series of Functions, is presented entirely in the context of functions of one variable, even though most of the results extend easily to the multivariable setting, or more generally to functions on metric spaces.

A good text should make the job of teaching easier. I hope I have succeeded in providing you with a text that you can easily teach from. I would appreciate receiving any comments or suggestions for improvements from you.

Acknowledgments

I would like to thank the students at Cornell who learned this material from me as I developed the preliminary versions of the text over the past dozen years. I am grateful for your criticism as well as your encouragement. I needed both, even if I was more gracious about the encouragement (special thanks to Chris Wittemann). I am also grateful to Graeme Bailey, Dan Barbasch, Eugene Dynkin, Archil Gulisashvili, and Oscar Rothaus, who also taught using this material.

I am especially grateful to Mark Barsamian, who went over the text line-by-line from the point of view of the student, and made me change just about everything. He helped me to improve the organization and style of presentation and pointed out possible sources of ambiguity and confusion. I know that the text is much stronger as a result of his criticism, and I feel more confident that I have fulfilled my promise to write a book that students can understand.

I am grateful to Carl Hesler, my editor at Jones and Bartlett, who has encouraged me throughout the long process of turning my rough lecture notes into a polished book, and I especially appreciate his confidence in the value of this work for the mathematical community.

I would like to thank June Meyermann for her outstanding job preparing the manuscript in LaTeX and David Larkin who produced many of the figures using Mathematica.

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