TRENDS AND CHALLENGES IN MATHEMATICS EDUCATION

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East China Normal University Press

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Trends and Challenges in Mathematics Education

（数学教育的趋势与挑战）

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PREFACE

Mathematics is the most highly internationalized language of science; it is also the common basis for a large number of branches of science and technology. Hence, it does not just belong to a few researchers of mathematics, but it should also be a part of the attainments required for the citizens of a country, as well as a practical instrument and a sharp weapon in the hands of the workers in all professions. Based on this understanding, mathematics education has become an area that professionals in various fields and people from all walks of life are concerned with. It holds a significant place in the educational system of any country in the world.

In China research and practice in mathematics education show distinctive characteristics. In some aspects we have achieved great successes. However, we have obviously had some drawbacks and there is much room for improvement in what we have done. To share China's successful experiences with educators the world over, and to absorb the quintessence of their experiences so as to remedy deficiencies in our work, mathematical educators of this country need to start and maintain cross-national dialogues and exchanges of views with their colleagues in other countries. Unfortunately, for quite a long time, Chinese mathematical educators' contacts with the outside world were far from being sufficient, as compared with the frequent exchanges and cooperative activities going on among researchers of mathematics around the world. Perhaps there are a few exceptions. For example, China's participation in international activities on mathematics education may date back at least to 1980 when Prof. Lokeng Hwa (华罗庚), a famous Chinese mathematician, was invited to make a speech at the 4th International Congress on Mathematical Education (ICME-4) held at Berkeley, California, USA. A Chinese delegation headed by Prof. Shisun Ding (丁石孙) from Peking University also attended

the Congress. Other members of the delegation included Ersheng Ding (丁尔陞) from Beijing Normal University, Xihua Cao (曹锡华) from East China Normal University and Rufu Zeng (曾如阜) from South China Normal University. At that time, for a country like China where the door was just open, this only indicated her interest in mathematics education in the outside world, but it was not yet an indication of any kind of dialogue or exchange of views between the Chinese mathematical educators and the world. Another remarkable example of the early Chinese-foreign exchange on mathematics education is the visit of Prof. H. Freudenthal to East China Normal University at Shanghai in 1987. Still, the exchanges in those years were limited in scope and depth, and our vision in the international perspective needs to be broadened.

Nevertheless, I am pleased to see, on the other hand, things have been changing since the 90s of the 20th century. The mutual visits and cooperations of Chinese mathematical educators with their colleagues in other countries are getting more frequent; more and more young scholars are returning to China after receiving degrees on mathematics education abroad. Moreover, Prof. Dianzhou Zhang and Jianpan Wang (both from East China Normal University) have assumed the responsibilities of Executive Committee (EC) members of International Commission on Mathematical Instruction (ICMI) successively; Chinese mathematical educators have taken an active part in organizing and/or participating ICMI Regional Conferences and Studies*. It is also worth mentioning that, taking the advantage of the convenience of the location, over a hundred mathematical educators from the Mainland of China went east to Japan and attended the 9th International Congress on Mathematical Education (ICME-9, 2000) in Makuhari, Tokyo. The delegation from the Mainland of China was the third largest in number, only behind the delegations of the host country Japan and the United States. During ICME-9 we even held a special forum on mathematics education and Chinese culture, gathering together all the Chinese scholars at ICME-9 — those from the Mainland of China, Hong Kong, Taiwan, and other countries in the world — and the foreign researchers who were interested in Chinese mathematics education. Comparing this with what we did at ICME-4, one may be amazed at the great changes that have taken place in China in the last two decades. With the expansion of activities in the dialogues and exchanges between the Chinese and foreign mathematical educators, the world has gradually deepened its understanding of the development and research in China's mathematics education. Meanwhile, through those dialogues and exchanges it has been made possible for the Chinese researchers to gain access to the latest developments of mathematics research across the world. The Chinese researchers have thus been able to accumulate their own

*According to our data, three ICMI Regional Conferences held in Beijing, Shanghai and Changchun, respectively.

experience as a starting point for work, broaden the scope of research activities, as well as develop their theory of mathematics education with the Chinese characteristics.

In an effort to further broaden the ties of the circle of Chinese mathematics-educational research with the world, I invited ICMI EC for its annual meeting held in Shanghai in 2001, hosted by East China Normal University. At the time when the meeting was held, the Department of Mathematics and the Institute of Curriculum and Instruction of East China Normal University took the opportunity to jointly host *The International Symposium on Mathematical Education.* All the ICMI EC members and representatives of the Chinese researchers in mathematics education reported the results of their research projects and discussed with great interest the hot issues on mathematics education that have attracted the world's attention. Particularly, the ICMI EC members encouraged the Chinese researchers to present in different ways what China has achieved in mathematics education, its research history and development. The teaching model of "Dual Basics" in mathematical instruction, the methods of changing equations in mathematical drills, skills training in mathematical instruction, the Chinese styles of design in solving mathematical problems, and mathematics curricula with the Chinese characteristics have aroused world-wide interest. Much more work needs to be done in these areas, though.

In the summer of 2002, the International Congress of Mathematicians (ICM) was held in Beijing. Entrusted by the ICM-2002 Organization Committee, East China Normal University and University of Tibet jointly sponsored a Satellite Conference on Mathematical Education at Lhasa. It was a very successful conference. More than 40 delegates from 20 countries and regions together with 25 representatives from 21 provinces and autonomous regions of mainland China exchanged their views and held interesting and fruitful discussions on issues of common interest in mathematics education from the perspectives of mathematicians or educators of mathematics. The conference provided wide-ranging coverage of issues: the curriculum reform in elementary and high schools, colleges and universities; pre- and post-service teacher training; comparative studies of cross-cultural mathematics education; teaching and learning mathematics; Chinese and world's history of mathematics and mathematics education; mathematics education and use of modern technology, etc. Views on these issues were not only expressed in the reports and discussed in the questions and answers that followed the reports, but also exchanged in the casual talks at the dinner table and on the journey after the conference. Through these interesting discussions and talks different ideas and experiences were exchanged, mutual understanding was achieved, and to some extent, common basis established.

This conference attracted the attention of mathematicians as well. Approximately half of the participants in the conference were mathematicians engaged

in professional mathematics research. They did not just show up at the meetings, but passed their experiences on to the audience, expressed their views on mathematics education, and were actively involved in the discussions. We noted particularly that Professor John M. Ball, then the ongoing president of the International Mathematical Union, attended the conference and made a speech, which inspired the mathematical educators to greater efforts. The development of mathematics education can never be separated from that of mathematics itself. Mathematics education calls for active involvement and participation of researchers of mathematics, so it can better represent the true nature and essence of the profession of mathematics, to keep up with the changing times and meet the needs of society and development of science and technology. Researchers of mathematics education can also draw inspiration from mathematicians' experiences and the proposals made by them.

Based on the fruitful results of the two conferences mentioned above, we have arranged the publication of this selection of research papers. We do not require that all the selected papers focus on certain topics, nor do we seek for consistency in their styles. The papers presented here by the mathematical educators and mathematicians from different countries and backgrounds are so rich and colorful in content and style that each of them has displayed its unique splendor. It could be a thesis concerning research of theory, or a summary of an experiment, or a paper about what has been learned from practice. If we must group the papers collected in this selection into different categories, they may fall into the following five aspects:

- ***Innovation of Mathematics Learning and Teaching***

Every research paper on innovation of mathematics learning and teaching presents the results of research from a unique perspective. The researchers from Australia construct a set of models of innovative assessment from the perspective of evaluating learning and teaching. They make a point that the emphasis of assessment activities should be laid on the analysis of the development of students' skills in mathematics, rather than their skills in doing the drills of mathematical formulas and reasoning. They also emphasize that assessment practices should inspire teachers to design innovative teaching activities in mathematics and provide more opportunities for their students to make use of mathematical knowledge. The Israeli researchers present their results of research in a contrastive analysis from the angle of mathematical communication, stressing the importance of meta-cognitive training and cooperative learning in the mathematical communication. They believe that it should be reflected in the continuous enhancement of fluency and flexibility of the mathematical interpretation on the part of the students through their cooperative learning integrated with meta-cognitive training. A Chinese scholar starts his paper with the discussion of the relationship between a student's involvement in the study and his achievements made in mathematics. His experiment indicates

that when a student solves a non-conventional problem (for example, an open-ended question), his achievement is highly correlated with his deep-structured learning strategies; whereas in solving a conventional problem, his achievement is correlated with his surface-structured learning strategies, such as diligence and confidence. The researchers from Russia, Singapore and China have explored systematically how to innovate the study of geometry, the science of statistics and probability in terms of the content of teaching. The Russian professor points out that students should study geometry from the perspectives of practical value, culture and the spirit of schools' mathematics teaching and the development of students' intelligence. From the perspectives of the development of innovations, that of aesthetics, and the cultivation of students' morals, he stresses that schools must teach geometry. The scholars from Singapore and China who have conducted long-term research projects in teaching the science of statistics and probability express their views on why schools should include education of statistics and probability, and how it should be carried out. They also analyze in detail the causes for the misconceptions that may exist in studying the science of probability and the strategies for clarifying them. A Chinese scholar has conducted her experiments in designing situational mathematics learning from the perspective of contemporary teaching and learning theory. By participating in these learning activities, the students have been able to develop competence in gaining information required by contemporary society and learn the skills in mathematical communications and reflections.

- ***Mathematics Education and Local Culture***

Researchers from different countries report in a vivid way their results of research in mathematics education against the cultural backgrounds of their own countries. The Italian professor has engaged in the research of mathematics and art as well as mathematics and culture for years. In a vivid case study, he discusses how to build up an eternal bridge between mathematics and culture. What he believes is that mathematics itself is part of culture. Actually it is not necessary to set up any bridge between them. The researcher from Argentina presents his program of teacher training in mathematics, and the courses taught at the stage of general education in Argentina. He says that any Argentine teacher teaching 1–6 graders has to teach all the subjects taught in schools, and mathematics is but one of the subjects. Generally these teachers have had two-year training in teacher-training schools, which only offer two courses in mathematics. As a result, their knowledge regarding the profession of mathematics is relatively limited. Since 1992 the Argentine law stipulates that all the teachers must receive continuous education. A series of mathematics courses organized in 12 modules have been worked out to raise the levels of professional knowledge and skills of the teachers. The researcher from the United States reports that as indicated in the results found in some international evaluation projects, American mathematics education has not been

able to develop satisfactorily. American students' performances in the study of mathematics included in those international evaluation projects as well as their performances in recent International Mathematics Olympics have not been up to par. He believes this is partly caused by the problems that exist in the philosophy of teaching, evaluation, innovation and technology in the field of mathematics education.

Some Malaysian and Chinese researchers present to us the reform of mathematics education with the characteristics of Asian culture. The researchers from Malaysia expresses their reflections on the mathematical curriculum reform started in 2000. They point out that the reform was concerned with that of culture in mathematics teaching. They emphasize that teachers of mathematics should respond to the challenges in the age of information and knowledge, and accomplish their mission in mathematics education so Malaysian mathematics education will develop in the right direction. In a case study, some other researchers analyze in detail the culture of mathematical study in Malaysia as a multi-national country, and discuss the differences and similarities in terms of culture in mathematical study due to different cultural backgrounds.

A large-scale curriculum reform in mathematics also started in China towards the end of the 20th century. In this respect, a Chinese researcher discusses comprehensively the reform carried out in compulsory education. His discussion shows that China's curriculum reform in mathematics is directed against the malady that existed in Chinese mathematics education for decades. The reform includes the formulation of curriculum standards and reproduction of textbooks, as well as the reform of the evaluation system and teacher training. Another researcher starts his discussion from the proverb "Practice makes perfect," a manifestation of culture in learning, which has been held in esteem in China and other Asian countries. In his in-depth analysis of the cognitive processes in studying mathematics, he expounds the relationship between "drilling" and "comprehension," and points out that facilitation of comprehension of mathematics should be the goal of "repeated drilling" in mathematical study. China is also a multi-national country. Each of the different nationalities has its own culture, which develops with its educational culture and learning culture. For example, the Tibetan researchers present to us the historical, present and future development of Tibetan mathematics education in a macro-level perspective. In a contrastive analysis of the historical and current mathematics education in Tibet, they cite the tremendous achievements made in Tibetan mathematics education. While analyzing the reasons why the achievements have been made, they have also found the problems and explored the ways to promote the development of Tibetan mathematics education. From a micro-level perspective, on the other hand, the researchers have made an analysis of the characteristics of Han and Tibetan students in the way they think in mathematics. After collecting samples from the Han and Tibetan students in University of Tibet, making investigations and analyzing the data gathered

in the questionnaires, they have been able to find the similar and different concepts and ways of thinking in mathematical study between Han and Tibetan students.

• ***Mathematics Education and Technology***

This is an issue that researchers in the world have shown concern for. The French researcher analyzes the functions of technology in secondary mathematics education systematically. She discusses the factors that have caused slow progress in the integration of computer technology and mathematics in secondary education, focusing on the internal relationship between the development of educational system and that of technology. She hopes that technology should win its legal status in the development of education. However, because the integration process is a comprehensive one, we should not underestimate the difficulties involved in its complexity and comprehensiveness. The Belgian researchers draw attention to problem solving and technology, which they think are the key points to effective and efficient mathematics teaching and learning. They point out that the major drawbacks in traditional mathematics teaching are rote learning and lack of transfer. Actually the primary emphasis in mathematics teaching should be placed on model construction and problem solving. More importantly, by using modern technology and developing software, students will be encouraged to discover facts and relationship between formulas, as well as develop their ability to construct mathematical models. For this the researcher cites MATHEMATICA as an example to illustrate the integration of modern technology and mathematical study. From the dimensions of history, culture and teaching, the German researcher explores the concept of algorithms as the basic one for mathematics and mathematics education. He stresses the dual nature of the field of algorithms: On the one hand, new algorithms are constructed or designed while the existing algorithms are comprehended and analyzed; on the other hand, the completed and existing algorithms are still operating, and sub-algorithms are being constructed. Based on the duality of the algorithms, he elaborates systematically on the inclusion of different categories of algorithms in the mathematical curricula in Germany and on how to teach them. Finally, the researcher from England focuses on how to design a computer-aided assessment system appropriately so as to create the learning environment that will contribute to the improvement of the high-level mathematical learning effectively. In addition, he introduces the ways to make use of computer algebra system — AIM (Alice Interactive Mathematics) — for providing opportunities to raise questions, propose hypotheses and solve problems, thus developing one's ability of self-reflection and self-monitoring.

• ***International Comparative Studies on Mathematics Education***

International comparative studies contribute to a better understanding of trends in the development of international mathematics education and display

the characteristics of mathematics education in different countries. On the other hand, they also help to identify the problems and deficiencies existing in each country's mathematics education. The researcher from Germany introduces PISA, a cross-national comparative study project conducted by OECD. He makes a detailed and insightful analysis of the characteristics of the mathematical tasks and questions included in PISA. Citing the examples of Japan and Finland, the two countries that have achieved fairly satisfactory results, he explains how the testing questions with different characteristics reflect students' achievements in mathematics. A Chinese researcher, on his part, introduces the methods of international comparative studies in mathematics education. He proposes that cross-national comparative studies in mathematics education should be carried out in two aspects — making comparisons of similarities as well as differences between the countries compared, and evaluation and analysis should be based on both of these two aspects. The Chinese researcher further points out that educators of mathematics should use the methods of cross-national contrastive study as reference to deepen the understanding of their tradition and form their own ways in reflecting continuously and moving on.

- ***Mathematics Education and Teacher Education***

In recent years, specialty development of mathematical teachers has become a hot topic in the research of mathematics education. In this respect, researchers have done much work related to teacher education from different perspectives. The researchers from the United States have conducted experiments and theoretical research in this area for more than a decade. They have explored the following issues systematically:

— What mathematics knowledge and resources are entailed by teaching?
— How are such mathematics knowledge and resources used in the course of teaching?
— How might teachers be helped to develop such usable mathematical knowledge and resources?

The results of research presented by the US researchers focus on the first two questions, with the emphasis on the analysis of the classroom activities of the teachers, the main tasks and issues in teaching.

A Canadian researcher addresses the issue on how to select mathematics themes in teacher education in order to arouse wider interest of the students studying in the teacher training programs. From the perspective of modern mathematics, the author of the paper selects some case studies to examine how elementary mathematics is related to the main content of school curricula and teacher preparation in elementary and secondary schools. He calls for the improvement of teachers' understanding of the nature of mathematics in the process of teaching. He notes particularly that training elementary and secondary

school teachers should be an important mission for colleges and universities, and mathematicians should also take up the responsibilities and obligations in teacher education. A German researcher remarks the multi-dimensions of mathematics, as reflected in applied versus pure, divergent versus successive, algebra versus geometry, etc. He feels strongly that with the development of mathematical science, the internal relationship of all the elements should not be overlooked. More importantly, in the mathematical curricula of higher education, it is imperative that attention should be paid to the construction of models in the fields of science, technology and engineering, so mathematics will be able to regain its unification. An Italian researcher expresses her views on how to teach mathematics for non-majors of mathematics in colleges and universities, such as the students majoring in chemistry or physics. From what she went through in her teaching, she believes we should attach importance to the organic integration of mathematics and science in designing curricula and teaching, be adept in motivating the students for better results of their studies, and make teaching more valid. Another Italian researcher points out that an inefficiency of mathematics teaching in Italy is due — above all — to the absence of direct and personal "mathematical experiences" for many mathematics teachers in the Italian compulsory schools. She expresses her views that "doing mathematics" is a crucial step in mathematical teacher's training. The Japanese researchers present the problem in Japan: With the trend that the level of mathematical knowledge is lowering, Japanese students' enthusiasm in studying mathematics is declining. Japanese educators have started the curriculum reform in pre- and post-service teacher education to help teachers develop their ability in dealing with different kinds of problems occurring in education. The specific measures they have taken include making full use of information technology to provide opportunities for in-service teachers in their life-long study, creating international exchange programs for cooperation (for example, cooperation with Argentine teachers), promoting specialty development of individual teachers, introducing comprehensive studies to teachers, and further coordinating the relationships between mathematics and mathematics teaching. From the perspective of reforming the content of textbooks, an American scholar discusses the reform of designing the content of teaching, which he thinks is the key to successful reform in mathematics education. He also stresses the internal coordination of all elements within mathematics. A Chinese scholar analyzes the important role played by mathematical concepts in mathematics teaching, stressing the importance of mathematical concepts, the bridge linking logic and mathematics directly perceived from the senses.

We are grateful to all ICMI EC members for contributing their wisdom to this selection of papers despite the busy schedules; we would like to express our gratitude to the delegates participating in the Satellite Conference in Lhasa. They have turned the results of their research into papers and made contributions to the completion of this book. Thanks go also to all those who have

joined us in the efforts in preparing the two conferences. We are also appreciative of the financial supports from the Tianyuan Mathematical Foundation of CNSF, from East China Normal University and from University of Tibet, which made the above-mentioned two conferences possible. Finally, we must thank the East China Normal University Press for the efforts they made in publishing this selection of papers. It is our hope that the publication of this selection of papers will play a moderate part in facilitating exchanges between the circles of mathematics education in China and overseas.

Jianpan Wang
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Winter, 2003

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