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Albert C.J. Luo

Nonlinear Deformable-body Dynamics

非线性变形体动力学

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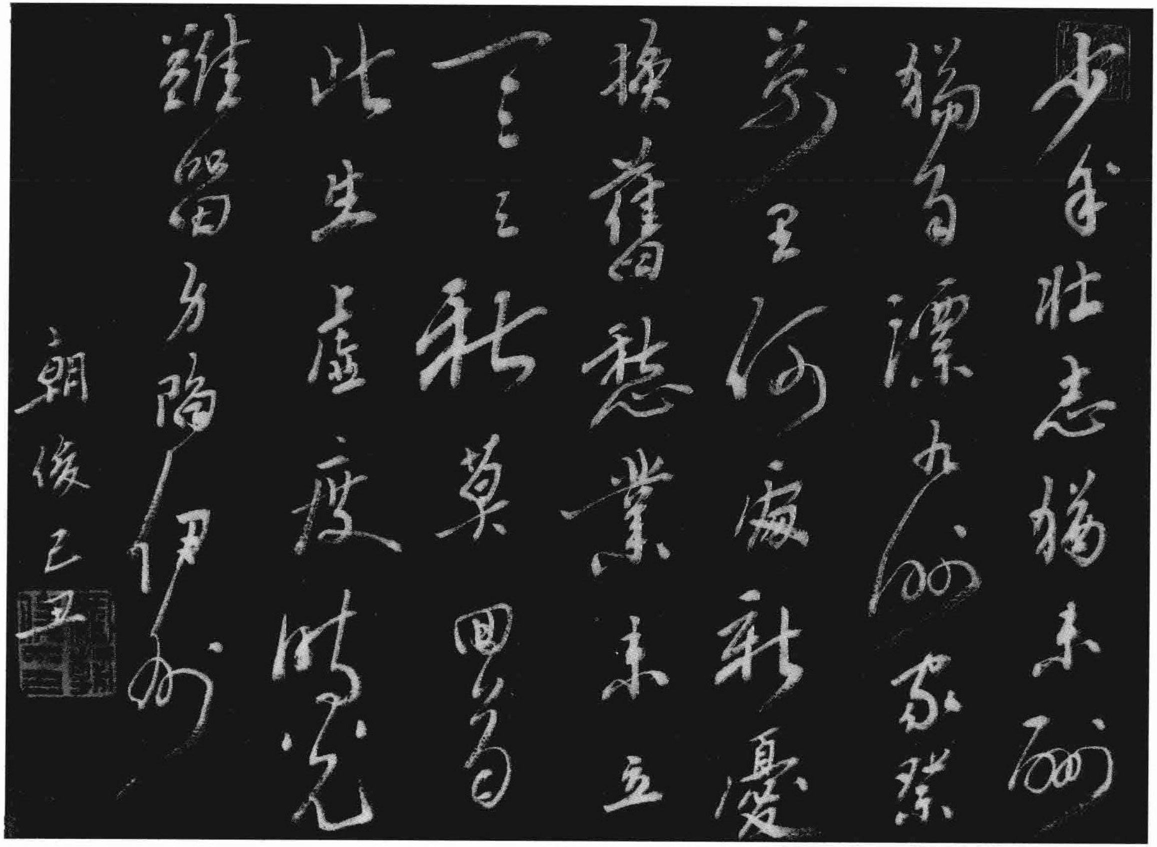

Homesick and disappointment without any achievements accompanies my life.

Preface

Deformable-body dynamics is a subject to investigate the states of strains and internal relative motions in deformable solids subject to the action of external forces. This is an old and interesting topic, and many problems still are unsolved or solved incompletely. Rethinking such problems in this topic may bring new vital to the modern science and technology. The first consideration of the nature of the resistance of deformable-bodies to rupture was given by Galileo in 1638. The theory of deformable-bodies, started from Galileo's problem, is based on the discovery of Hooke's Law in 1660 and the general differential equations of elasticity by Navier in 1821. The Hooke's law is an experimental discovery about the stress and strain relation. This law provides the basis to develop the mathematical theory of deformable bodies. In 1821, Navier was the first to investigate the general equations of equilibrium and vibration of elastic solids. In 1850, Kirchhoff proposed two assumptions: (i) that linear filaments of the plate initially normal to the middle-surface remain straight and normal to the middle-surface after deformed, and (ii) that all fibers in middle surface remain unstretched. Based on the Kirchhoff assumptions, the approximate theories for beams, rods, plates and shells have been developed for recent 150 years. From the theory of 3-dimensional deformable body, with certain assumptions, this book will present a mathematical treatise of such approximate theories for thin deformable-bodies including cables, beams, rods, webs, membranes, plates and shells. The nonlinear theory for deformable body based on the Kirchhoff assumptions is a special case to be discussed. This book consists of eight chapters. Chapter 1 discusses the history of the deformable body dynamics. Chapter 2 presents the mathematical tool for the deformation and kinematics of deformable-bodies. Chapter 3 addresses the deformation geometry, kinematics and dynamics of deformable body. Chapter 4 discusses constitutive laws and damage theory for deformable-bodies. In Chapter 5, nonlinear dynamics of cables is addressed. Chapter 6 discusses nonlinear plates and waves, and the nonlinear theories for webs, membranes and shells are presented in Chapter 7. Finally, Chapter 8 presents the nonlinear theory for thin beams and rods.

The purpose to write this book is to answer a question from Professor Huancun Sun (my thesis advisor) during my master thesis defense in 1990. In my master thesis, I considered the higher order terms to correct the strains in the von Karman plate theory. However, such a correction did not consider curvature effects on the balance equations. Professor Sun asked me what is the error compared to the exact theory of the 3-dimensional deformable bodies. After about 20 years, I believe that I can give an appropriate answer to his question. In fact, after my thesis defense, I almost place such a question away. However, in 1996, I worked with Professor C.D. Mote, Jr. at UC Berkeley on nonlinear dynamical behaviors of high speed rotating disks in disk drives. Such a problem drove me to rethink about the accurate plate theory. To express my indebtedness to both of them for their guidance and advice, this book is my presence for the 80th birthday of Professor Sun and the 70th birthday of Professor Mote, Jr.. This book is also dedicated to my friend and colleague, Professor Zhongheng Guo. His book on *Nonlinear Elasticity* stimulated my research interest in nonlinear deformable solids 25 years ago. His book was an excellent book for graduate students. Some inspirations of this book originated from the book of *Nonlinear Elasticity*. In addition, I would like to thank Professor Youjin Che for lending his books "*Tensor Analysis*" and "*Variational Principles*" to me during my sophomore year in 1981. After almost 30 years, I cannot find both of books to return to him. I sincerely hope this book can bring my apology and appreciation to him.

Herein, I would like to thank my wife (Sherry X. Huang) and my children (Yanyi Luo, Robin Ruo-Bing Luo, and Robert Zong-Yuan Luo) for their tolerance, patience, understanding and support.

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Chapter 1
Introduction

To investigate deformable-body dynamics, it is very important to learn a development history of the mathematical theory of deformable solids. From such a development history, one can find how the deformable-body dynamics to stimulate the development of modern physical science, which will give people a kind of indication for new discoveries. In this chapter, a brief history for establishing the approximate theories of deformable solids will be given. Especially, the cable dynamics will be discussed first, and a mathematical treatise of nonlinear beams and rods will be presented. In addition, the past and current status of plates and shell theory will be discussed, and the current status of soft web theory and applications will be presented. Finally, the book layout will be presented, and a brief summarization for each chapter will be given.

1.1. Deformable-body dynamics

Deformable-body dynamics is a subject to investigate the states of strain and internal relative motions in deformable solids subject to the action of the external forces. The first consideration of the nature of the resistance of solids to rupture was given by Galileo (1638). He treated the deformable body as inelastic without any laws and hypotheses between the displacement and forces. Galileo studied the resistance of a beam clamped at one end into the wall under its own weight or applied weight. He concluded that the beam rotates about the axis perpendicular to its length and in the plane of the wall. The determination of this axis is known as the Galileo's problem. The theory of deformable-bodies started from the Galileo's problem is based on the discovery of Hooke's Law in 1660 and the general differential equations of elasticity by Navier in 1821. The Hooke's law (Hooke, 1678) is an experimental discovery about the stress and strain relation. This law provides the basis to develop the mathematical theory of deformable-bodies. In 1821, Navier was the first to investigate the general equations of equilibrium and vibration of elastic solids, as presented in Love (1944). Although equilibrium and vibrations of plates and shells were treated before the general theory of elasticity was developed, one was interested in reduction of the general theory of elasticity to the theory of plates and shells by the power series of the distance from the middle surface. The problem is that the resultant forces and moments at the edge must be equal to the internal forces and moments generated by the strain. However, too many unknowns cannot be solved. Kirchhoff (1850a,b) proposed two assumptions: (i) that linear filaments of the plate initially normal to the middle-surface remain straight and normal to the middle-surface after deformed, and (ii) that all fibers in middle surface remain unstretched. Independent of the general equation of elasticity, the theory of the bending and twisting of thin rods and wires was developed by methods akin to those employed by Euler. One thought how to connect the general theory of elasticity to the theory of thin rods. Kirchhoff (1859) pointed out that the general equations of elasticity are strictly applicable to any small portion of a thin rod if all the linear dimensions of the portion are of the same order of magnitude as the diameters of the cross section. The equation of motion for such a portion of the rod could be simplified from the first approximation of deformation and kinematics. Based on the Kirchhoff assumptions, the approximate theories for beams, rods, plates and shells have been developed for recent 150 years. From 3-dimensional deformable body theory, with certain assumptions, this book will present a mathematic frame to develop such approximate theories for thin deformable-bodies including cables, beams, rods, webs, membranes, plates and shells. The theory for deformable body based on the Kirchhoff assumptions is a special case to be discussed. In this chapter, the development history for the theory of nonlinear cable dynamics will be discussed first.

1.1.1. Cable dynamics

Cables are used as one of the simplest structures for human being at least thousands of years. The cable configuration attracted scientists to investigate since it was used for the suspension bridge in the early human-being history. Based on the historical record, the sophisticated suspension bridges in China appeared before the start of the Christian era. The iron chain suspension bridge was built in Yunnan, China in A.D. 65 in Needham (1954). In 1586, Stevin established the triangle forces experimentally with loaded string to understand the catenary and the collapse mechanism in a voussoir arch, as reported in Hopkins (1970). From Truesdell (1960), Beeckman, in 1615, for the first time solved the suspension bridge problem that the configuration of hanged cables with the in-plane, uniformly distributed loading is a parabolic arc. Galileo mused on the shape of a hanging chain and concluded that it is parabolic primarily by analogy to the flight of a projectile, which was published in *Discourse on Two New Sciences* in 1638. However, it was proved that this view was incorrect as Bernoullis (James and John), Leibnitz and Huygens jointly discovered the catenary in Truesdell (1960). To solve the catenary, Huygens relied on the geometrical principle, and Leibnitz and Bernoullis used the calculus and Hooke's law to develop the general differential equations of equilibrium of a chain element under various loading. In addition, Bernoullis provided the basic fundamental of the calculus of variations to keep the center of gravity of the chain as low as possible. Furthermore, the principle of virtual work was developed. In the early 18th century, the vibration of taut string was extensively investigated to get the nature of the solution of partial differential equations. In 1738, Daniel Bernoulli (son of John Bernoulli) published a solution for natural frequencies of a chain that hangs from one end, and the solution was in the form of an infinite series (Watson, 1966). In 1764, Euler obtained the equation of motion for the vibrating taut membrane and obtained the infinite series solution through the variable separation. The partial solution was given by Poisson in 1829 and Clebsch in 1862. In addition, Lagrange in 1760 used the discretized string of beads model of the taut string as an illustration of the application of his equations of motion in Whittacker (1970). This work was done for the first time on the solution of vibration problems by the difference equations.

The equilibrium configuration, tension and displacement of elastic cables under arbitrary loading are needed in the design of cable structures. Rohrs (1851) first modeled the vibration of a uniform, inextensible suspended chain hanging freely under its own weight and obtained the approximate natural frequencies and responses of the cable. Routh (1884) considered the symmetric transverse vibration of a heterogeneous chain hanging in the form of a cycloid, and application of this chain model to the uniform chain yielded the Rohrs' model when the sag ratio is small. The chain was still modeled as inextensible. Pugsley (1949) developed a semi-empirical theory for the in-plane natural frequencies of the first three modes of a uniform, inextensible suspended chain. Saxon and Cahn (1953) developed an asymptotic method for the natural frequencies of the chain for large sag to span ratios. Simpson (1966) investigated the in-plane vibration of a stretched cable through its equilibrium and also determined the natural frequencies of multispan, sagged transmission lines using the transfer matrix method. Irvine and Caughey (1974) used a similar approach to investigate the free vibrations of a sagged, stretched cable hanging under its own weight. Hagedorn and Schafer (1980) showed that geometrical nonlinearity is significant in the computation of natural frequencies of in-plane vibration of an elastic cable. Luongo et al. (1984) analyzed the planar, non-linear, free vibrations of sagged cables through a perturbation method. Perkins (1992) considered the nonlinear vibrations of 3-dimensional, elastic, sagged cables analytically and experimentally, and gave a brief review of recent developments in cable dynamics. For translating cables, Simpson (1972) investigated planar oscillations by the linearized equations of motion around the equilibrium. Triantafyllou (1985) used an alternative approach to derive the linearized equations of motion at the equilibrium. Perkins and Mote (1987, 1989) developed a 3-dimensional cable theory for traveling elastic cables. The natural modes for the vibration and stability of translating cable at equilibria were obtained from the eigensolutions of discretized continuum models, and also some experimental results were reported. Luo et al (1996) presented the analytical solution and resonant motion for a stretched, spinning, nonlinear tether.

The non-straight equilibria of the cable have been determined by approximate means. For the stationary cables or strings, Dickey (1969) investigated the nonlinear string under a vertical force and gave tensile and compressive equilibrium solutions. Antman (1979) extended the Dickey investigation and investigated comprehensively the existence, multiplicity and qualitative behaviors of equilibrium for nonlinear elastic strings under different loads. The translating, sagged string possesses two non-trivial equilibrium states because of centrifugal loading. O'Reilly and Varadi (1995) investigated the equilibria of translating elastic cables. O'Reilly (1996) showed that if one used an observation due to Routh (1884) for inextensible strings, then the work of Antman (1979) and Dickey (1969) on static equilibria for strings can be extended to examine the steady motions of these strings. Healey and Papadopoulos (1990) extended the inextensible cable results to all the elastic strings. O'Reilly (1996) obtained the steady motion and stability of elastic and inextensible strings, and it was also shown that multiple steady motions were possible. In the quantitative investigation of elastic cables, Irvine (1981) used the method of Dickey (1969) to determine the exact equilibrium configuration and the approximate displacements of 2-dimensional cables under positive tension. For a single concentrated vertical load, the predicted displacement is constrained by the assumption that the equilibrium configuration is parabolic and that the ends of the cable are fixed. For multiple concentrated masses, the solutions given by Irvine (1981) require specificity of the initial configuration. To overcome these limitations, Yu et al. (1995) followed Irvine's procedure and computed the tension and equilibrium configuration of a 3-dimensional cable under uniform and concentrated transverse loading. The aforementioned exact solutions describe the equilibrium but not the deformation displacement because the initial configuration is not known. Luo and Mote (2000a) developed a nonlinear theory for traveling, arbitrarily sagged, elastic cables, and the closed-form equilibrium solution and existence were developed analytically. To investigate dynamics of nonlinear sagged, elastic cable, the dynamics of the inextensible cables should be investigated. Luo and Wang (2002) gave a series solution for the oscillation of the traveling, inextensible cable. Wang and Luo (2004) presented an alternative analytic solution for the motion of the in-plane, traveling inextensible cable. This analytical solution is also valid for the traveling speed over the critical speed. Based on dynamics of the inextensible cables, the dynamics of the sagged cables can be determined.

1.1.2. Beams and rods

Galileo (1638) studied the resistance of a beam clamped at one end into the wall under its own weight or applied weight, which caused modern science to develop. Through waves and vibrations in deformable-bodies, one understood the light and sound propagations. Before the theory of elasticity based on the Hooke's law and Navier's general differential equations for deformable-bodies, one investigated the theory of the bending and twisting of thin rods and wires. To obtain the solutions and extension of the Galileo's problems, the related, approximate theories for the vibration of bars and plates and the stability of columns were developed between 1638 and 1821. The 1-dimensional rod theory from 3-dimensional models by averaging the stress on the cross section was introduced by Leibniz in 1684. Since then, the first investigation of the elastic line or elastica was presented by James Bernoulli in 1705. In that research, the resistance of the bent rod is assumed to arise from the extension and contraction of its longitudinal fibers in the elastica, and the equation of the curve assumed by the axis is given, in which the resistance to bending is a bending moment proportional to curvature of the rod as bent. Once the concept about the bending moment perpendicular to curvature was established, the work done in bending a rod is proportional to the square of its curvature. Danial Bernoulli suggested to Euler that the differential equation of the bent rod can be obtained by minimizing the integral of the square of the curvature along the rod. From that suggestion, in 1744, Euler obtained the differential equation of the bent rod and classified the various form for such a problem. From this problem, Euler worked on what is the least length of elastica to bend under its own weight or applied weight (distributed force). Following the Euler theory, Lagrange determined the strongest form of column. Such an idea is a base for the variational principle, and such research is the earliest research on elastic stability. In the Euler's investigation, the rod was assumed as a line of particles to resist bending. In 1776, Coulomb considered the cross section of rod to present the flexure theory of beams and investigated the torsion of the thin rods. The theory improved the rod theory presented by Daniel Bernoulli and Euler. The concept of shear was proposed for the first time. From variation of energy function, the differential equations for the transverse vibration of bars were obtained by Euler and Daniel Bernoulli, and the vibration of rods with different boundary conditions was discussed. In 1802, Chldni presented an investigation of those modes of vibrations, and discussed the longitudinal and torsional vibrations of the bar. Based on the Hooke's law, in 1821, Navier developed the general differential equation for the theory of elasticity. Since the theory of rods was independently developed, one thought how to connect the general theory of elasticity to the theory of thin rods. Kirchhoff (1859) pointed out that the general equations of elasticity are strictly applicable to any small portion of a thin rod if all the linear dimensions of the portion are of the same order of magnitude as the diameters of the cross section. The equation of motions for such a portion of the rod could be simplified from the first approximation of deformation and kinematics. The earlier beam theories were developed by Kirchhoff (1859) and Clebsch (1862), as also presented in Love (1944). The comprehensive history of elasticity can be found in Todhunter and Pearson (1960) and Truesdell (1960).

Since 1940's, one has been interested in the systematic development of rod theories from 3-dimensional continuum mechanics. Hay (1942) obtained the strain from the power series in a thickness parameter. Novozhilov (1948) developed nonlinear theory for a rod with a large deformation. The other approximate theories for 1-dimensional rods or bars were presented by Midlin and Herrmann

(1952), Volterra (1955, 1956, 1961), Midlin and McNiven (1960) and Medick (1966). The theories were used to investigate the wave propagation and vibrations. On the other hand, to develop a theory of rod based on a 1-dimensional continuum model, E. and F. Cosserat (1909) introduced a concept of four vector fields to describe deformable vectors (directors) at a point of the directed and oriented curve. Ericksen and Truesdell (1958) used the Cosserat approach to develop a nonlinear theory of stress and strain in rods and shells through the oriented bodies. Cohn (1966) developed a static, isothermal theory of elastic curves. Whitman and DeSilva (1969) followed the Cohn's work to obtain the dynamical case and gave an explicit expression for the director inertia terms, and DeSilva and Whitman (1971) presented a thermo-dynamical theory for the directed curves with constitutive equations of materials. Such a theory can reduce to classic elastica and the linear theory of the Timoshenko beam when the assumptions were introduced to the corresponding theories, and an exact solution for such a nonlinear theory rods was presented (e.g., Whitman and DeSilva, 1970, 1972, 1974). On the other hand, Green (1959) presented the exact equilibrium equations for resultant force and moments by integration of the 3-dimensional equations over the cross section. Green and Laws (1966) extended this concept and developed a general theory of rods through two directors at each point in rods which requires specification of three vector fields. Antman and Warner (1966) used the polynomials in transverse coordinates to express the location of particle in rods and obtained the equation of motion with powers of the transverse coordinates for hyperelastic rods. Green, Laws and Naghdi (1967) used the idea of Green and Laws (1966) to present a linear theory of straight elastic rods, and Green, Knops and Laws (1968) used the same treatment for small deformation superimposed on finite deformation of elastic rods. A more detailed discussion of rod theories with directors can be referred to Antman (1972). Reissner (1972, 1973) developed a 1-dimensional finite-strain, static beam theory but how to treat the moment was not given. Wempner (1973) presented mechanics of curved rods, but the strain is the Almansi-Hamel strain. The strain energy of nonlinear rods was presented in Berdichevsky (1982). Maewal (1983) gave strain-displacement relations in nonlinear rods and shells. Danielson and Hodges (1987) discussed nonlinear beam kinematics through the deposition of the rotation tensor, and a mixed variational formulation for dynamics of moving beams was presented in Hodges (1990). Simo and Vu-Quoc (1987, 1991) used the exact strain to develop a theory for geometrically-nonlinear, planar rods, and several higher-order approximate theories were also given. Recently, this approach was used for development of the 3-D composite beam theory and numerical approaches were developed for prediction of dynamic responses in Vu-Quoc and Ebcioglu (1995, 1996) and Vu-Quoc and Deng (1995, 1997). The other derivation of equations of motion for geometrically-nonlinear rods can be referred to Crespo da Silva and Glynn (1978a), Crespo da Silva (1991), Pai and Nayfeh (1990, 1992, 1994).

The vibration of nonlinear, planar rods based on an accurate beam theory was investigated through a perturbation approach in Verma (1972). The free, nonlinear transverse vibration of beams was investigated in Nayfeh (1973) when the beam properties varied along with length. Ho et al. (1975, 1976) discussed the nonlinear vibration of rods through a single mode model and a perturbation approach. The forced vibration of nonlinear, torsional, inextensional beams was investigated in Crespo da Silva and Glynn (1978b). The planar, forced oscillations of shear in deformable beams were investigated through a specific, single-mode response and perturbation method in Luongo et al. (1986) and the planar motion of an elastic rod under a compressive force was analyzed in Atanackovic and Cveticanin (1996). Holmes and Marsden (1981) used the Melnikov method to investigate the chaotic oscillation of a forced beam. Maewal (1986) investigated chaotic motion in a harmonically excited elastic beam through the perturbation approach and Lyapunov exponent method. The dynamical potential for the nonlinear vibration of cantilevered beams was discussed in Berdichevsky et al. (1995), and the numerical simulations of chaotic motions in non-dampened nonlinear rods were also presented. Luo and Han (1999) presented the nonlinear equations of an in-plane rod to investigate its chaos. In practical applications, one often used the approximate theories to discuss the deformations and vibration of nonlinear rods and beams. In recent decades, in order to more accurately describe DNA structures and micro-electromechanical-systems (MEMS), one tried to revisit the theory of rods. The nonlinear theory of rods in Kirchhoff (1859) was revisited. Tsuru (1987) discussed equilibrium shape and vibrations of thin elastic rods. Coleman and Dill (1992) discussed the flexure waves in elastic rods (also see, Coleman et al., 1993). Tobias and Olson (1993) used a homogeneous inextensible elastic rod with a uniform cross section to describe a segment of DNA (also see, Coleman et al., 1995, 1996; Swigon et al., 1998). Lembo (2001) discussed the free shapes of elastic rods, and Colemen and Swigon (2004) presented the theory of self-contact in Kirchhoff rods with applications in supercoiling of knotted and unknotted DNA plasmids. Recently, the Cosserat theory of elastic rods was used to model MEMS (e.g., Cao et al., 2005; 2006), and the systematic description of elastic rod based on the Cosserat theory was presented in Cao and Tucker (2008). From the aforementioned survey, it is very important to develop an accurate theory for beams and rods. This book will present a theoretic frame for one to develop accurate theories for beams and rods.

1.1.3. Plates and shells

In the 17^{th} century, based on special hypotheses, the theories of thin rods were developed. In the same fashion, the theory for plates and shells could be developed. Euler was the first to consider the plate consisting of annuli bars. In fact, the linear bending theory of plates was really developed by Kirchhoff (1850a,b) from his assumptions for the theory of thin rods. Love (1888) developed the linear theory of shells from the 3-dimensional equation of linear elasticity, as also presented in Love (1944). The nonlinear strains were determined by the first-order approximation of the extension. Such a theory originated from the small free vibration of a thin elastic shell in Love (1888). Such a work drew the criticism from Rayleigh (1888) because such an extensional deformation theory of shells is against the inextensional deformation theory. Lamb (1890) used the alternative way to derive the same equations as in Love (1888) and Basset (1890) considered the higher-order terms of the extension for thin cylindrical and spherical shells. To solve this argument, around 1940, with the framework of the Kirchhoff-Love assumption, Chien (1944 a,b) presented an intrinsic theory of plates and shells. Gol'denveizer (1944) discussed the applicability of the general theorems of the theory of elasticity to the thin shells. Reissner (1944) introduced the deformation caused by shear strain into the bending of elastic plates through an assumed displacement field. Discussion on the developments of the linear theory can be referred to Naghdi (1972), and other books.

The 3-dimensional thin continuous medium can be described by a 2-dimensional surface with a director. Such a concept of the continuous and oriented media was initialed by Duhem (1906). E. and F. Cosserat (1909) extended such concepts to develop the theory for shells and rods. Such a concept provides a base for development of the field theory for plates and shells. In addition, the existing approximate nonlinear theories for plates and shells have been derived from the 3-dimensional equations. In the early stage, it was assumed that the strain is very small but the rotation is large or moderately large, and the linear constitutive equations are assumed to be valid. von Karman (1910) extended the Love's strain based on the first order approximation of extension and developed an approximate theory for plates, and von Karman and Tsien (1939, 1941) used such approximate theory to investigate the buckling of thin spherical and cylindrical shells by external pressure. However, Galerkin (1915) discussed series solutions of some problems of elastic equilibrium of rods and plates. Novozhilov (1941) presented a general theory for stability of thin shells, and followed Galerkin's idea systematically presented the nonlinear theory for elasticity in Nolvozhilov (1948) or Nolvozhilov (1953) (English version). Following the von Karman theory, Reissner (1957) presented his nonlinear plate theory including shear deformation. Herrmann (1955) derived a plate theory governing dynamic motion with small elongation and shear deformation but moderately large rotation. Wang (1990) developed the 2-dimensional theory reduced from the 3-dimensional theory for transversely isotropic plates. Hodges et al. (1993) developed the geometrically nonlinear plate theory through the warping displacement. Since von Karman (1910) developed a nonlinear theory for thin plates with large deflection, ones used that nonlinear theory to investigate the buckling stability (e.g., Levy, 1942) and the nonlinear vibration of a spinning disk (e.g., Nowinski, 1964, 1981).

Based on the concept of continuous and oriented media, Ericksen and Truesdell (1958) presented a general development of the kinematics of the oriented media through n-stretchable directors in the n-dimensional space. The concept of directors was introduced. Truesdell and Toupin (1960) gave an exposition of the kinematics of the theory of oriented bodies. The 3-dimensional theory of an oriented medium with a single deformable director at all points of the body was developed in Green, Naghdi and Rivlin (1965). Cohen and DeSilva (1966) used the kinemat-