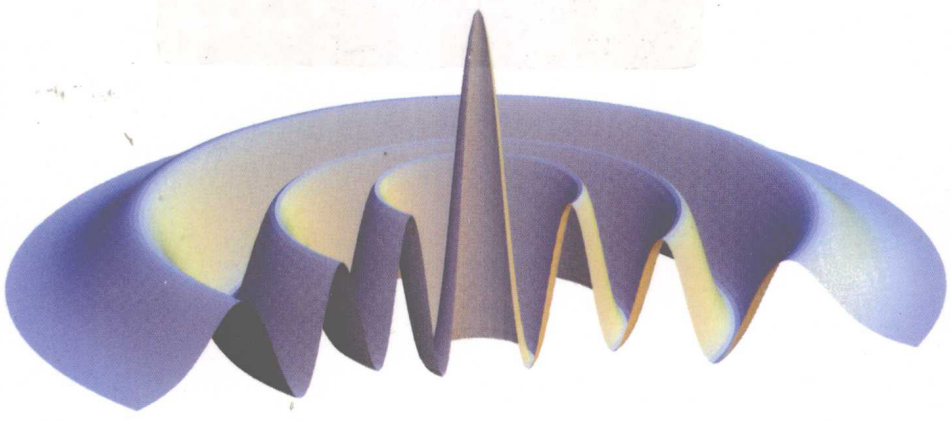
Wolfgang P. Schleich

Quantum Optics in Phase Space

相空间中的量子光学



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"FAITH MAINTAINED IS ONE OF THE GREAT GIFTS BESTOWED BY FELLOW MAN"

Dedicated to the two people who always had faith that this book would be completed

Kathy and Michael

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Preface

During the winter semester of 1992/93 I taught for the first time the course *Quantum Optics I* at the University of Ulm, which was followed by part II in the summer semester of 1993. When I offered the course a second time the University was kind enough to financially support two diplom students, Erwin Mayr and Daniel Krähmer, who had already taken this class in the previous year to transform my hand-written notes and sketches of drawings into a legible form. Erwin and Daniel have done a tremendous job. Since then I have taught this course many times and collected more and more material which was included into this manuscript by other graduate students of the Abteilung. It has served many generations of students at the University of Ulm as a first introduction to the field of quantum optics.

During one of his many visits to Ulm, Michael Poulson, a close friend from the VCH-Wiley publishing house saw the manuscript on my desk. "I want to publish these notes" was his immediate reaction. Michael had complete faith that this manuscript would eventually be turned into a publishable book. He wanted the material to be expanded to include problems, experiments and an exhaustive list of references. The goal was to convert the existing manuscript of about 150 pages into a book of about 250 pages. His trust in me was so great that he started advertising *Quantum Optics in Phase Space* before we had even signed a contract. I believe the present result satisfies the criteria Michael had put forward with one exception – the number of pages.

At Christmas of 1996 we finally signed a contract and Michael was extremely relieved. I still remember his words "now I have finally succeeded in signing you up for the book". A week later his untimely death during Christmas vacation added a new meaning to this sentence and a purposeful dimension to his faith and expectation; I was determined more than ever to deliver what I had promised.

Eventually Erwin and Daniel graduated and their new professional life did not allow them to devote more time to continue the project. Since that fateful Christmas of 1996, many students have helped me transform my class notes into various sections of the book continuing the work that Erwin and Daniel had begun. Stephan Meneghini took over and for several years he was instrumental in typing the manuscript. But also he graduated during the course of the project. In the final phase of the book his role was taken over by Florian Haug. I am enormously grateful to all of them for their assistance. What started out with 200 pages at Erwin and Daniel's departure eventually expanded and reached its present 700 page size.

Similarly, the field of quantum optics has expanded enormously over the last 10 years. This fact reflects itself in the variety of textbooks that have been published

on this topic. It is impossible to represent all branches of this rapidly moving field in one single book. As a consequence many current topics are left out in the present one, such as quantum information or Bose-Einstein condensation. The main theme of the book is quantum phase space and the application of semi-classical concepts, such as WKB techniques to problems of quantum optics. In the present American hype of the "e-mail and the information highway" some people have suggested to call the book "phase-space.com."

Many friends and colleagues have read through various parts of the book and have made useful comments. In this regard I want to mention especially I. Bialynicki-Birula, J.H. Eberly, H.J. Kimble, D. Kobe, R.F. O'Connell, H. Walther, K. Wódkiewicz and E. Wolf. Special thanks go to M. König who has worked very carefully through the whole book and has made numerous constructive remarks. In the final stage all members of the Abteilung have proofread the entire book. Many thanks to G. Alber, M. Bienert, M. Cirone, O. Crasser, A. Delgado, D. Fischer, M. Freyberger, F. Haug, V. Kozlov, H. Mack, W. Merkel, G. Metikas, M. Mussinger, K. Vogel, J. Wichmann and V.P. Yakovlev. K. Vogel was also instrumental in putting the index together. I am grateful to my secretaries B. Casel, R. Knöpfle and U. Thomas who were helpful in collecting the literature.

Various chapters of the book have been tested in two lecture series given at the University of Texas at Austin. The penetrating questions of J.H. Eberly, M. Fink, D. Heinzen, J. Keto, M. Raizen, W.C. Schieve and E.C.G. Sudarshan have helped to sharpen my arguments through the extremely lively discussions during and after the classes. They have enormously helped to improve the presentation of the material. The kind hospitality of and the always friendly atmosphere at the physics department at UT Austin are greatly appreciated.

Many science organisations have supported the research summarized in the present book. In this context I want to mention especially the Deutsche Forschungsgemeinschaft and the Leibniz Program, the European Community, the Heraeus Foundation, the Humboldt Foundation and the University of Ulm. All have graciously financed my students, assistants and visitors. Many thanks to all of them.

The quiet periods in Denton, Texas with my understanding father-in-law, H.C. Phillips who always refers to me as his "blue electron son-in-law" were very conducive to completing this book. Moreover, I greatfully acknowledge the kind hospitality at the physics department at North Texas State University, Denton.

Last, but not least, I want to express my sincere thanks to my teachers. G. Süßmann, whose lectures at the Ludwig-Maximilians-Universität in München woke my interest in theoretical physics and made me change my degree from high school teacher to physicist. Süßmann's broad and deep interest in the whole field of physics and not just a special area has always impressed me and hopefully this book reflects his influence. M. O. Scully and H. Walther have introduced me to the field of quantum optics 20 years ago. I was fortunate enough to closely work with them on various problems of quantum optics and they have strongly influenced my view of the field. Through my collaborations with them I have gained many insights. A different angle of physics came through my years in Texas working with J.A. Wheeler. He taught me that many phenomena in physics become transparent when viewed using WKB techniques combined with the concept of phase space. In this sense the origin of this

book stems from my years in Austin, Texas working with John on interference in phase space.

Special thanks go to my publishers Wiley-VCH and, in particular, to the innocent successor of Michael Poulson, Michael Bär, for his patience in awaiting the final outcome of *Quantum Optics in Phase Space*. Indeed they have suffered along with me in my trials of writing a comprehensive textbook on the application of phase space to quantum optics.

Above all I want to thank my parents, who encouraged me to think deeply and who made it possible for me to get the education necessary to pursue my studies. A special thanks goes to my wife Kathy and Michael Poulson, who never gave up their faith in me that this book would ever be finished even when other people close to me have made bets that the book would never (or not) be completed before the year 2050. Michael Poulson once said "I am not worried about the book being finished because Kathy will make sure you get it done for both of us". With these fateful words he was right; may he rest in peace.

Wolfgang P. Schleich
Ulm, November 2000

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