



高等学校土木建筑工程类系列教材

# 结构力学 (双语教材·第二版)

■ 袁文阳 周剑波 编



WUHAN UNIVERSITY PRESS

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## 序

建筑业是国民经济的支柱产业,就业容量大,产业关联度高,全社会 50% 以上的固定资产投资要通过建筑业才能形成新的生产能力或使用价值,建筑业增加值占国内生产总值较高比率。土木建筑工程专业人才的培养质量直接影响建筑业的可持续发展,乃至影响国民经济的发展。高等学校是培养高新科学技术人才的摇篮,同时也是培养土木建筑工程专业高级人才的重要基地,土木建筑工程类教材建设始终应是一项不容忽视的重要工作。

为了提高高等学校土木建筑工程类课程教材建设水平,由武汉大学土木建筑工程学院与武汉大学出版社联合倡议、策划,组建高等学校土木建筑工程类课程系列教材编委会,在一定范围内,联合多所高校合作编写土木建筑工程类课程系列教材,为高等学校从事土木建筑工程类教学和科研的教师,特别是长期从事土木建筑工程类教学且具有丰富教学经验的广大教师搭建一个交流和编写土木建筑工程类教材的平台。通过该平台,联合编写教材,交流教学经验,确保教材的编写质量,同时提高教材的编写与出版速度,有利于教材的不断更新,极力打造精品教材。

本着上述指导思想,我们组织编撰出版了这套高等学校土木建筑工程类课程系列教材,旨在提高高等学校土木建筑工程类课程的教育质量和教材建设水平。

参加高等学校土木建筑工程类系列教材编委会的高校有:武汉大学、华中科技大学、南京航空航天大学、南昌航空大学、湖北工业大学、汕头大学、南通大学、江汉大学、三峡大学、孝感学院、长江大学、昆明理工大学、江西理工大学、江西农业大学、江西蓝天学院 15 所院校。

高等学校土木建筑工程类系列教材涵盖土木工程专业的力学、建筑、结构、施工组织与管理等教学领域。本系列教材的定位,编委会全体成员在充分讨论、商榷的基础上,一致认为在遵循高等学校土木建筑工程类人才培养规律,满足土木建筑工程类人才培养方案的前提下,突出以实用为主,切实达到培养和提高学生的实际工作能力的目标。本教材编委会明确了近 30 门专业主干课程作为今后一个时期的编撰、出版工作计划。我们深切期望这套系列教材能对我国土木建筑事业的发展 and 人才培养有所贡献。

武汉大学出版社是中共中央宣传部与国家新闻出版署联合授予的全国优秀出版社之一,在国内有较高的知名度和社会影响力。武汉大学出版社愿尽其所能为国内高校的教学与科研服务。我们愿与各位朋友真诚合作,力争使该系列教材打造成为国内同类教材中的精品教材,为高等教育的发展贡献力量!

高等学校土木建筑工程类系列教材编委会

2008 年 8 月

## 前 言

本书被列入武汉大学“十五”规划教材，系武汉大学出版社组织出版的高等学校土木建筑工程类系列教材之一，在2002年底投入编写。本着勇于探索、有所创新的原则，书中在以下几个方面做了新的尝试和安排：

一、以原结构力学专业英语教材为基础，增加了实质性的解题内容和理论分析，有别于原结构力学专业英语教材的阅读材料性质。

二、参考了大量的国内外同类教材，在编排上尽量与本科结构力学教学的格局相同，并做了比较性的探索，对于同一教学内容，加进了国外教材与本国教材不同的解题方法。

三、把提高学生的英语水平作为本教材的目的之一，加进了科技英语的口语表达，此外还增加了数学公式和图表的文字表述（这由教师在课堂教学中体现）。

教材建设工作需要长期积累，同时又需要不断总结和翻新。本书作为正式教材出版还是第一次，其中多有不尽如人意的地方，恳请各方指正。

本书由袁文阳统筹，周剑波选材，历时一年编写完成。陈亚鹏和周艳国参与了校对工作。

感谢武汉大学出版社的李汉保老师、谢群英老师的热情支持和大力协助。

作 者

2004年3月于珞珈山

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# CHAPTER 1 Introduction

## 1.1 Statics of Structures Defined

By definition<sup>①</sup>, a structure (especially, an engineering structure) is anything built by man. This covers buildings, bridges, power-line supports, storage tanks, railway carriages and wagons, trucks, airplanes, and a multitude of other things. In the narrower sense, a structure is the load-bearing part of a building, bridge, etc.

In this text, we shall treat *a structure* as any system of interconnected rigid bodies (members).

The requirements that a structure must satisfy may be summed up as follows. Above all, it must be immovable with respect to the ground (or its equivalent) and retain its original geometry throughout its service life. Also, it must be sufficiently strong, stiff, and stable so as to offer adequate resistance to the imposed loads and to keep deformations within safe limits. Finally, it must be economical of materials and inexpensive to erect.

To meet the above requirements the structural engineer must be able to propose a suitable structure, to examine its overall stability and, finally, to calculate structural forces and deformations, no matter what materials (elastic or nonelastic), loads (static or dynamic), and calculation techniques are involved or used. This procedure comes under the heading of structural engineering. It widely draws on the techniques and mathematics of strength of materials and the theory of elasticity and plasticity.

The subject dealing with the calculation of reactions (that is, forces and moments) and deformations (that is, translations and rotations) in structures due to applied loads is known as structural analysis.

The branch of structural analysis concerned with the methods of analyzing structures for strength, stiffness, and stability under loads applied statically<sup>②</sup> is termed *statics of structures*. This will be the subject-matter of the present book.

Statics of structures is closely associated with engineering mechanics and, as already noted, strength of materials. Strength of materials is, in turn, based on knowledge supplied by engineering mechanics and is concerned with the analysis of structural members for strength,

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① See *A dictionary of Civil Engineering* by John S. Scott. — Translator's note.

② In more detail this is discussed in Sec. 1.6.

stiffness, and stability. Statics of structures applies the techniques carried over from strength of materials and engineering mechanics to the statical analysis of structures, and serves as the foundation for related subjects in civil engineering.

To sum up, the structural analyst's contribution consists in the choice of optimal structural configurations, preliminary analysis of likely alternatives and final analysis for internal forces, elastic deformations caused by external factors, and for overall stability.

Statics of structures is not concerned with stress-strain relations as such. Nor is it supposed to assign sections to members; both are assumed to be found from strength of materials in the course of design.

It should be clear that statics of structures is an applied science and is primarily a tool for good design rather than an end in itself. This above all implies orientation towards advanced techniques in both analysis and design, and towards economy in both materials.

## 1.2 Basic Simplifying Assumptions

The basic simplifying assumptions employed in structural analysis cover the structure as a whole. They are as follows:

1. Within certain loading limits, the material of a structure is assumed to be ideally elastic. In other words, once a load has been removed, it leaves behind no deformation.

2. The displacements of various points of a structure, caused by elastic deformation, are assumed to be rather small compared with the size of the structure itself.

This implies that any change in the distribution of forces due to deformation may be ignored when setting up equilibrium equations (that is, in finding constraint reactions and/or internal forces). The analytical theory based on the premise of small structural deformations is called small deformations theory.

3. Within certain loading limits, the displacements of various points of an elastic structure are assumed to be directly proportional to the forces that cause these displacements. Such structures are referred to as linearly elastic.

4. Linearly elastic structures obey the principle of superposition which is an outcome of Hooke's law. This principle states that:

The forces and the deformations, in a linearly elastic structure, caused by the joint action of loads, are the algebraic sum of the effects of the same loads produced individually, irrespective of the sequence of their application.

## 1.3 The Structural Model

Real structures are usually much too complex for rational analysis; often, they have to be reduced to simplified models prior to quantitative treatment. This modeling is one of the most important jobs of the analyst and requires experience and judgement so that the resulting model



strikes a happy compromise between reality and simplicity. In this book, the numerous "structures" discussed and shown are really only models of the real things.

Basically, a model of a structure is a simplified picture of the main factors governing its behavior under load. The correct choice of a model is a complex and critical task, and depends on the accuracy of analysis required.

As an illustration, consider a single-span railway bridge (Fig. 1.1(a)) which generally consists of two vertical plane trusses joined together by lateral braces and a deck. The deck is assembled from floor beams with their ends fixed to the trusses, and stringers which are connected to and rest on the floor beams. The truss members are rigidly welded or riveted to one another at the ends. The bridge carries a vertical load due to the self-weight of the train, and a horizontal wind load.

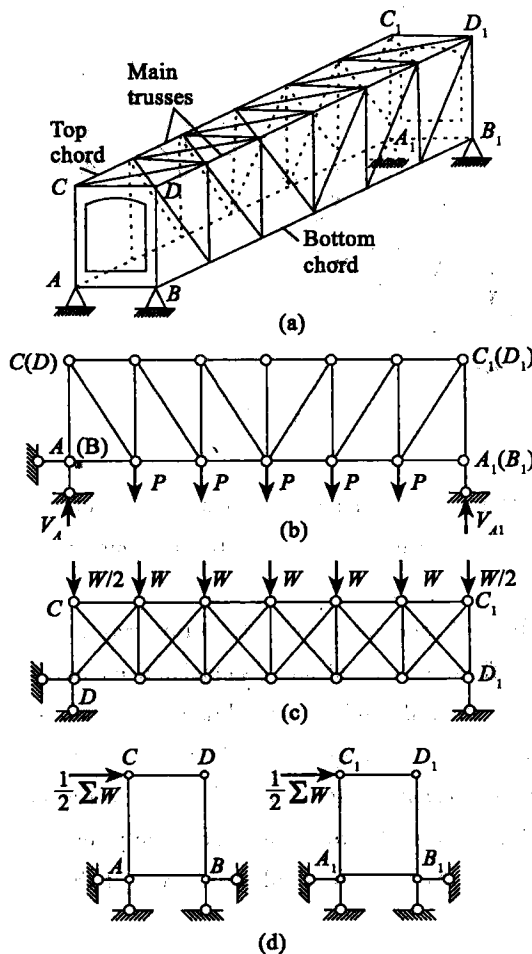


Fig. 1.1

(1) two vertical trusses  $ACC_1A_1$  and  $BDD_1B_1$  whose structural models are shown in Fig. 1.1(b);

- (2) a horizontal truss  $CC_1D_1D$  lying between the top chords of the main vertical trusses and resisting wind loads (its structural model is shown in Fig. 1.1(c));
- (3) two lateral supporting frames  $ACDB$  and  $A_1C_1D_1B_1$ , whose structural models are shown in Fig. 1.1(d).

## 1.4 Classification of Structures

Structures may be classified in various ways:

(1) According to their components which lie in a plane or in space, structures may be categorized as:

(a) *Plane structures*. These lie in one plane which also contains their loads (Fig. 1.2). Only plane structures are discussed in this text.

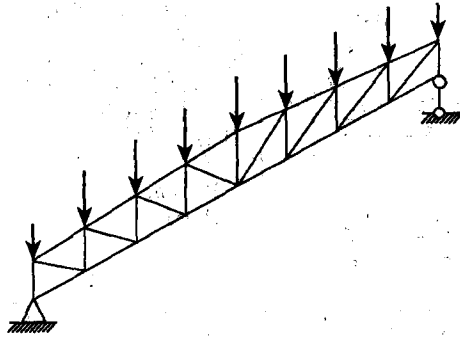


Fig. 1.2

(b) *Space structures*. These lie in space, and loads may act on them along any direction (Fig. 1.3(a) and (b)). Such structures have only been mentioned to make the discussion complete.

(2) According to the type of their members, structures may further be classified into:

(a) *Framed structures*. These consist of one-dimensional members, that is, those for which one dimension is much larger than the other two. Framed structures include beams, trusses, frames, and arches (Fig. 1.4(a), (b), (c), and (d)).

(b) *Thin-walled (two-dimensional) structures*. The thickness of such structures is much smaller than the other two dimensions. There are also commonly referred to as plate structures, if their members are plates (Fig. 1.5(a)), or shell structures, if their members are shells (Fig. 1.5(b)).

(c) *Massive structures*. For similar reasons, these are also termed three-dimensional structures. They include retaining walls (Fig. 1.6), masonry vaults (Fig. 1.7), dams, and footings analyzed and designed per metre run.

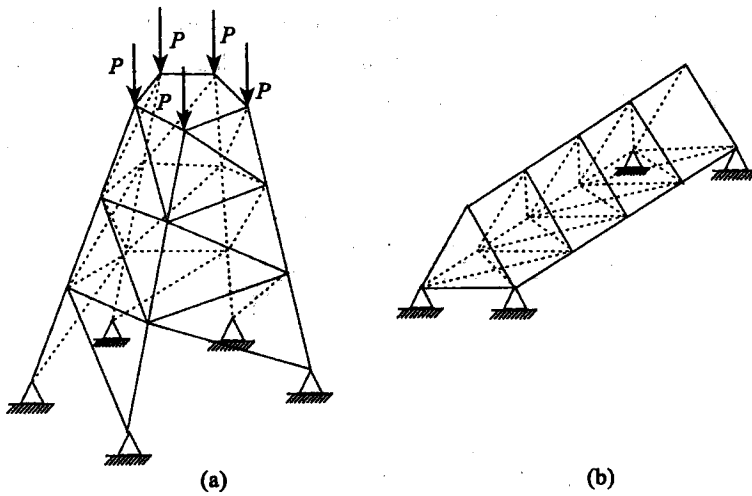


Fig. 1.3

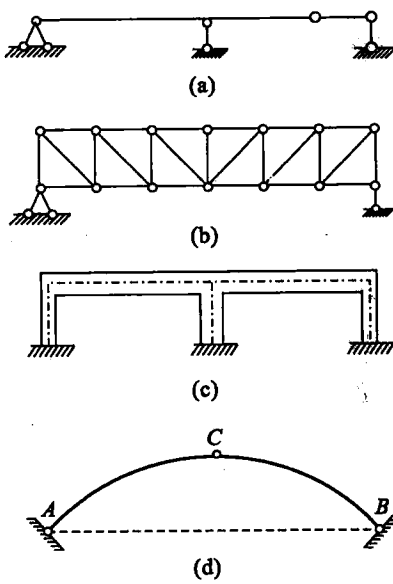


Fig. 1.4

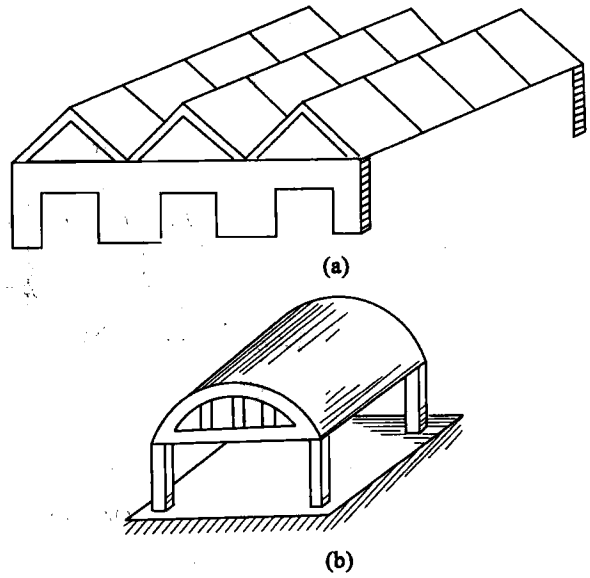


Fig. 1.5

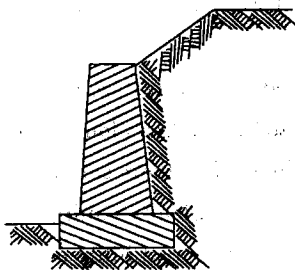


Fig. 1.6

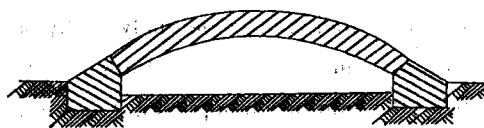


Fig. 1.7

(3) According to the direction of their support reactions, structures may be classified into:

(a) *Thrust-free structures*. When subjected to a vertical load, these develop only vertical supports reactions (Fig. 1.8(a), (b), and (c)).

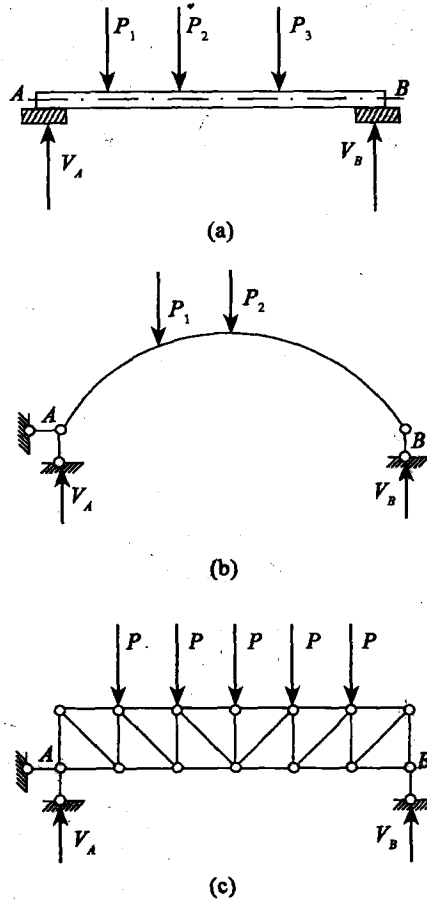


Fig. 1.8

(b) *Thrust-developing structures*. These develop inclined support reactions which may be resolved into vertical and horizontal components. The latter component is called the thrust. Thrust-developing structure includes arches, vaults, frames (Fig. 1.9(a) and (b)), and arched and cable-stayed trusses (Fig. 1.9(c) and (d)).

(4) According to the manner in which they can be analyzed, structures are divided into: (a) *Statically determinate structures* which can completely be analyzed by statics alone. (b) *Statically indeterminate structures* which cannot be analyzed by statics alone. For their solution, redundant structures (as statically indeterminate structures are also frequently called) require that the three equations of statics (the equilibrium equations) be supplemented by compatibility equations which take care of their geometry.

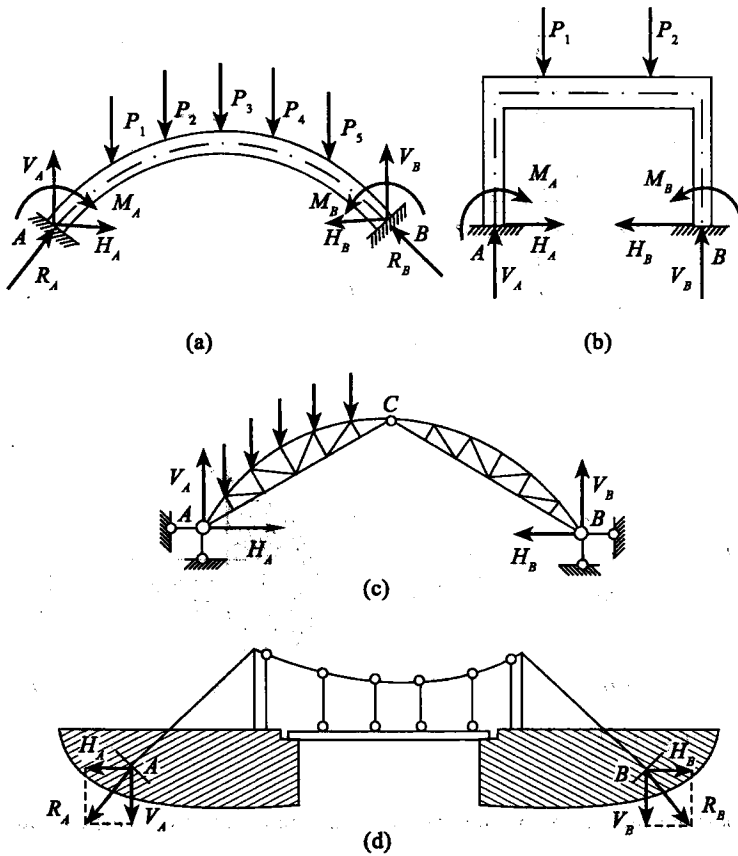


Fig. 1.9

## 1.5 Supports for Plane Structures

All structures have to be supported suitably. The following three types of support are in common use:

1. A roller support (also called a movable hinged support) (Fig. 1.10(a)).
2. A hinged support (also called an immovable hinged support) (Fig. 1.11(a)).
3. A fixed support (Fig. 1.12(a)).

The above supports are symbolized as shown in Figs. 1.10(b) and (c), 1.11(b) and (c), and 1.12(b) and (c), respectively.

A *roller support* offers no resistance either to the rotation of the supported body about an axis perpendicular to its plane through  $C$ , or to its displacement along the support base. Friction at the support is customarily ignored, so the only reaction  $R$  possible at a roller support is along the perpendicular to its base through  $C$  (see Fig. 1.10(a)).

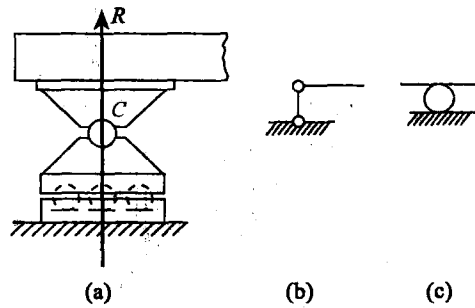


Fig. 1.10

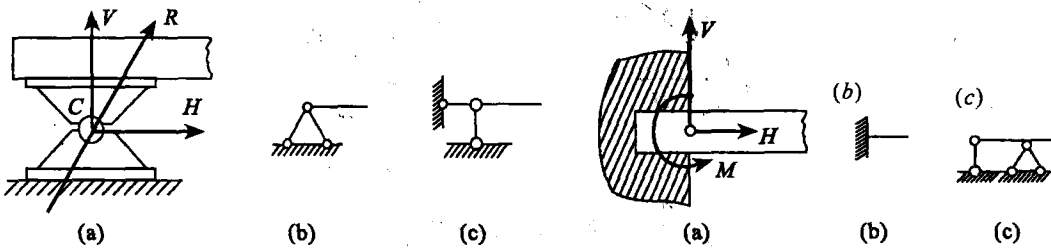


Fig. 1.11

Fig. 1.12

A *hinged support* permits free rotation of the body about an axis perpendicular to its plane through  $C$ , but does not permit its displacement either along or perpendicular to the base. The reaction  $R$  developed at such a support may be in any direction (see Fig. 1.11(a)). As a rule, it is resolved into two components, namely a component  $H$  along and a component  $V$  perpendicular to the base.

A *fixed support* allows the supported body neither in-plane rotation nor translation in any direction. The three possible support reactions — one moment and two reactive forces—are indicated in Fig. 1.12(a).

## CHAPTER 2 Stability

### 2.1 Stable and Unstable Structures

As already mentioned in Sec. 1.1, any structure must retain its original geometry throughout its service life. This requirement is met by what are called stable structures.

To gain insight into this matter, consider a structure made up of three bars hinged to one another at the ends to form a triangle  $ABC$  (Fig. 2.1(a)). The geometry of such a triangle will obviously remain unchanged, whatever position it may occupy in space, because three bars of constant length can form only one triangle. If we load the triangle by a force as shown in Fig. 2.1(b), it will nevertheless change its shape, although very insignificantly (see triangle  $AB_1C_1$ ) — a fact which can be attributed to the elastic deformation of its members alone.

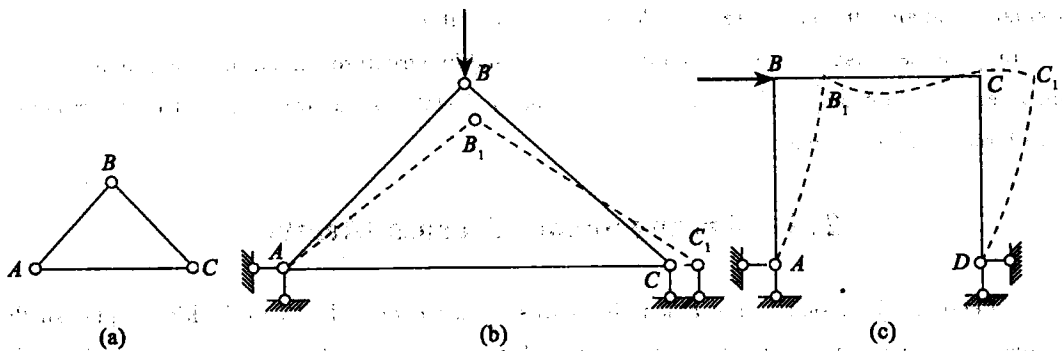


Fig. 2.1

Thus, a structure may be called *stable* if its shape remains unchanged, whatever the position in space.

A characteristic feature of a stable structure loaded within reasonable limits is the ability to change its shape only insignificantly as a result of elastic deformation of its members. By elastic deformation is meant a change in the size of the constituent members as shown in Fig. 2.1(b), or a change in both the size and shape of the members as shown in Fig. 2.1(c).

A stable structure is amenable to small deformations theory and, as a consequence, to the principle of superposition.

The simplest stable structure is a triangle (sometimes called a basic triangle).

A structure whose shape changes suddenly when its position in space is altered or when it is subjected to a load, however small, is *unstable*.

A characteristic feature of an unstable structure is that any change in its shape is associated with finite displacements of its members without deformation.

As an example, consider the pin-jointed rectangle  $ABCD$  shown in Fig. 2.2 (a). It is unstable, because even an infinitesimal load will force its members  $AB$ ,  $BC$ ,  $CD$ , and  $DA$  to change their position without any change in length or shape. At first, the loaded rectangle becomes the shape of a parallelogram ( $AB'C'D$ ). Then, its sides collapse, as it were, on one another to form an almost straight line  $AC''$  composed of segments  $AB''$ ,  $B''C''$ ,  $C''D$ , and  $DA$ .

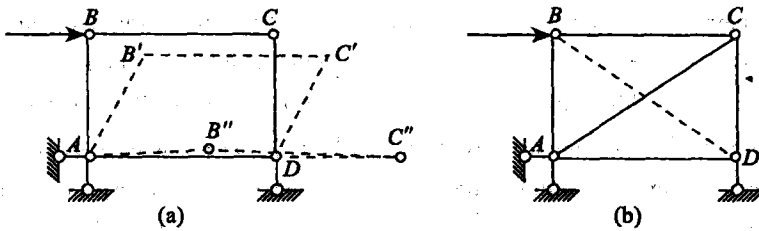


Fig. 2.2.

If we add a diagonal bar  $AC$  (Fig. 2.2 (b)) or  $BD$  (shown by the dashed line) to the original rectangle, the structure thus derived will be stable.

In practice, use is predominantly made of stable structures fixed to the ground (or its equivalent) so that they will not move, or internally unstable structures attached to the ground so as to form a stable system.

## 2.2 Arrangement of Truss Members

A detailed discussion of the assembly of trusses has been delayed until this chapter so that the reader will have had some contact with the elementary types. The background should enable him or her to understand the material to follow more easily.

The triangle has been shown to be the basic shape from which trusses are developed because it is the only stable shape. Other shapes such as the ones shown in Figs. 2.3 (a) and (b) are obviously unstable and may possibly collapse under load. Structures such as these can, however, be made stable by one of the following methods.

1. Adding members so that the shapes are made to consist of triangles. The structures of Fig. 2.3a and (b) are stabilized in this manner in (c) and (d), respectively.

2. Using a member to tie the unstable structure to a stable support. Member  $AB$  performs this function in Fig. 2.3 (e).

3. Making some or all of the joints of an unstable structure rigid, so they become moment resisting. A figure with moment-resisting joints, however, does not coincide with the definition of



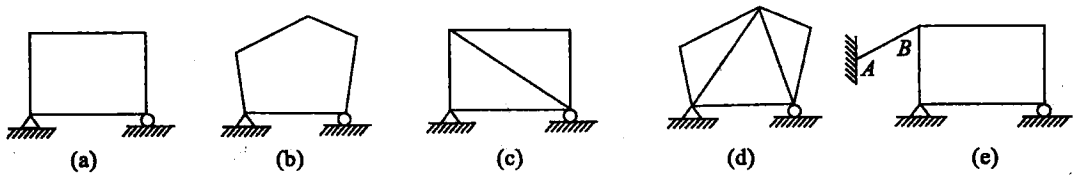


Fig. 2.3

a truss (that is members connected with frictionless pins, and so on).

### 2.3 Statical Determinacy of Trusses

The simplest form of truss, a single triangle, is illustrated in Fig. 2.4 (a). To determine the unknown forces and reaction components for this truss, it is possible to isolate the joints and write two equations,  $\sum H = 0$  and  $\sum V = 0$ , for each. From experience obtained before there should be little difficulty in making the necessary calculations.

The single-triangle truss may be expanded into a two-triangle one by the addition of two new members and one new joint. In Fig. 2.4 (b), triangle  $ABD$  is added by installing new members  $AD$  and  $BD$  and the new joint  $D$ . A further expansion with a third triangle is made in part (c) of the figure by the addition of members  $BE$  and  $DE$  and joint  $E$ . For each of the new joints,  $D$  and  $E$ , a new pair of equations is available for calculating the two new-member forces. As long as this procedure of expanding the truss is followed, the truss will be statically determinate internally. Should new members be installed without adding new joints, such as member  $CE$  in Fig. 2.4 (d), the truss will become statically indeterminate because no new joint equations are made available to find the new member forces.

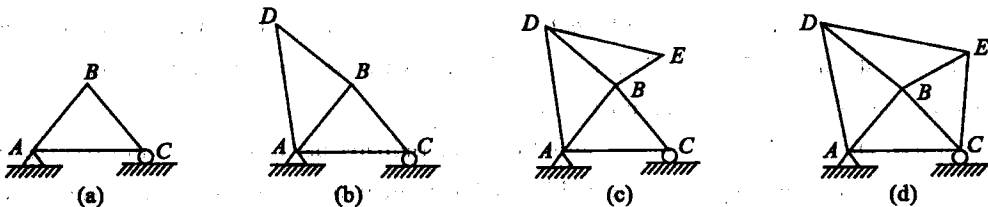


Fig. 2.4

From the information above an expression can be written for the relationship that must exist between the number of joints and the number of members and reaction components for a particular truss if it is to be statically determinate internally. In the following discussion,  $m$  is the number of member,  $j$  is the number of joints, and  $r$  is the number of reaction components.

If the number of equation available ( $2j$ ) is sufficient to obtain the unknowns, the structure is