



刘高联文选

Selected Works of Liu Gaolian



【下卷】

上海大学出版社

刘高联文选

Selected Works of Liu Gaolian

(下卷)

上海大学出版社

· 上海 ·

目 录

New Research and Concepts in Turbo-Jet Engine Design	553
二维跨声速有旋流动反命题的赝势函数变分原理.....	565
Derivation and Transformation of Variational Principles with Emphasis on Inverse and Hybrid Problems in Fluid Mechanics; A Systematic Approach	571
流体力学反命题变分有限元非线性规划法.....	594
旋成面叶栅气动力学反命题解的唯一性定理.....	601
Aerodynamic Design Method of Cascade Profiles Based on Load and Blade Thickness Distribution	605
三维管道内有旋流动的涡势函数拟变分原理.....	614
The First Exact Variational Formulation of 3-D Navier-Stokes Equations	621
基于变域变分有限元的翼型反设计.....	630
跨音速翼型杂交问题的赝势函数变域变分有限元法.....	637
Hybrid Design Methods of Cascade Profile Based on Variational Principles	643
应用多块分区搭接网格技术和变域变分有限元法设计分流叶栅.....	656

第二部分 流热固耦合问题

Numerical Methods for Inverse Problem of Heat Conduction with Unknown Boundary Based on Variational Principles with Variable Domain	667
旋转叶片扭转恢复的气动弹性力学耦合变分理论.....	680
The Generalized Untwist Problem of Rotating Blades; A Coupled Aeroelastic Formulation	689
在大位移变形下叶片扭转恢复的气动弹性力学耦合变分理论.....	703
耦合热弹性动力学的统一变分原理族.....	710



Variational Formulation of Inverse Shape Design Problem of Heat Conductors in an Image Plane and Finite Element Solutions	720
A New Generation of Inverse Shape Design Problem in Aerodynamics and Aerothermoelasticity: Concepts, Theory and Methods	737
流体力学和气动热弹性力学新一代反命题的研究	756
A Coupled Aerothermoelastic Theory of Vibrating Blade-Fluid Interaction in Fully 3-D Transonic Rotor Flow	761

第三部分 非定常与多工况问题

二维振荡机翼含激波跨声速非定常势流的变分原理	777
二维振荡机翼含激波跨声速非定常绕流广义变分原理的普遍形式及其派 生族	787
General Variational Principles Family for Fully 3-D Unsteady Transonic Flow with Shocks Around Oscillating Wings	797
Variational Finite Element Calculation of 2-D Unsteady Compressible Flow Around Oscillating Airfoils	809
近水面二维振荡水翼非定常绕流的反命题与杂交命题的变分理论	816
机翼和叶栅非定常流的 Kutta 条件	824
On Variational Crisis and Generalized Variational Principles for Inverse and Hybrid Problems of Free Surface Flow	830
二维振荡机翼跨声速非定常绕流的变域变分有限元解	835
Variational Formulation of 3-D Unsteady Transonic Flow Past Oscillating Rotor Bladings; Part I — Potential Flow	841
A General Variational Theory of Multipoint Inverse Design of 2-D Transonic Airfoils Based on an Artificial Airfoil-Oscillation Concept	852
二维水翼多工况点反命题的通用变分理论: 人工振荡法	861
Variational Principles for 1-D Unsteady Compressible Flow in a Deforming Tube of Variable Cross Section	867
A Variable-Domain Variational Formulation of Inverse Problem I_A of 2-D Unsteady Transonic Flow Around Oscillating Airfoils	875
A General Variational Theory of Multipoint Inverse Design of 2-D Transonic Cascades Based on an Artificial Flow-Oscillation Model	893

机翼跨声速非定常绕流 I_A 型反命题变域变分有限元解	905
跨声速平面叶栅多工况点反命题变分理论: 人工来流振荡模型	914
Multipoint Inverse Shape Design of Airfoils Based on Variable-Domain Variational Principle	919
Exact Variational Principle For 3-D Unsteady Heat Conduction With Second Sound	934
二维非定常 Navier-Stokes 方程精确的对偶变分原理族	941
基于欧拉方程的二维振荡机翼非定常气动设计反命题方法(II)	948

第四部分 其 他

正齿轮的最优啮合理论	961
水坝溢流等问题中的新变分原理族	978
关于固体力学中的应力函数、应变函数及其推广	988
The Hybrid Problem of Free-Surface Gravity Spillway Flow Treated by Variational Principles with Variable Domain; Potential Function Formulation	997
论弹性力学广义变分原理的临界变分状态	1003
应用变域变分原理与有限元法求解泵喷射推进器外流场	1015
径向轴承含空泡流体润滑问题的变分原理族(二)	1021
动态差分变换及其对最优控制论和变分原理的应用	1026
论基础数学工具的创新运用	1031
水波泵的流体力学原理与设计计算	1040
水波泵内流体径向平衡问题的解法	1048
Mesh Free Method Based on Local Cartesian Frame	1055
钱伟长——我国近代力学和国际奇异摄动理论的奠基人	1062
康托洛维奇-里茨杂交法及其应用	1068
有限元-康托洛维奇杂交法及其应用	1080

New Research and Concepts in Turbo-Jet Engine Design*

Abstract Presents a brief overview of some new concepts and research results concerning aerodynamic computation and design of jet-propulsion engines with emphasis on turbomachinery (TM) developed in China, without any attempt to be exhaustive.

Improved Method for Aerodynamic Design of Wave Rotors in Turbofan Engines

The experimental results gained in the USA and Europe show that turbofan engines using wave rotors (WR) as the high pressure stage have merits such as: high specific power output, low specific fuel consumption, quick response to fuel regulation together with a smooth transient process, capability of stable operating near the stall boundary of the compressor, etc. For these reasons this new kind of jet engine is considered to be one of the potential, perspective engine types for cruise missiles and/or aircraft.

It is well recognized that in the WR part the most salient factors for realizing the practical utility of the turbofan jet engine using WR are: improvement of aerodynamic design method of WR and suggesting a new sealing device to diminish the leakage loss through the rotor-end clearance. So far, however, the aerodynamic design of WR has been confined to 1-D unsteady flow calculation at the mean radius of the rotor without accounting for the radial variation of the flow field, which for rotors with small hub-tip radius ratio becomes so significant that its effect on flow losses owing to very large angles of attack at the rotor inlet and outlet in the tip and hub regions, can

* In collaboration with Ji-Huan He. Reprinted from *Aircraft Engineering and Aerospace Technology*, 1997, 69(6): 527-533.



never be ignored. To bridge this gap, Liu^[1] addressed the radial equilibrium R-E problem of flow in WR and suggested a method for its solution. This method enables the radial distribution of all flow parameters such as velocities and flow angles to be computed so as to shape the blade and port walls optimally for reducing losses at the rotor inlet and outlet.

For axial-flow WR the basic flow equations for the R-E problem are:

Radial-momentum equation:

$$\frac{1}{\rho} \frac{\partial p}{\partial r} = \frac{v_{\theta}^2}{r}, \quad (1)$$

Energy equation:

$$\frac{1}{\gamma-1} \left(\frac{p_0}{\rho_0^\gamma} \right) \rho^{\gamma-1} + \frac{1}{2} (v_{\theta}^2 + v_z^2) = H(\Psi), \quad (2)$$

State-characteristics equation:

$$w \pm \frac{2}{\gamma-1} a = K_{\pm}, \quad (3)$$

where v and w are the absolute and relative velocities respectively; a is sound speed; Ψ is stream function; K_{\pm} is Riemann's invariants along the right and left-running characteristic lines $\partial z / \partial t = w \pm a$ respectively.

It should be noted that the present R-E problem differs from that of turbomachinery in that:

(1) Both the stagnation enthalpy H and the rothalpy R are not constant along the path line across the rotor, so that the compatibility equation (3) must be used to relate flow parameters at the rotor inlet and outlet; and

(2) For all inlet/outlet ports and pockets, R-E calculations must be carried out for every wall separately in order to determine the shape of all port/pocket walls.

From a numerical example of WR design of pressure-exchanger type given in Liu^[1] Fig. 1 is taken, where the wave patterns at the tip, hub and mean radii are shown. Of special importance is that at the tip of the air inlet port the flow angle of attack $\Delta\alpha$ will be as large as 23° if this port wall is made, as usual, without accounting for the R-E effect. Obviously, such a large $\Delta\alpha$ gives rise to significant losses. Thus, we come to the conclusion that a complete gas dynamic design procedure of WR should consist of two parts:

(1) 1D unsteady relative flow analysis in rotor cells at the mean radius; and

(2) R-E solution at the rotor inlet and outlet.

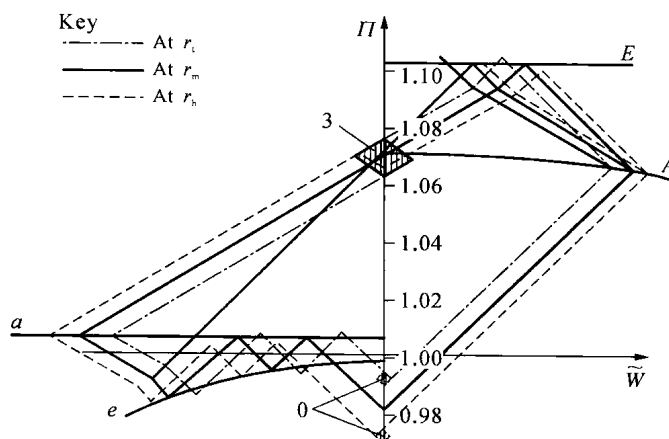


Fig. 1 Wave patterns from R-E solution at the tip, mean and hub radii in WR

An Inverse Problem in Aeroacoustics of TM and Propeller

In practice we often encounter serious difficulties in conducting direct measurements of some parameters, for instance, measuring the pressure distribution along the rotating blade surface. This is due to the fact that the blade is in high speed rotation and is too thin (especially in the tip region) to install transducers in it. So, it is very desirable and valuable to gain some information from contactless measurements and then, based on this information, to compute the blade surface pressure distribution by the inversion method.

Such an inversion method has been suggested in Li and Zhou^[2]. As we know, in the conventional (direct) problem of aeroacoustics, where using the wall pressure distribution obtained from computational fluid dynamics or direct measurements as input, the Ffowcs Williams-Hawking's equation is solved to obtain the acoustic field around the moving body (blade). Conversely, in the present inverse problem we do just the opposite, namely, using the sound pressure data measured at a number of selected points distributed in space as input, the pressure distribution along the moving wall is computed from the Ffowcs Williams-Hawking's equation by inversion. This is an ill-posed problem and must be solved by the Tikhonov's regularization method. For illustration of the effectiveness of the method, the numerical results of the blade surface pressure distribution of a propeller taken from Li and Zhou^[2] is depicted and compared with the exact solution



in Fig. 2, where N/S stands for the signal-noise ratio and the accuracy of the inversion solution increases with decreasing N/S ratio.

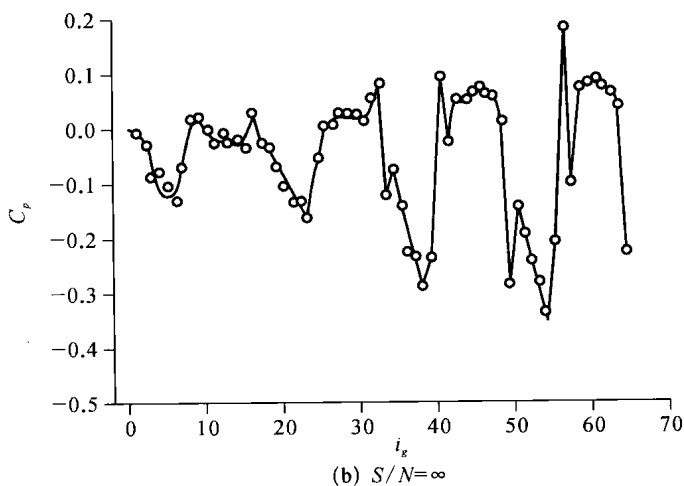
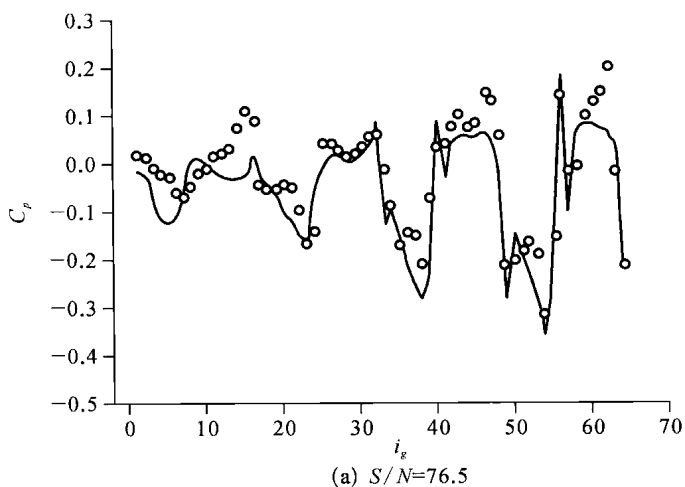


Fig. 2 Pressure distribution along propeller blade surface computed by inversion

Generalized Blade Untwist Problem and other Aerothermoelasticity Problems

TM blades (especially long twisted-curved rotating blades), aircraft wings and propfan and rotorcraft blades operate under the action of centrifugal and aerodynamic forces and undergo an untwist that gives rise to changes in flow condition (angle of attack, flow rate etc.) in blade rows. The TM actually

operates at off-design conditions, spoiling harmonious matching between successive blade rows and leading in turn to a remarkable increase in flow losses, power change as well as operation instability (stall, surge, etc.). It follows that the untwist is a very important problem for blade design and must be studied in-depth and predicted with high accuracy.

Up to now, the blade untwist problem has been solved in the practical blade design by employing Ohtsuka's method^[3] that accounts only for the centrifugal force, disregarding the "blade-fluid" interaction. The reason for the apparent agreement of Ohtsuka's calculated and experimental results lies in the fact that his experiment was conducted in vacuum. A qualitative analysis given in Ref. [4] shows that the effect of the aerodynamic force on untwist is not always negligible in comparison with that of the centrifugal force.

Recently, Liu^[4] generalized, from the view-point of aeroelasticity, the untwist problem in such a way that not only the centrifugal force but also the blade-fluid interaction are allowed for rigorously. He also suggests the following two kinds of untwist problem:

(1) Direct (analysis) problem: given the shape of the blading at rest and the inlet flow conditions, the resulting untwist along with the flow field in it during operation needs to be calculated.

(2) Inverse (design) problem: given the desired (as calculated by aerodynamic method) blading shape during operation and the inlet flow conditions, the blading shape at rest (i. e., the shape required by the machining drawing) needs to be calculated.

Obviously, the inverse untwist problem is most important and thus most useful for practical blade design.

In Ref. [4] a family of variational principles (VP) and generalized VP (GVP) has been established for the direct untwist problem and from each of these VP one can derive naturally and simultaneously not only the basic equations of both gas dynamics and elasticity, but also the "fluid-solid" interface matching conditions via variable-domain (VD) variations. Based on these VP, a 3-D finite element method (FEM) has been worked out for an axial turbine and an axial compressor. The preliminary results obtained so far^[5] showed that in some situations the untwist due to aerodynamic force may be



comparable with or even larger than that due to centrifugal force.

This aeroelastic theory of blade untwist problem is now being extended further to include also the thermal deformation effect to form an aerothermoelasticity theory by VP in a joint research programme with the Gas and Steam Turbine Institute of Aachen Technical University, Germany. In addition, other coupled “fluid-solid” interaction problems (e. g. . vibration system of rotor-bladings and wings in 3-D transonic flow) are also formulated by VPs with variable domain, e. g. , [6].

Inverse Design of Aerofoils and Cascades in Unsteady Flow

In recent years, ever-increasing interest of scientists and engineers has been attracted to the inverse design problem of aerodynamics, but all papers in the literature on the inverse problem up to now are still confined to steady flow. In Ref. [7] the inverse problem of unsteady flow is investigated for the first time, suggesting proper ways of problem posing and mathematical formulation. A special feature of the unsteady inverse problem that makes it quite different from the steady one is that the pressure distribution along the wall can never be specified over the whole oscillation period so as to keep the airfoil geometry unchanged with time, while allowing the airfoil to move as a rigid body. Two types of the unsteady inverse problem, I_A and I_B , have been studied in detail:

- I_A problem: given the time-averaged pressure over the airfoil contour, the corresponding airfoil contour is required.

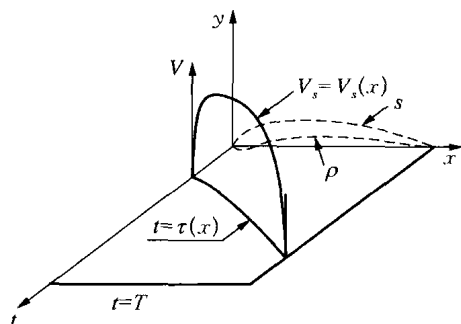


Fig. 3 Specification of suction surface velocity $V_s(x)$ along a selected line in $t-x$ plane: $t = \tau(x)$ in unsteady inverse problem I_B

- I_B problem: given the contour pressure along a specified line in the $t-x$ plane: $t = \tau(x)$ (Fig. 3), the corresponding airfoil shape is required.

Two families of VP with variable domain are established for these I_A and I_B problems respectively and thus a new sound theoretical basis for unsteady aerodynamic design and for

FEM is provided.

A New Artificial Oscillation Concept for Multipoint Inverse Design of Aerodynamic Shapes

Almost all aerodynamic shapes (wings, bladings, etc.) have to operate at off-design conditions and hence they should have favourable performance at more than one operating condition, e. g. at several angles of attack. This can be achieved only by the multipoint inverse design method. However, so far in the literature there exists only one such method, proposed by Eppler and Somers^[8] and improved by Selig and Maughmer^[9] and this has found some application in hydro-foils and ship propeller design. Unfortunately, Eppler's method is based on conformal mapping and hence inherently suffers from the following restrictions:

- (1) 2D flow;
- (2) incompressible flow; and
- (3) potential flow.

So, its applicability range is severely limited. Recently, Liu suggested a novel artificial oscillation concept and, based on this as well as on Ref. [7], developed a general variational theory of multipoint inverse design of 2-D transonic airfoils and cascades^[10]. The essential advantages of this new approach over Eppler's are the capability of covering 3D, compressible (transonic flow with shocks) and rotational flows.

According to the artificial oscillation concept^[10] the steady airfoil flow at different angles of attack can be thought of as unsteady flow at corresponding oscillating airfoil positions. Obviously, as the oscillation period T becomes very large, the unsteady flow approaches asymptotically to the steady one, so that the specified unsteady contour velocity distribution (Fig. 4) becomes just the desired steady one.

Flow Field Diagnosis in Multistage Axial Compressor

As is well known, in general, the designed performance characteristics of a compressor can only be achieved after a series of measurements, data analysis, readjusting of blade angles and tip-clearances and so on. Owing to the very limited measuring space available, complicated construction in compressors and imperfect measuring apparatus and techniques, the

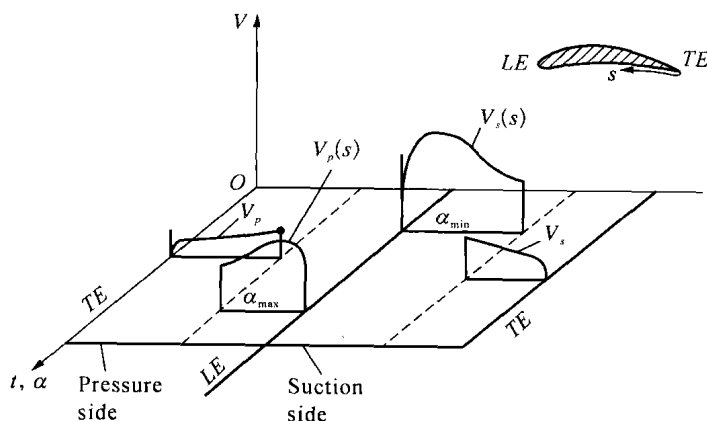


Fig. 4 Specification of airfoil surface velocity V_s and V_p at different selected angles of attack α (or t) in multipoint inverse design

information obtained by direct measurement is insufficient to get a full picture of the flow field, which is essential for compressor readjustment. This makes the compressor readjustment extremely difficult and time consuming. True, the flow field can be calculated from the Navier-Stokes equations by computational fluid dynamic methods. However, its accuracy and practical value are limited by the capability of computers and the unreliability of turbulence models available as well as very complicated flow phenomena in TM. As a result, the flow field analysis and readjusting process of compressors still rely on simplified flow models in conjunction with some empirical correcting methods. To improve this situation, in [11] the flow field diagnosis problem is addressed and a mathematical model for the flow diagnosis in multistage axial compressors is also developed. This model consists of:

- (1) radial equilibrium equations of flow;
- (2) boundary conditions;
- (3) supplementary measured data: static pressure distribution along the outer casing, stagnation temperature and pressure at compressor outlet;
- (4) diagnostic parameters (i. e., parameters to be diagnosed): it is suggested that the blockage factors KB, the deviation angles δ and the stagnation pressure loss coefficient $\bar{\omega}$ in every blade row are used as the diagnostic factors.

To simplify the model and make it less illposed, the Roberts' (ASME paper 85-GT-189, 1985) correlations for $\bar{\omega}$ and δ are adopted and assumed to be the

same for all middle stages. We can realize that this is just the problem of system identification.

It transforms into an optimization problem that is solved by a regularization method. The rationality of the flow diagnosis problem and the effectiveness of the solution method are validated in [11] by numerical results shown in Fig. 5.

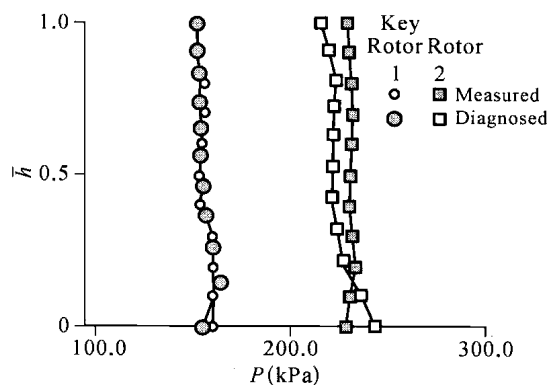


Fig. 5 Stagnation pressure distribution at outlet of two rotor-blade rows

Mechanism of Loss Reduction in Stators with Curved-Twisted Blades

It was suggested in the early 1960s in [12] that the turbine stator blades should be curvedtwisted in order to control and reduce the secondary losses. The mechanism was explained in [13] and the positively curved blades (Fig. 6) are recommended for use in turbine stators owing to their remarkable lossreducing effect. Recent experimental investigations^[14], however, showed that for blade cascades with considerable diffusion (pressure rise) and cascades with essentially constant average flow velocity but with high flow-turning angle (such as guide vanes), the positively curved blade cascade causes higher energy

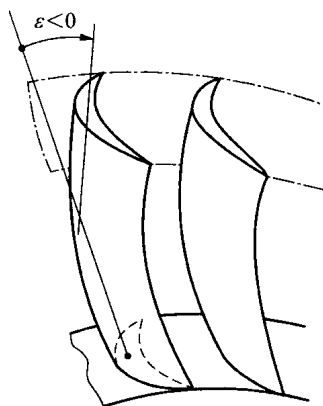


Fig. 6 Negatively curved blades

losses than cascades with a straight or negatively curved blade (Fig. 7). The reason for that is that in such cascades the boundary layer on the suction surface in the midspan region is very thick or even separated and under the radial pressure gradient (positive near the tip and negative near the hub) caused by the positively curved blades the low energy fluid moves from both ends to the midspan, leading to a very high loss region there. Thus, it can be concluded that for expansion (nozzle type) cascades positively curved blades should be used, while for diffuser cascades and guide vanes



negatively curved blades (Fig. 6) may be used to advantage.

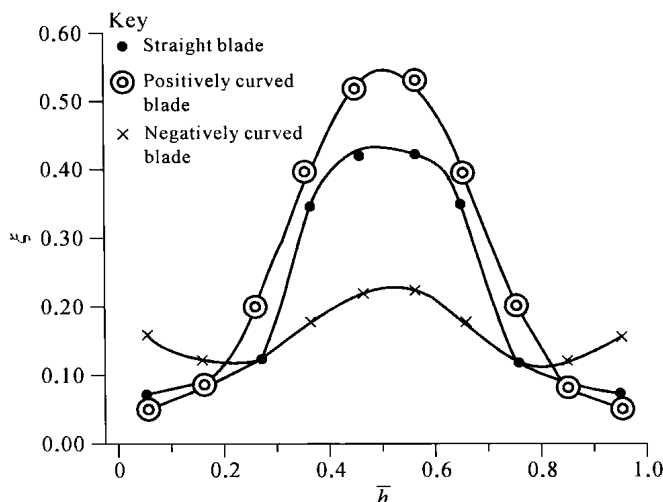


Fig. 7 Loss coefficient along blade span

Generalized Euler's Turbomachine Equation and Generalized Kuta's Condition in 3D Rotor-Flow

With the rapid development of the computational fluid dynamics of TM towards fully 3D flow with ever-increasing speed and loading and with separation and/or cavitation, some new basic problems arise and need to be explored in a more rigorous theoretical way, others need to be reconsidered and handled in a more general setting. For instance, as is well known, the TM power equation derived in 1755 by Euler plays a fundamental role in TM theory and is very widely employed in design and analysis. Up to now, it has been commonly accepted that this equation is of general validity for TM with both shrouded and unshrouded impellers. Actually, however, as is shown in Ref. [15], this equation is valid only for shrouded impellers. A generalized Euler's TM equation valid for shrouded and unshrouded rotors has been derived in the following form^[15]:

$$\begin{aligned}
 L &= \omega \{ (rV_{\theta})_2 - (rV_{\theta})_1 \} - K_0 L_i, \\
 L_f &= \omega M_{st} / G, \\
 K_0 &= \begin{cases} 0, & \text{for shrouded rotor,} \\ 1, & \text{for unshrouded rotor.} \end{cases}
 \end{aligned} \tag{4}$$

The casing friction moment M_{st} can be determined either by experiments or by theoretical methods based on the theory of annulus wall boundary layers.

In addition, in dealing numerically or analytically with fully 3D flow in rotors/stators the proper understanding and formulation of the flow conditions at the free trailing vortex sheets (especially when a separated or cavitated region exists) and of the Kuta condition are of great importance for correctly imposing downstream boundary conditions. These problems have been solved rigorously in Ref. [15].

Flow Loss Reduction in Cascades Based on a Bionics Concept

Recent research reveals that the turbulent drag is directly related to a tiny coherent structure inside the boundary layer and the drag (loss) of a specially designed non-smooth surface can be less than that of a smooth surface. A similar situation has been observed on the body surface of some sea animals (e. g. , seals). Based on this bionics concept, six compressor cascades with different wrinkle patterns carved on the blade surface have been tested in the near-sonic wind tunnel^[16]. Compared with the reference smooth cascade, the best one of six tested cascades raised the critical Mach number by 7.3 per cent. The air flow deflection angle at the design conditions increased by 0.9° . Correspondingly, the deviation angle decreased by 0.9° and the maximum static compression ratio rose by 0.0177.

References

- [1] Liu G L. The radial-equilibrium problem of flow in wave machinery. Proc, IMechE, Part A, Journal of Power & Energy, 1993, 207: 23 - 30.
- [2] Li X D, Zhou S. Spatial transformation of discrete sound field from a propeller. AIAA J, 1996, 34, (6): 1097 - 1102.
- [3] Ohtsuka M. Untwist of rotating blades. ASME Paper 74 - GT - 2, 1974.
- [4] Liu G L. The generalized untwist problem of rotating blades: a coupled aeroelastic formulation. International Journal of Turbo & Jet Engines, 1995, 12: 109 - 117.
- [5] Gao J H. Aeroelastic analysis of the untwist problem of rotating blades by a variational FEM. Ph.D. dissertation, Institute of Mech. , Shanghai University (to be completed at the end of 1997).
- [6] Liu G L. A variational aeroelasticity theory of "fluidblade" vibration system in fully 3-D transonic rotor flow. presented at the lecture session 'Fluid-Structure Interaction Problems 2" of the 19th International Congress of Theoretical and Applied Mechanics,



Kyoto, Japan, August 1996.

- [7] Liu G L. Formulation of inverse problem of 2-D unsteady flow around oscillating airfoils by VP. *Acta Aerodynamica Sinica*, 1996, 14(1): 1 - 6 (in Chinese); *Proc. Intl. Conf. on Aerohydroelasticity*, Beijing, China, October, 1993.
- [8] Eppler R, Somers D M. A computer program for the design and analysis of low-speed airfoils. NASA TM - 80210, 1980: 82 - 87.
- [9] Selig M S, Maughmer M D. Multipoint inverse airfoil design method based on conformal mapping. *AIAA Journal*, 1992, 30(5): 1162 - 1170.
- [10] Liu G L. A general variational theory of multipoint inverse design of transonic airfoils and cascades based on an artificial oscillation concept//to be presented at the International Symposium on Inverse Problems in Engineering Mechanics, Nagano, Japan, March 1998.
- [11] Zhang Z C, Liu J, Ye D J. Numerical solution of flow field diagnosis in multistage axial compressors. *Journal of Engineering Thermophysics*, 1996, 17: 180 - 183 (in Chinese).
- [12] Deich M E, Gubarev A W, Filippov G A, Wang Z Q. New method for profiling turbine stator blades with small hub/tip radius ratio. *Thermal Energetics*, 1963, 8 (in Russian).
- [13] Wang Z Q, et al. Aerodynamic calculation of turbine stator cascades with curvilinear leaned blades and some experimental results. *Symposium Paper of 5th International Symposium on Air-Breathing Engines (ISABE)*, 1981.
- [14] Wang Z Q, Su J X, Zhong J J. New progress of investigation into mechanism of reducing loss in cascades with curved-twisted blades. *Journal of Engineering Thermophysics*, 1994, 15(2): 147 - 152 (in Chinese).
- [15] Liu G L. Generalized Euler's turbomachine equation and free vortex sheet conditions in separated/cavitated turbo-flows. *International Journal of Turbo & Jet Engines*, 1996, 13: 1 - 11.
- [16] Miao R T, et al. Experimental investigation on turbulent drag reduction in compressor cascades. *Journal of Aerospace Power*, 1991, 6: 13 - 16 (in Chinese).

Further reading

Liu G L. Research on inverse, hybrid and optimization problems in engineering science with emphasis on turbomachine aerodynamics: a review of Chinese advance. *International Journal of Turbo & Jet Engines*, 1994, 11: 53 - 70.