



普通高等教育“十五”规划教材
PUTONG GAODENG JIAOYU SHIYIWU GUIHUA JIAOCAI

STABILITY OF STEEL STRUCTURES THEORY AND DESIGN

钢结构稳定—理论与设计

Ji, Chen(Editor-in-Chief); Haojun, Chen

陈 骥 (主编) 陈浩军



中国电力出版社

<http://jc.cepp.com.cn>



普通高等教育“十一五”规划教材
PUTONG GAODENG JIAOYU SHIYIWU GUIHUA JIAOCAI

STABILITY OF STEEL STRUCTURES THEORY AND DESIGN

钢结构稳定—理论与设计

Ji,Chen(Editor-in-Chief); Haojun,Chen

陈 骥(主编) 陈浩军



中国电力出版社
<http://jc.cepp.com.cn>

内 容 简 介

本书为普通高等教育“十一五”规划教材，全书共八章，主要内容包括以下几个方面：

(1) 失稳分类：分岔失稳的类型，极值点失稳和跃越失稳。

(2) 轴心受压柱，梁柱，刚接和半刚接刚架的平面弯曲屈曲性能和实用设计方法。

(3) 柱，梁和梁柱的平面外弯扭屈曲性能和实用设计方法。

(4) 薄板的凸曲和屈曲后性能，冷弯薄壁板件的局部屈曲，畸变屈曲，整体屈曲和它们之间的相关屈曲，有效宽度和直接强度两种设计方法。

(5) 弹性和弹塑性钢结构的能量法和数值法以及其试验验证。

全书内容注重钢结构材料和构件几何非线性的特点，使之符合实际的结构设计。同时，书中还附有依照国内外钢结构设计规范设计的许多钢结构构件和刚架有关理论研究和设计方法的实例。

本书可作为普通高等院校工程结构、工程力学专业研究生的教材，也可作为结构工程师和研究人员的参考用书。

图书在版编目 (CIP) 数据

钢结构稳定：理论与设计 = Stability of Steel Structures: Theory and Design: 英文/陈骥, 陈浩军主编. —北京: 中国电力出版社, 2009. 8

普通高等教育“十一五”规划教材

ISBN 978-7-5083-8906-6

I. 钢… II. ①陈…②陈… III. 钢结构-结构稳定性-高等学校-教材-英文 IV. TU391.01

中国版本图书馆 CIP 数据核字 (2009) 第 087135 号

中国电力出版社出版、发行

(北京三里河路 6 号 100044 <http://jc.cepp.com.cn>)

北京盛通印刷股份有限公司印刷

各地新华书店经售

*

2010 年 3 月第一版 2010 年 3 月北京第一次印刷

787 毫米 × 1092 毫米 16 开本 57 印张 1333 千字

定价 110.00 元

敬告读者

本书封面贴有防伪标签，加热后中心图案消失

本书如有印装质量问题，我社发行部负责退换

版权专有 翻印必究

Preface

The theory of structural stability is, strictly speaking, a branch of structural mechanics. But, from the process of the development, it can be observed that this learning is closely linked with the progress of structural engineering. S. P. Timoshenko, in his classic monograph **Theory of Elastic Stability** published in 1936, wrote: “The modern use of steel and high-strength alloys in engineering structures, especially in bridges, ships and aircraft, has made elastic instability a problem of great importance”. This statement made more than 60 years ago is still holding true nowadays, whereas buildings and offshore platforms are added to the rank of above structures and plasticity is more involved in stability issues. Constructional steel by itself is an elasto-plastic material welding, the contemporary art of connection, gives rise to residual stresses which accentuate the inelastic behavior of steel members.

This book **Stability of Steel Structures—Theory and Design** by Professors CHEN Ji and CHEN Haojun not only reflects the up-to-date achievement of this discipline, but also proves to be a good integration of stability theory with structure design. The topics have been expanded from traditional analysis of elastic members and plate elements to that of inelastic ones, from perfect structural components to imperfect ones and from buckling load to post-buckling capacity as well. In the first chapter, the concept of stable and unstable bifurcation is introduced and post-buckling behavior of both perfect and imperfect members are revealed through analysis of a series of rigid-elastic models, thus building a wide field of vision to the readers. On the other hand, inelastic properties of steel, including in the strain-hardening range, and the distribution of residual stresses (of both rolled and welded sections) are discussed thoroughly in this chapter, laying a solid foundation for seeking the actual behavior of members in practical engineering structures.

In the succeeding chapters, inelastic problems are solved by considering the effect of the specific residual stresses. This approach is certainly more realistic and rational than simply using tangent modulus in lieu of Young's modulus. Frames subject solely to loading on top of columns have been typical model for buckling analysis, although real structures always have gravity load on the girder. Due consideration of primary moment caused by girder loading in frame buckling analysis is discussed in this book, which is another example of close link between theory and practice. Besides comprehensive treatment of the basic knowledge of the structural stability theory, the authors pay adequate attention to the application of the theory. As design work always needs recourse to design specification, provisions of design code of several countries are introduced to suit the tendency of world-wide economic unification.

Stability assurance to structural members and systems is a predominant task of steel designers. But, at the present time, the learning of stability theory is not yet widespread among

designers. This gap should be filled the soon the better. The publication of this book provides a useful tool toward this end.

CHEN Shaofan, 5 August, 2008

Foreword

Steel is a material whose stress-strain relationship is practically symmetrical in both tension and compression, but for a steel structural element, its deformation may behave asymmetrically due to buckling of compressed parts. Such buckling phenomena affect the load carrying capacities of axially loaded columns, beam-columns, frames, beams, and web-panels etc. They produce a lowering effect on the load carrying capacity of these elements. The use of steel leads naturally to types of structures embodying slender compression members, thin shells and space slender grid structures. Analytical and experimental investigations show that such structures may fail suddenly not on account of high stresses, surpassing the strength of material, but owing to the insufficient buckling strength of slender or thin-walled members, which often influence the steel structural design. Knowledge of structural stability theory is of paramount importance to the practicing structural engineer. In many cases, buckling is the primary consideration in the design of various steel structures.

This book presents a comprehensive introduction to the principles and analysis of structural stability which constitute the basis for steel structural stability design. It also provides the necessary back ground for the transition from the fundamental theories of structural stability of axial compression members, beams, plate girders, beam-columns, frames and thin plates to the practical design rules.

This book can be used by graduate students in colleges and universities as a text book and also can be used by structural engineers and researchers, since the various design provisions of specifications are discussed in this book.

There are eight chapters in this book. Chapter 1 includes the concepts and principles of structural stability, such as the classification of instability types which are equilibrium bifurcation instability, limit point instability and snap-through instability, stability behavior of some simplified mechanical models, consideration of initial imperfections, nonlinear material properties and residual stress patterns. All these above form the basis of the elastic and inelastic theories on stability of steel structures.

In this book, importance is equally attached to the distinction of two categories of structural stability, the one with planar deformations and the other with spatial deformations. In the first category, the flexural buckling of axially loaded solid and latticed columns, the development of single as well as multiple column design curves, the in-plane stability of beam-columns including the ultimate load and interactive design formulas all are derived, the in-plane stability of braced and unbraced frames referring to the behavior of beam-to-column connections and both individual member design method and overall structural system design method (advanced, direct analysis) of

rigid and semi-rigid frames are considered. These are discussed in chapters 2, 3 and 4 respectively. In the second category, the out-of plane torsional and flexural-torsional buckling of axially loaded columns, the flexural-torsional buckling of beam-columns and beams, the stability analysis of biaxial beam-columns and beams, the buckling of thin plates and their post-buckling behavior including the requirements of plate element width-thickness ratios of compression members and beams, the ultimate capacities of plate girders, the effective area method of thin-walled steel structures and the strength reduction factors of slender compression members are developed. These are analyzed in chapter 6, 7 and 8 separately.

Besides the general discussion of the theoretical elastic behavior, much attention is paid to the inelastic behavior of structural members for both the in-plane stability and out-of-plane stability. Various conceptual problems used to explain the inelastic instability are presented, discussed and compared with experimental data. In the final section of each chapter, the design procedures are examined by different specifications. These specifications referred in this book are as follows :

Chinese GB 50017 – 2003 Code for Design of Steel Structures and GB 50018 – 2002 Technical Code of Design of Cold-Formed Thin-Walled Steel Structures; EN 1993 – 1 – 1 Euro code 3-2005, Design of Steel Structures, General Rules and Rules for Buildings; American National Standard ANSI/AISC 360 – 2005 Specification for Structural Steel Buildings Combining Allowable Stress Design(ASD) and Load and Resistance Factor Design(LRFD); North American NAS AISI 2001 Specification for the Design of Cold-Formed Steel Structural Members; AISI 2004, Supplement to NAS AISI 2001, Appendix, Design of Cold-Formed Steel Structural Members Using Direct Strength Method; AS/NZS 4600 – 2005, Standards Australian/Standards New Zealand on Cold-Formed Steel Structures; Canadian CAN/CSA S16 – 2001 Limit State Design of Steel Structures; International Organization for Standardization ISO/167/SC – 1 – 1995; British Standards BS 5950 – Part 1 – 2000 Structural Use of Steelwork in Building Code of Practice for Design-Rolled and Welded Sections; BS 5950 – Part 5 – 1998 Code of Practice for Design of Cold Formed Thin Gauge Sections and Japanese AIJ 2002 Recommendation for Limit State Design of Steel Structures.

The energy approach and numerical methods such as finite difference method, finite integral method and finite element method of analyzing a structure for its stability limit load are described in chapter 5. The continued rapid development of computer and its use in structural calculation have made it possible for engineers and designers to predicate the structural stability behavior quite accurately. Computer programs used to determine the in-plane capacity of beam-columns are given in chapter 3, those used to determine the flexural-torsional capacities of elastic-plastic beam-columns and beams are given in chapters 6 and 7, those also used to determine the ultimate capacity of plate girder and the utilization of post-buckling strength in slender beam-columns are given in chapter 8.

The discussion on stability behavior is supplemented throughout by numerous examples to

illustrate the applications of stability theory. A comprehensive list of references is appended to the end of each chapter. These references provide an excellent source for future studies, for better understanding of certain specific concepts and for detailed information about specific applications. Problems for exercise and corresponding answers are also provided.

Appendices are attached to the end of this book to deal with six special complex stability problems, such as the buckling of compression members on elastic foundation; total flexural-torsional potential energy of beams and beam-columns; numerical method to determine the flexural-torsional capacities of beams and beam-columns and direct analysis method for design of cold-formed members including the interactions of plate element convex buckling, section distortional buckling and member flexural or flexural-torsional buckling. These problems are also illustrated by practical examples.

This book may be used for teaching purpose as a graduate student textbook emphasizing on the fundamental principles and theory of structural stability, which are elastic and inelastic stability theory of centrally compressed members, flexural members, beam-columns, unbraced frames and thin plates. The master degree students may obtain the new information of stability theory and design from this textbook, they may, depending upon their interest, proceed with the study of the stability analysis of other structural configurations such as shells and grid structures. A number of fresh reference books are provided for teachers, researchers, engineers and constructional designers.

The editor-in-chief would like to express his appreciation and gratitude to Professor Shaofan, Chen, who encouraged him to write this book, gave him an invaluable help and wrote a preface for this book. The editor-in-chief also acknowledge with thanks to the comments and suggestions by their colleagues and students at the Xi'an University of Architecture and Technology.

The authors deeply cherish the memory of the late Professor Baokang, He. We would like to thank him and Professor Ying hua, Yang, who read the manuscript and made many valuable suggestions and corrections. We also would like to thank doctor Xuechun, Liu for his revision of this manuscript.

Readers of this book are urged to communicate with authors regarding all aspects of this book, particularly on identification of errors and suggestions for improvement.

The senior author gives a special credit to his wife Shen Xiuzhen on her patience and encouragement.

Ji, Chen, Editor-in-Chief
E-mail chenji_jichen@163.com
Haojun, Chen
E-mail chen-haojun2007@163.com
January 2009

Notation

The following notation is used in this book. Usually only one meaning is assigned to each symbol, but in those cases where more meanings than one are possible, then the correct one will be evident from the context in which is used.

A	Cross-sectional area
A_1	$P - \delta$ moment amplification factor of beam-columns
A_2	$P - \Delta$ moment amplification factor of unbraced frames
A_c	Area of corner
A_e	Elastic element area
	Effective cross-sectional area
A_f	Cross-sectional area of one flange
A_m	Moment amplification factor
A_n	Net cross-sectional area
A_w	Cross-sectional area of web
B	Flange width
B_1	$P - \delta$ moment amplification factor of beam-column as defined in ANSI/AISC 360 - 2005
B_2	$P - \Delta$ moment amplification factor of unbraced frame as defined in ANSI/AISC 360 - 2005
B_ω	Restrained warping bi-moment = $\int_A \sigma \omega dA = -EI_\omega \phi''$
C	Near end rotation stability function of beam-column
C_1	Modified near end rotation stability function of beam-column with elastically restrained ends
C_v	Web shear coefficient as defined in ANSI/AISC 360 - 2005
D	Elastic flexural stiffness of plate
	Cross-section depth of rafter
	Lip length
D_s	Elastic-plastic flexural stiffness of plate
	Rafter depth allowing for its slope
	Structural characteristic coefficient
E	Elastic modulus
E_r	Reduced modulus
E_s	Secant modulus

E_{st}	Strain-hardening modulus
E_t	Tangent modulus
E	Reduced tangent modulus
F	Function of stress
G	Elastic shear modulus
G_A	Relative stiffness factor at joint A as defined in ANSI/AISC 360 – 2005
G_B	Relative stiffness factor at joint B as defined in ANSI/AISC 360 – 2005
G_{st}	Strain-hardening shear modulus
G_t	Elastic-plastic shear modulus
H	Horizontal load
	Equivalent notional lateral load
	Fictitious horizontal force
	Column height
	Height of section
I	Moment of inertia
I_1	Moment of inertia of compression flange about y axis
I_2	Moment of inertia of tension flange about y axis
I_b	Moment of inertia of a beam
I_c	Moment of inertia of a column
I_e	Moment of inertia of elastic core of cross-section
	Moment of inertia of effective cross-section
I_{et}	Moment of inertia of torsional resistance of elastic core
I_{ex}	Moment of inertia of elastic core about x axis
I_{ey}	Moment of inertia of elastic core about y axis
$I_{e\omega}$	Warping moment of inertia of elastic core
I_{min}	Minimum moment of inertia of intermediate stiffness
I_{pt}	Moment of inertia of torsional resistance of plastic core
I_s	Moment of inertia of lip section
I_t	Moment of inertia of torsional resistance of cross-section
	Saint-Venant torsion constant
I_x	Moment of inertia of cross-section about x axis
I_{xy}	Product of inertia = $\int_A xy dA$
I_y	Moment of inertia of cross-section about y axis
	Moment of inertia of longitudinal stiffener
I_{yc}	Moment of inertia about y – axis referred to compression flange as defined in ANSI/AISC 360 – 2005
I_{yt}	Moment of inertia about y -axis referred to tension flange

I_z	Moment of inertia of transverse stiffener
I_ω	Warping moment of inertia
	Moment of inertia of sector section
K	Complete elliptic integral of first kind
	Linear stiffness
	Torsion parameter = $\sqrt{\pi^2 EI_\omega / (GI_t l^2)}$
K_1	Ratio of sum of beam linear stiffness to sum of column linear stiffness at joint <i>A</i> as defined in GB 50017 – 2003
K_2	Ratio of sum of beam linear stiffness to sum of column linear stiffness at joint <i>B</i> as defined in GB 50017 – 2003
\bar{K}	Wagner effect = $\int_A \sigma \rho^2 dA = \int_A \sigma [(x - x_0)^2 + (y - y_0)^2] dA$
L	Span in pitched-roof frame
L_b	Effective span in pitched-roof frame
L_h	Haunch length in pitched-roof frame
L_r	Total developed length of rafters
M	Bending moment
M_1	Larger end moment of beam-column as defined in GB 50017 – 2003 and also in AIJ – 2002
M_2	Smaller end moment of beam-column as defined in GB 50017 – 2003 and also in AIJ – 2002
M_A	Absolute value of moment at quarter point of unbraced segment
M_B	Absolute value of moment at centerline of unbraced segment
M_b	Buckling resistance moment as defined in BS 5950 – 1 – 2000
M_C	Absolute value of moment at three-quarter point of unbraced segment
M_{cr}	Critical moment
M_{crd}	Elastic distortional buckling moment
M_{crl}	Elastic local interactive buckling moment
M_d	Distortional interactive ultimate moment
M_e	Elastic critical moment
	End moment of connection
	External applied moment
M_{eq}	Equivalent critical moment
M_f	Warping bending moment of flange
M_{FA}, M_{FB}	Fixed-ended bending moment of member <i>A B</i>
M_i	Internal resisting moment
M_l	Local interactive ultimate moment
M_{lt}	First-order elastic moment of framed column of lateral translation

M_{\max}	Absolute value of maximum moment of unbraced segment Maximum bending moment
M_{nt}	First-order elastic moment of framed column of no lateral translation
M_{ocr}	Elastic-plastic critical moment under uniform bending
M_{p}	Full plastic moment
M_{pc}	Plastic moment under compression and bending
M_{r}	Limit state of inelastic flexural-torsional buckling moment
M_{s}	Saint-Venant torque Uniform torque Effective plastic moment as defined in AS 4100 – 1998
M_{t}	Service moment of a Partially restrained connection Applied torque
M_{u}	Ultimate moment capacity
M_x	Bending moment about major x axis Unit moment of plate about y axis
M_y	Bending moment about minor y axis Yield moment Unit moment of plate about x axis
M_{xy}	Unit twisting moment of plate
M_z	Torque about longitudinal z axis
M_{ξ}, M_{η}	Bending moments about ξ and η axes in a deformed cross-section
M_{ζ}	Torsional moment about ζ axis in a deformed cross-section
M_{ω}	Warping torque
$[N]$	Integration operator
N_x, N_y	Middle surface forces of plate in x and y directions respectively
N_{xy}	Middle surface shear force of plate
N'_x, N'_y, N'_{xy}	Membrane forces
P	Load Axial compression force
P_{cr}	Buckling load
P_{crd}	Elastic distortional buckling load
P_{crl}	Elastic local interactive buckling load
$P_{\text{crx}}, P_{\text{cry}}$	Buckling loads about x and y axes respectively
P_{d}	Design load Distortional interactive ultimate load
P_{E}	Euler buckling load
P_{e}	Extreme fiber yield load of section
P_{l}	Local interactive ultimate load

P_p	First-order rigid-plastic collapse load
P_r	Reduced modulus buckling load
P_t	Tangent modulus buckling load
P_u	Ultimate load
P_x	Elastic buckling load about x axis
P_y	Elastic buckling load about y axis
	Yield load
$P_{x\omega}, P_{y\omega}, P_{xy\omega}$	Flexural-torsional buckling load
P_ω	Torsional buckling load
Q	Transverse force
Q_i	Shear force at framed column end of i th story
	Gravity load acting on i th story
Q_x, Q_y	Shear forces of thin-walled open section in x and y axes
	Unit shear forces of plate
R	Outer radius at corner of cold-formed section
\bar{R}	Wagner effect coefficient due to residual stress distribution = $\int_A \sigma_r \rho^2 dA =$ $\int_A \sigma_r [(x - x_0)^2 + (y - y_0)^2] dA = \int_A \sigma_r (x^2 + y^2) dA$
S	Far end rotation stability function of beam-column Parameter of stiffened element with lip of thin-walled cross-section for limit of full section effective, $S = 1.28 \sqrt{E/\sigma_{\max}}$ Slenderness factor as defined in AS/NZS 4600 – 2005
	Shear center
	Statical moment about neutral axis
S_1	Modified far end rotation stability function of beam-column with elastically restrained ends
S_{bi}	Translational stiffness induced by unit translation angle at i th story of frame as defined in GB 50017 – 2003
S_ω	Warping statical modulus
U	Strain energy
V	Potential energy of external load
W	Generalized force
	Section modulus
	Work done by external load
W_e	Effective section modulus
W_{px}, W_{py}	Plastic section modulus about x and y axis
W_{xc}	Section modulus corresponding to compression flange

W_{xt}	Section modulus corresponding to tension flange
W_x, W_y	Elastic section modulus about x and y axis
W_ω	Warping section modulus
Z	Shape factor
a	Clear distance between transverse stiffeners Element length Plate dimension
a_w	Ratio of two times web area in compression due to application of major axis bending moment alone to area of compression flange components as defined in ANSI/AISC 360 – 2005
b	Flange width Plate dimension Section width
b_1	Outstanding width of flange
b_e	Effective width of slender compression element
b_{fc}	Width of compression flange
b_{ft}	Width of tension flange
d	Depth of section Diameter Diameter of web hole Flat depth of lip
d_e	Effective flat depth of lip
e	Eccentricity
e_0	Initial eccentricity
e_x, e_y	Eccentricities on major and minor axes
f	Design strength of steel Deflection of plate
f_b	Allowable stress for flexure Design bending stress of beam under uniform bending as defined in BS 5950 – 1 – 2000
f_c	Allowable stress for compression
f_e	Effective stress as defined in ANSI/AISC 360 – 2005
f_0	Initial deflection of plate
f_p	Proportional limit stress
f_u	Allowable ultimate stress fore shear
f_v	Design shear stress of steel, allowable stress for shear
f_{vy}	Yield shear stress
f_{ya}	Average yield strength

f_{yc}	Yield point at corner of cold-formed section
f_{yf}, f_{yw}	Yield stresses of flange and web respectively
h	Depth of cross-section
	Depth of web hole
	Distance between flange centroids
h_0	Depth of web
h_1	Distance between centroid of cross-section and upper flange
h_2	Distance between centroid of cross-section and lower flange
h_c	Twice distance from centroid to inside face of compression flange when welds are used as defined in ANSI/AISC 360 – 2005
h_e	Effective height of slender bending web
h_p	Twice distance from plastic neutral axis to inside face of compression flange when welds are used as defined in ANSI/AISC 360 – 2005
h_{1s}, h_{2s}	Distances from shear center of cross-section to upper and lower flanges
k	Parameter = $\sqrt{P/EI}$
	Spring stiffness under compression
	Buckling coefficient of plate
k_0	Buckling coefficient of single plate
k_1	Stiffness distribution factor at joint A as defined in BS 5950 – 1 – 2000
k_2	Stiffness distribution factor at joint B as defined in BS 5950 – 1 – 2000
k_c	Coefficient for slender unstiffened elements as defined in ANSI/AISC 360 – 2005
	Correction factor on strength reduction factor of beam under non-uniform bending as defined in EN 1993 – 1 – 1 EC 3 – 2005
k_e	Elastic buckling coefficient of plate
k_f	Interactive buckling coefficient of flange
k_p	Elastic-plastic buckling coefficient of plate
k_s	Buckling coefficient of plate under shear
k_x	Translational spring constant, translational stiffness as defined in AS/NZS 4600 – 2005
k_w	Interactive buckling coefficient of web
k_φ	Rotational spring constant, rotational stiffness as defined in AS/NZS 4600 – 2005
l	Length of member
l_1	Unsupported length of compression flange
l_b	Length of beam
l_c	Length of column
l_0	Effective length of member

l_p	Limiting laterally unbraced length for limit state of yielding as defined in ANSI/AISC 360 – 2005
l_r	Limiting laterally unbraced length for limit state of inelastic flexural-torsional buckling as defined in ANSI/AISC 360 – 2005
l_x, l_y	Effective lengths about x and y axes
l_ω	Effective torsion-buckling length
m	Number of half buckling waves in x direction
n	Number of half buckling waves in y direction
	Number of corners in cold-formed section
	Shape parameter in three-parameter power model connection
o	Origin of coordinates
	Centroid of cross-section
p	Parameter = $\sin\theta_0/2$
$P_{crx}, P_{cry}, P_{crxy}$	Unit buckling loads of plate
p_x, p_y	Unit middle surface loads in x and y axes
p_{xy}	Unit middle surface shear load of plate
q	Unit distributed load
q_{cr}	Unit distributed critical load
r	Inter radius at corner of cold-formed section
	Rotational spring constant
	Radius of gyration
	Spring stiffness under bending
r_0	Polar radius of gyration = $\sqrt{(I_x + I_y)/A + x_0^2 + y_0^2} = \sqrt{i_x^2 + i_y^2 + x_0^2 + y_0^2}$
r_b	Secant stiffness of connection determined by beam-line
r_{fe}	Flexural stiffness provided by compression flange in lipped channel
r_{fg}	Geometric stiffness provided by compression flange in lipped channel
r_i	Initial stiffness of connection
r_{i0}	Initial connection secant stiffness
r_s	Secant stiffness of connection due to service moment M_s of a Partially restrained connection
r_t	Tangent stiffness of connection
	Radius of gyration of flange components in flexural compression plus one-third of web area in compression due to application of major axis bending moment alone as defined in ANSI/AISC 360 – 2005
r_{ts}	Effective radius of gyration used in determination of l_r for flexural-torsional buckling limit state for major axis bending of doubly symmetric compact I-shaped members and channels as defined in ANSI/AISC 360 – 2005
r_{we}	Flexural stiffness provided by web in lipped channel

r_{wg}	Geometric stiffness provided by web in lipped channel
r_x, r_y	Radii of gyration about x and y axes
s	Coordinate along the middle line of thin-walled section
$[r_0]$	Rotational spring constant limitation
t	Thickness of element
	Thickness of thin-walled cross-section
t_1	Compression flange thickness
t_s	Stiffener thickness
t_{fc}	Compression flange thickness as defined in ANSI/AISC 360 – 2005
t_{ft}	Tension flange thickness
t_w	Web thickness
u	Displacement of shear center in x direction
	Displacement of any point at plate in x direction
u_B	Displacement of point B at cross-section in x direction
u_0	Displacement of any point at middle surface of plate in x direction
v	Displacement of shear center in y direction
v_B	Displacement of point B at cross-section in y direction
v_0	Displacement of any point at middle surface of plate in y direction
	Amplitude of initial deflection
v_s	Displacement of any point at cross-section in tangent direction
w	Displacement of any point at plate in z direction
x_0, y_0	Coordinates of shear center of cross-section
x_i, y_i	Coordinates of an element
y_{max}	Maximum deflection
z_{ei}	Distance between elastic element and y axis
z_i	Distance between element and y axis
Δ	Displacement of unbraced frame
Δ_0	First-order displacement of unbraced frame
Φ	Curvature
Π	Total potential energy
Ω	Arching ratio
Ω_b	Safety factor for compression as defined in ANSI/AISC 360 – 2005
Ω_c	Safety factor for flexure as defined in ANSI/AISC 360 – 2005
α	Amplification factor for compression as defined in ANSI/AISC 360 – 2005
	Aspect ratio of plate
	Imperfection factor
	Modification factor of linear stiffness of beam
	Modification coefficient of plate element in thin-walled cross-section