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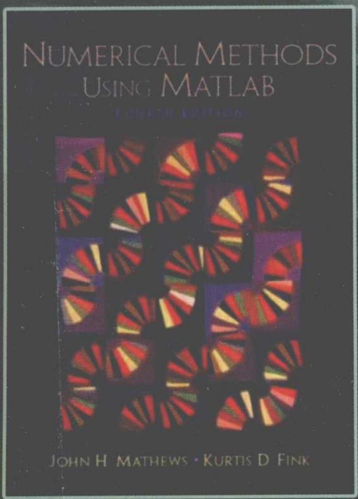
数值分析

Numerical Methods Using MATLAB

Fourth Edition



英文版

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数值分析

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（英文版）

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Publishing House of Electronics Industry
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内容简介

本书为适合国内双语教学需求对原著进行了缩编（适合一学期课程采用），新增了一些必要的证明过程以及改编者在长期教学实践中积累的1000个常用英汉数学词汇以及一些数学表达式的读法。书中介绍了数值方法的基本理论和计算方法，并讲述如何利用MATLAB软件实现各种数值算法。它的突出特点是把经典的数值方法内容与现代MATLAB计算软件相结合，强调利用MATLAB的内置函数命令进行数值方法（算法）的程序设计，程序语句简短，算法优化。另外，利用MATLAB软件的图像处理功能，给出各种数值问题的近似解及误差的可视化解释，图文并茂。书中的每个概念均以实例说明，同时还包含大量的习题与编程练习，通过这些实例进一步说明数值方法的实际应用，从而提高读者的实践能力并加深对数值计算方法的理解，以便为读者今后的学习打下坚实的数值分析与科学计算基础。

本书经过缩编后适合一学期课程使用，可作为大专院校数学、计算机及工程各专业双语教学的教材和参考书。

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出 版 说 明

21世纪初的5至10年是我国国民经济和社会发展的重要时期，也是信息产业快速发展的关键时期。在我国加入WTO后的今天，培养一支适应国际化竞争的一流IT人才队伍是我国高等教育的重要任务之一。信息科学和技术方面人才的优劣与多寡，是我国面对国际竞争时成败的关键因素。

当前，正值我国高等教育特别是信息科学领域的教育调整、变革的重大时期，为使我国教育体制与国际化接轨，有条件的高等院校正在为某些信息学科和技术课程使用国外优秀教材和优秀原版教材，以使我国在计算机教学上尽快赶上国际先进水平。

电子工业出版社秉承多年来引进国外优秀图书的经验，翻译出版了"国外计算机科学教材系列"丛书，这套教材覆盖学科范围广、领域宽、层次多，既有本科专业课程教材，也有研究生课程教材，以适应不同院系、不同专业、不同层次的师生对教材的需求，广大师生可自由选择和自由组合使用。这些教材涉及的学科方向包括网络与通信、操作系统、计算机组织与结构、算法与数据结构、数据库与信息处理、编程语言、图形图像与多媒体、软件工程等。同时，我们也适当引进了一些优秀英文原版教材，本着翻译版本和英文原版并重的原则，对重点图书既提供英文原版又提供相应的翻译版本。

在图书选题上，我们大都选择国外著名出版公司出版的高校教材，如Pearson Education培生教育出版集团、麦格劳－希尔教育出版集团、麻省理工学院出版社、剑桥大学出版社等。撰写教材的许多作者都是蜚声世界的教授、学者，如道格拉斯·科默(Douglas E. Comer)、威廉·斯托林斯(William Stallings)、哈维·戴特尔(Harvey M. Deitel)、尤利斯·布莱克(Uyless Black)等。

为确保教材的选题质量和翻译质量，我们约请了清华大学、北京大学、北京航空航天大学、复旦大学、上海交通大学、南京大学、浙江大学、哈尔滨工业大学、华中科技大学、西安交通大学、国防科学技术大学、解放军理工大学等著名高校的教授和骨干教师参与了本系列教材的选题、翻译和审校工作。他们中既有讲授同类教材的骨干教师、博士，也有积累了几十年教学经验的老教授和博士生导师。

在该系列教材的选题、翻译和编辑加工过程中，为提高教材质量，我们做了大量细致的工作，包括对所选教材进行全面论证；选择编辑时力求达到专业对口；对排版、印制质量进行严格把关。对于英文教材中出现的错误，我们通过与作者联络和网上下载勘误表等方式，逐一进行了修订。

此外，我们还将与国外著名出版公司合作，提供一些教材的教学支持资料，希望能为授课老师提供帮助。今后，我们将继续加强与各高校教师的密切联系，为广大师生引进更多的国外优秀教材和参考书，为我国计算机科学教学体系与国际教学体系的接轨做出努力。

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改编者序

John H. Mathews与Kurtis D. Fink合著的Numerical Methods Using MATLAB, Fourth Edition是为数学、计算机和工程各专业本科生学习“数值分析”（计算方法）课程而写的一本经典教材，在美国及世界各地许多著名大学开设的“数值分析”课程都把它作为教材使用，已被翻译成多种文字（如中文、俄文和西班牙文）在世界各地使用。原书最鲜明的特点是把经典的数值方法内容与现代MATLAB计算软件结合起来，讲述如何利用MATLAB软件实现各种数值算法，强调利用MATLAB的内置函数命令进行数值方法（算法）的程序设计，编程语句简短，算法优化，并利用MATLAB软件的图像处理功能，给出各种数值问题的近似解及误差的可视化解释，图文并茂。

Mathews的书的内容是为一学年的课程设置的，内容丰富，全书600多页。但是，书中的某些章节对于中国学生来说是不必要的，比如第1章的微积分预备知识，3.1节和3.2节的高等代数预备知识等。中国学生在学“数值分析”课程之前已经在其他单独开设的课程中学过这些知识。并且，在我国绝大多数大学的数学、计算机和工程等各专业培养方案中，基础的“数值分析”课程仅为一学期课程。鉴于这个原因，我在安徽大学多年使用此教材的基础上，征得原书作者和Pearson Education出版公司的同意后，对此书进行了改编。

改编后的书适合一学期课程教学使用，更加适合中国学生。在改编过程中，使用Mathews教材的其他老师们也提出了许多改编的建设性建议。在尽可能尊重原书风貌的基础上，具体改编如下：

- 将原书内容压缩成6章，300多页，使其内容完全覆盖我国理工科各专业“数值分析”课程教学大纲的全部内容。建议54学时的理论课，18学时的编程上机操作；
- 对于原书中有些只叙述而未给出证明的定理和结论，给出了详细的证明，使改编后的版本从理论上更加严谨而完整；
- 对于原书中出现的错误，通过与作者联络和网上下载勘误表等方式，逐一进行了更正；
- 改编后的版本在附录中增加了改编者在长期双语教学实践中积累的1000个常用英汉数学词汇和一些数学表达式的读法，以便更加适合双语教学。

改编后的版本虽经反复推敲，但由于改编者水平有限，书中的不当之处仍难免，恳请广大同行和读者给予指正（敬请致函 flhuang@ahu.edu.cn），以便进一步提高本书的质量，更好地推动双语教学。

黄仿伦
安徽大学数学科学学院
2009年7月

CONTENTS

Chapter 1

Solution of Nonlinear Equations $f(x) = 0$

Consider the physical problem that involves a spherical ball of radius r that is submerged to a depth d in water(see Figure 1.1). Assume that the ball is constructed from a variety of longleaf pine that has a density of $\rho = 0.638$ and that its radius measures $r = 10$ cm. How much of the ball will be submerged when it is placed in water?

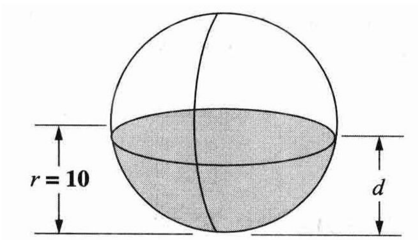

Figure 1.1 The portion of a sphere of radius r that is to be submerged to a depth d.

The mass M_w of water displaced when a sphere is submerged to a depth d is

$$M_w = \int_0^d \pi(r^2 - (x-r)^2)\,dx = \frac{\pi d^2(3r - d)}{3},$$

and the mass of the ball is $M_b = 4\pi r^3 \rho/3$. Applying Archimedes' law, $M_w = M_b$, produces the following equation that must be solved:

$$\frac{\pi(d^3 - 3d^2 r + 4r^3\rho)}{3} = 0.$$

In our case (with $r = 10$ and $\rho = 0.638$) this equation becomes

$$\frac{\pi(2552 - 30d^2 + d^3)}{3} = 0.$$

The graph of the cubic polynomial $y = 2552 - 30d^2 + d^3$ is shown in Figure 1.2 and from it one can see that the solution lies near the value $d = 12$.

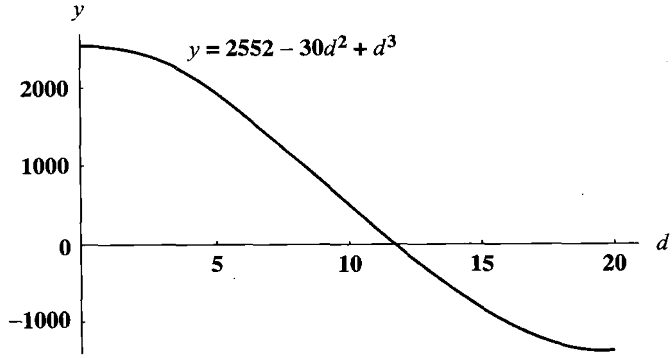

Figure 1.2 The cubic $y = 2552 - 30d^2 + d^3$.

The goal of this chapter is to develop a variety of methods for finding numerical approximations for the roots of an equation. For example, the bisection method could be applied to obtain the three roots $d_1 = -8.17607212$, $d_2 = 11.86150151$, and $d_3 = 26.31457061$. The first root d_1 is not a feasible solution for this problem, because d cannot be negative. The third root d_3 is larger than the diameter of the sphere and it is not the solution desired. The root $d_2 = 11.86150151$ lies in the interval $[0, 20]$ and is the proper solution. Its magnitude is reasonable because a little more than one-half of the sphere must be submerged.

1.1 Iteration for Solving $x = g(x)$

A fundamental principle in computer science is *iteration*. As the name suggests, a process is repeated until an answer is achieved. Iterative techniques are used to find roots of equations, solutions of linear and nonlinear systems of equations, and solutions of differential equations. In this section we study the process of iteration using repeated substitution.

A rule or function $g(x)$ for computing successive terms is needed, together with a starting value p_0. Then a sequence of values $\{p_k\}$ is obtained using the iterative rule $p_{k+1} = g(p_k)$. The sequence has the pattern

$$\begin{aligned}
&p_0 && \text{(starting value)}\\
&p_1 = g(p_0)\\
&p_2 = g(p_1)\\
&\quad\vdots\\
&p_k = g(p_{k-1})\\
&p_{k+1} = g(p_k)\\
&\quad\vdots
\end{aligned} \tag{1.1}$$

What can we learn from an unending sequence of numbers? If the numbers tend to a limit, we feel that something has been achieved. But what if the numbers diverge or are periodic? The next example addresses this situation.

Example 1.1. The iterative rule $p_0 = 1$ and $p_{k+1} = 1.001p_k$ for $k = 0, 1, \ldots$ produces a divergent sequence. The first 100 terms look as follows:

$$\begin{aligned}
p_1 &= 1.001p_0 &&= (1.001)(1.000000) &&= 1.001000,\\
p_2 &= 1.001p_1 &&= (1.001)(1.001000) &&= 1.002001,\\
p_3 &= 1.001p_2 &&= (1.001)(1.002001) &&= 1.003003,\\
&\;\;\vdots && \;\;\vdots && \;\;\vdots\\
p_{100} &= 1.001p_{99} &&= (1.001)(1.104012) &&= 1.105116.
\end{aligned}$$

The process can be continued indefinitely, and it is easily shown that $\lim_{n\to\infty} p_n = +\infty$. In Chapter 6 we will see that the sequence $\{p_k\}$ is a numerical solution to the differential equation $y' = 0.001y$. The solution is known to be $y(x) = e^{0.001x}$. Indeed, if we compare the 100th term in the sequence with $y(100)$, we see that $p_{100} = 1.105116 \approx 1.105171 = e^{0.1} = y(100)$.

In this section we are concerned with the types of functions $g(x)$ that produce convergent sequences $\{p_k\}$.

1.1.1 Finding Fixed Points

Definition 1.1. A ***fixed point*** of a function $g(x)$ is a real number P such that $P = g(P)$.

Geometrically, the fixed points of a function $y = g(x)$ are the points of intersection of $y = g(x)$ and $y = x$.

Definition 1.2. The iteration $p_{n+1} = g(p_n)$ for $n = 0, 1, \ldots$ is called ***fixed-point iteration***.

Theorem 1.1. Assume that g is a continuous function and that $\{p_n\}_{n=0}^{\infty}$ is a sequence generated by fixed-point iteration. If $\lim_{n\to\infty} p_n = P$, then P is a fixed point of $g(x)$.

Proof. If $\lim_{n\to\infty} p_n = P$, then $\lim_{n\to\infty} p_{n+1} = P$. It follows from this result, the continuity of g, and the relation $p_{n+1} = g(p_n)$ that

$$g(P) = g\left(\lim_{n\to\infty} p_n\right) = \lim_{n\to\infty} g(p_n) = \lim_{n\to\infty} p_{n+1} = P. \tag{1.2}$$

Therefore, P is a fixed point of $g(x)$. ∎

Example 1.2. Consider the convergent iteration

$$p_0 = 0.5 \quad \text{and} \quad p_{k+1} = e^{-p_k} \quad \text{for} \quad k = 0, 1, \ldots.$$

The first 10 terms are obtained by the calculations

$$
\begin{aligned}
p_1 &= e^{-0.500000} = 0.606531\\
p_2 &= e^{-0.606531} = 0.545239\\
p_3 &= e^{-0.545239} = 0.579703\\
&\ \ \vdots \qquad\qquad\ \ \vdots\\
p_9 &= e^{-0.566409} = 0.567560\\
p_{10} &= e^{-0.567560} = 0.566907
\end{aligned}
$$

The sequence is converging, and further calculations reveal that

$$\lim_{n\to\infty} p_n = 0.567143\ldots.$$

Thus we have found an approximation for the fixed point of the function $y = e^{-x}$.

The following two theorems establish conditions for the existence of a fixed point and the convergence of the fixed-point iteration process to a fixed point.

Theorem 1.2. Assume that $g \in C[a, b]$.

If the range of the mapping $y = g(x)$ satisfies $y \in [a, b]$ for all $x \in [a, b]$, then g has a fixed point in $[a, b]$. (1.3)

Furthermore, suppose that $g'(x)$ is defined over (a, b) and that a positive constant $K < 1$ exists with $|g'(x)| \le K < 1$ for all $x \in (a, b)$; then g has a unique fixed point P in $[a, b]$. (1.4)

Proof of (1.3). If $g(a) = a$ or $g(b) = b$, the assertion is true. Otherwise, the values of $g(a)$ and $g(b)$ must satisfy $g(a) \in (a, b]$ and $g(b) \in [a, b)$. The function $f(x) \equiv x - g(x)$ has the property that

$$f(a) = a - g(a) < 0 \quad \text{and} \quad f(b) = b - g(b) > 0.$$

Now apply the intermediate value theorem, to $f(x)$, with the intermediate value 0, and conclude that there exists a number P with $P \in (a, b)$ so that $f(P) = 0$. Therefore, $P = g(P)$ and P is the desired fixed point of $g(x)$.

Proof of (1.4). Now we must show that this solution is unique. By way of contradiction, let us make the additional assumption that there exist two fixed points P_1 and P_2. Now apply the Lagrange mean value theorem, and conclude that there exists a number $d \in (a, b)$ so that

$$g'(d) = \frac{g(P_2) - g(P_1)}{P_2 - P_1}. \tag{1.5}$$

Next, use the facts that $g(P_1) = P_1$ and $g(P_2) = P_2$ to simplify the right side of equation (1.5) and obtain

$$g'(d) = \frac{P_2 - P_1}{P_2 - P_1} = 1.$$

But this contradicts the hypothesis in (1.4) that $|g'(x)| < 1$ over (a, b), so it is not possible for two fixed points to exist. Therefore, $g(x)$ has a unique fixed point P in $[a, b]$ under the conditions given in (1.4).

•

Example 1.3. Apply Theorem 1.2 to show rigorously that $g(x) = \cos(x)$ has a unique fixed point in $[0, 1]$.

Clearly, $g \in C[0, 1]$. Also, $g(x) = \cos(x)$ is a decreasing function on $[0, 1]$; thus its range on $[0, 1]$ is $[\cos(1), 1] \subseteq [0, 1]$. Thus condition (1.3) of Theorem 1.2 is satisfied and g has a fixed point in $[0, 1]$. Finally, if $x \in (0, 1)$, then $|g'(x)| = |-\sin(x)| = \sin(x) \leq \sin(1) < 0.8415 < 1$. Thus $K = \sin(1) < 1$, condition (1.4) of Theorem 1.2 is satisfied, and g has a unique fixed point in $[0, 1]$.

We can now state a theorem that can be used to determine whether the fixed-point iteration process given in (1.1) will produce a convergent or a divergent sequence.

Theorem 1.3 (Fixed-Point Theorem). Assume that (i) $g, g' \in C[a, b]$, (ii) K is a positive constant, (iii) $p_0 \in (a, b)$, and (iv) $g(x) \in [a, b]$ for all $x \in [a, b]$.

If $|g'(x)| \leq K < 1$ for all $x \in [a, b]$, then the iteration $p_n = g(p_{n-1})$ will converge to the unique fixed point $P \in [a, b]$. In this case, P is said to be an attractive fixed point. (1.6)

If $|g'(x)| > 1$ for all $x \in [a, b]$, then the iteration $p_n = g(p_{n-1})$ will not converge to P. In this case, P is said to be a repelling fixed point and the iteration exhibits local divergence. (1.7)

Remark 1. It is assumed that $p_0 \neq P$ in statement (1.7).
Remark 2. Because g is continuous on an interval containing P, it is permissible to use the simpler criterion $|g'(P)| \leq K < 1$ and $|g'(P)| > 1$ in (1.6) and (1.7), respectively.
Proof. We first show that the points $\{p_n\}_{n=0}^{\infty}$ all lie in (a, b). Starting with p_0, we apply the Lagrange mean value theorem. There exists a value $c_0 \in (a, b)$ so that

$$\begin{aligned} |P - p_1| &= |g(P) - g(p_0)| = |g'(c_0)(P - p_0)| \\ &= |g'(c_0)||P - p_0| \leq K|P - p_0| < |P - p_0|. \end{aligned} \tag{1.8}$$

Therefore, p_1 is no further from P than p_0 was, and it follows that $p_1 \in (a, b)$ (see Figure 1.3). In general, suppose that $p_{n-1} \in (a, b)$; then

$$\begin{aligned} |P - p_n| &= |g(P) - g(p_{n-1})| = |g'(c_{n-1})(P - p_{n-1})| \\ &= |g'(c_{n-1})||P - p_{n-1}| \leq K|P - p_{n-1}| < |P - p_{n-1}|. \end{aligned} \tag{1.9}$$

Therefore, $p_n \in (a, b)$ and hence, by induction, all the points $\{p_n\}_{n=0}^{\infty}$ lie in (a, b).

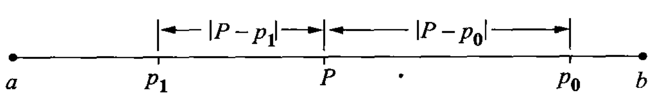

Figure 1.3 The relationship among P, p_0, p_1, $|P - p_0|$, and $|P - p_1|$.

To complete the proof of (1.6), we will show that

$$\lim_{n\to\infty} |P - p_n| = 0. \tag{1.10}$$

First, a proof by induction will establish the inequality

$$|P - p_n| \leq K^n |P - p_0|. \tag{1.11}$$

The case $n = 1$ follows from the details in relation (1.8). Using the induction hypothesis $|P - p_{n-1}| \leq K^{n-1}|P - p_0|$ and the ideas in (1.9), we obtain

$$|P - p_n| \leq K|P - p_{n-1}| \leq KK^{n-1}|P - p_0| = K^n|P - p_0|.$$

Thus, by induction, inequality (1.11) holds for all n. Since $0 < K < 1$, the term K^n goes to zero as n goes to infinity. Hence

$$0 \leq \lim_{n\to\infty} |P - p_n| \leq \lim_{n\to\infty} K^n|P - p_0| = 0. \tag{1.12}$$

The limit of $|P - p_n|$ is squeezed between zero on the left and zero on the right, so we can conclude that $\lim_{n\to\infty} |P - p_n| = 0$. Thus $\lim_{n\to\infty} p_n = P$ and, by Theorem 1.1, the iteration $p_n = g(p_{n-1})$ converges to the fixed point P. Therefore, statement (1.6) of Theorem 1.3 is proved. We leave statement (1.7) for the reader to investigate.

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Corollary 1.1. Assume that g satisfies the hypothesis given in (1.6) of Theorem 1.3. Bounds for the error involved when using p_n to approximate P are given by

$$|P - p_n| \leq K^n|P - p_0| \quad \text{for all} \quad n \geq 1 \tag{1.13}$$

and

$$|P - p_n| \leq \frac{K^n|p_1 - p_0|}{1 - K} \quad \text{for all} \quad n \geq 1. \tag{1.14}$$

Proof of (1.14.) For all $n \geq 1$, using the Lagrange mean value theorem, we have

$$\begin{aligned} |p_{n+1} - p_n| &= |g(p_n) - g(p_{n-1})| = |g'(c_n)(p_n - p_{n-1})| \\ &\leq K|p_n - p_{n-1}| \leq \cdots \leq K^n|p_1 - p_0|. \end{aligned}$$

For any $m > n$,

$$\begin{aligned} |p_m - p_n| &= |p_m - p_{m-1} + p_{m-1} - p_{m-2} + \cdots + p_{n+1} - p_n| \\ &\leq |p_m - p_{m-1}| + |p_{m-1} - p_{m-2}| + \cdots + |p_{n+1} - p_n| \\ &\leq K^{m-1}|p_1 - p_0| + K^{m-2}|p_1 - p_0| + \cdots + K^n|p_1 - p_0| \\ &= K^n|p_1 - p_0|(K^{m-n-1} + K^{m-n-2} + \cdots + K^2 + K + 1) \\ &= K^n|p_1 - p_0|\frac{1 - K^{n-m}}{1 - K} \leq \frac{K^n|p_1 - p_0|}{1 - K}. \end{aligned}$$

Therefore,

$$|P - p_n| = \lim_{m\to\infty} |p_m - p_n| \leq \frac{K^n|p_1 - p_0|}{1 - K}.$$

•

1.1.2 Graphical Interpretation of Fixed-Point Iteration

Since we seek a fixed point P to $g(x)$, it is necessary that the graph of the curve $y = g(x)$ and the line $y = x$ intersect at the point (P, P). Two simple types of convergent iteration, monotone and oscillating, are illustrated in Figure 1.4(a) and (b), respectively.

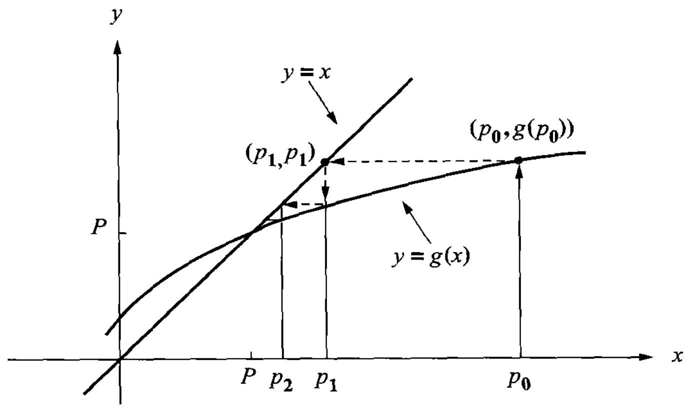

Figure 1.4 (a) Monotone convergence when $0 < g'(P) < 1$.

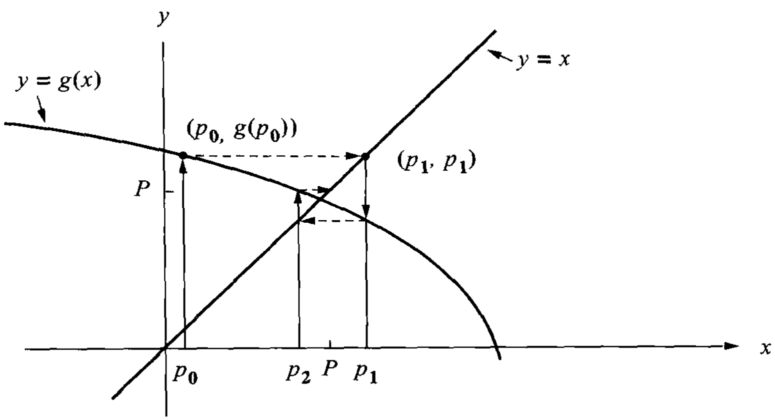

Figure 1.4 (b) Oscillating convergence when $-1 < g'(P) < 0$.

To visualize the process, start at p_0 on the x-axis and move vertically to the point $(p_0, p_1) = (p_0, g(p_0))$ on the curve $y = g(x)$. Then move horizontally from (p_0, p_1) to the point (p_1, p_1) on the line $y = x$. Finally, move vertically downward to p_1 on the x-axis. The recursion $p_{n+1} = g(p_n)$ is used to construct the point (p_n, p_{n+1}) on the graph, then a horizontal motion locates (p_{n+1}, p_{n+1}) on the line $y = x$, and then a vertical movement ends up at p_{n+1} on the x-axis. The situation is shown in Figure 1.4.

If $|g'(P)| > 1$, then the iteration $p_{n+1} = g(p_n)$ produces a sequence that diverges away from P. The two simple types of divergent iteration, monotone and oscillating, are illustrated in Figure 1.5(a) and (b), respectively.

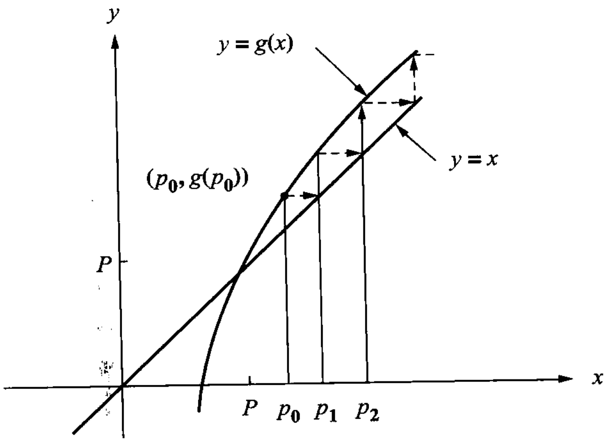

Figure 1.5 (a) Monotone divergence when $1 < g'(P)$.

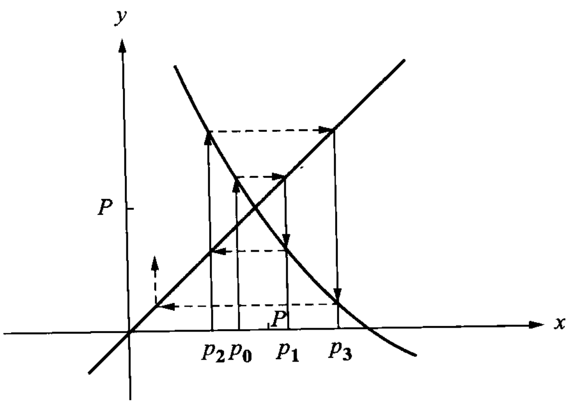

Figure 1.5 (b) Divergent oscillation when $g'(P) < -1$.

Example 1.4. Consider the iteration $p_{n+1} = g(p_n)$ when the function $g(x) = 1 + x - x^2/4$ is used. The fixed points can be found by solving the equation $x = g(x)$. The two solutions (fixed points of g) are $x = -2$ and $x = 2$. The derivative of the function is $g'(x) = 1 - x/2$, and there are only two cases to consider.