Springer 大学数学图书——影印版

Matrix Groups

An Introduction to Lie Group Theory

矩阵群

李群理论基础

Andrew Baker 著

TSINGHUA UNIVERSITY PRESS

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内 容 提 要

本书讲述李群和李代数基础理论，内容先进，讲述方法科学，易于掌握和使用。书中有大量例题和习题（附答案或提示），便于阅读。适合用作大学数学系和物理系高年级本科生选修课教材、研究生课程教材或参考书。

Andrew Baker
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序　言

在学校教书多年，当学生（特别是本科生）问有什么好的参考书时，我们所能推荐的似乎除了教材还是教材，而且不同教材之间的差别并不明显、特色也不鲜明。所以多年前我们就开始酝酿，希望为本科学生引进一些好的参考书，为此清华大学数学科学系的许多教授与清华大学出版社共同付出了很多心血。

这里首批推出的十余本图书，是从 Springer 出版社的多个系列丛书中精心挑选出来的。在丛书的筹划过程中，我们挑选图书最重要的标准并不是完美，而是有特色并包容各个学派（有些书甚至有争议，比如从数学上看也许不够严格），其出发点是希望我们的学生能够吸纳百家之长；同时，在价格方面，我们也做了很多工作，以使得本系列丛书的价格能让更多学校和学生接受，使得更多学生能够从中受益。

本系列图书按其定位，大体有如下四种类型（一本书可以属于多类，但这里限于篇幅不能一一介绍）。

一、适用面比较广、有特色并可以用作教材或参考书的图书。例如：

- Lovász et al.: Discrete Mathematics. 2003

  该书是离散数学的入门类型教材。与现有的教材（包括国外的教材）相比，它涵盖了离散数学新颖而又前沿的研究课题，同时还涉及信息科学方面既基本又有趣的应用；在着力打好数学基础的同时，也强调了数学与信息科学的关联。姚期智先生倡导和主持的清华大学计算机科学试验班，已经选择该书作为离散数学课程的教材。

二、在目前国内的数学教育中，课程主要以学科的纵向发展为主线，而对数学不同学科之间的联系讨论很少。学生缺乏把不同学科视为一个数学整体的训练，这方面的读物尤其欠缺。这是本丛书一个重要的着力点。最典型的是：

- Fine/Rosenberger: The Fundamental Theorem of Algebra. 1997

  该书对数学中最重要的定理——代数基本定理给出了六种证明，方法涉及到分析、代数与拓扑；附录中还给出了 Gauss 的证明和 Cauchy 的证明。全书以一个数学问题为主线展开，纵横数学的核心领域；结构严谨、文笔流畅、浅显易懂、引人入胜，是一本少见的能够让读者入迷的好读物，用它来引导学生欣赏和领会“数学的美”绝对不会落于空谈。该书适于自学、讨论，也是极好的短学期课程教材。

- Baker: Matrix Groups. 2001

  就内容而言，本书并不超出我国大学线性代数、抽象代数和一般拓扑学课程的内容，但是本书所讲的是李群和李代数的基础理论——这是现代数学和物理学非常重要的工具。各种矩阵群和矩阵代数是李群和李代数最典型和

最重要的例子，同时也能帮助学生建立数学不同学科之间的联系。从矩阵出发，既能把握李群和李代数的实质，又能学会计算和运用，所以这是一本不可多得的好书。

三、科学与技术的发展不断为数学提出新的研究课题，因此在数学学科的发展过程中，来自其他学科的推动力是不能忽视的。本系列中第三种类型的读物就是强调数学与其他学科的联系。例如：

● Woodhouse: Special Relativity. 2003

该书将物理与数学有机结合，体现了物理学家伽利略的名言："大自然是一部用数学语言写成的巨著。" 不仅如此，本书作者还通过对线性代数、微积分、场论等数学的运用进一步强调并贯穿这样的观点：数学的真谛和发展存在并产生于物理或自然规律及其发现中。精读此书有助于理解物理学和数学的整体关系。

● Britton: Essential Mathematical Biology. 2003

生命科学在本世纪一定会有很大发展，其对数学的需求和推动是可以预见的。因此生物数学在应用数学中占有日益重要的地位，数学系培养的学生至少一部分人应当对这个领域有所了解。随着生命科学的迅速发展，生物数学也发展很快。本书由浅入深，从经典的问题入手，最后走向学科前沿和近年的热点问题。该书至少可以消除学生对生物学的神秘感。

四、最后一类是适合本科学生的课外读物。这类图书对激发和引导学生学习数学的兴趣会非常有帮助，而且目前国内也急需这样的图书。例如：

● Daepp/Gorkin: Reading, Writing and Proving. 2003

该书对初学高等数学的读者来说特别有意义。它的基本出发点是引导读者以研究的心态去学习，让读者养成独立思考的习惯，并进而成为研究型的学习者。该书将一个学习数学的过程在某种意义下程序化，努力让学习者养成一个好的学习习惯，以及学会如何应对问题。该书特色鲜明，类似的图书确实很少。

● Brzezniak/Zastawniak: Basic Stochastic Processes. 1998

随机过程理论在数学、科学和工程中有越来越广泛的应用，本书适合国内的需要。其主要特点是：书中配有的习题是巩固和延伸学习内容的基本手段，而且有十分完整的解答，非常适合自学和作为教学参考书。这是一本难得的好书，它 1999 年出版，到 2000 年已经是第 3 次印刷，到 2003 年则第 6 次重印。

● Anglin/Lambek: The Heritage of Thales. 1995

该书的基本内容是数学的历史和数学的哲学。数学历史是该书的线索，数学是内容的主体，引申到的是数学哲学。它不是一本史论型的著作，而是采用专题式编写方式，每个专题相对独立，所以比较易读、易懂，是本科生学习数学过程中非常好的课外读物。

本系列丛书中的大部分图书还将翻译为中文出版，以适应更多读者的需要。丛书筹划过程中，冯克勤、郑志勇、卢旭光、郑建华、王殿军、杨利军、叶俊、扈志明等很多清华大学的教授都投入了大量精力。他们之中很多人也将是后面中文版的译者。此外，我们今后还将不断努力丰富引进丛书的种类，同时也会将选书的范围在可能情况下进一步扩大到其他高水平的出版机构。

教育是科学和技术发展的基石，数学教育更是基石的基础。因为是基础所以它重要；也因为是基础所以它显示度不高，容易不被重视。只有将人才培养放到更高的地位上，中国成为创新型国家的目标才会成为可能。

本系列丛书的正式推出，圆了一个我们多年的梦，但这无疑仅仅是开始。

白峰杉

2006 年 6 月于清华园

Preface

This work provides a first taste of the theory of Lie groups accessible to advanced mathematics undergraduates and beginning graduate students, providing an appetiser for a more substantial further course. Although the formal prerequisites are kept as low level as possible, the subject matter is sophisticated and contains many of the key themes of the fully developed theory. We concentrate on *matrix groups*, *i.e.*, closed subgroups of real and complex general linear groups. One of the results proved is that every matrix group is in fact a Lie group, the proof following that in the expository paper of Howe [12]. Indeed, the latter, together with the book of Curtis [7], influenced our choice of goals for the present book and the course which it evolved from. As pointed out by Howe, Lie theoretic ideas lie at the heart of much of standard undergraduate linear algebra, and exposure to them can inform or motivate the study of the latter; we frequently describe such topics in enough detail to provide the necessary background for the benefit of readers unfamiliar with them.

Outline of the Chapters

Each chapter contains exercises designed to consolidate and deepen readers' understanding of the material covered. We also use these to explore related topics that may not be familiar to all readers but which should be in the toolkit of every well-educated mathematics graduate. Here is a brief synopsis of the chapters.

Chapter 1: The *general linear groups* $\mathrm{GL}_n(\Bbbk)$ for $\Bbbk = \mathbb{R}$ (the real numbers) and $\Bbbk = \mathbb{C}$ (the complex numbers) are introduced and studied both as groups and as topological spaces. *Matrix groups* are defined and a number of standard examples discussed, including *special linear groups* $\mathrm{SL}_n(\Bbbk)$, *orthogonal groups* $\mathrm{O}(n)$ and *special orthogonal groups* $\mathrm{SO}(n)$, *unitary groups* $\mathrm{U}(n)$ and *special unitary groups* $\mathrm{SU}(n)$, as well as more exotic examples such as *Lorentz groups* and *symplectic groups.* The relation of complex to real matrix groups is also studied. Along the way we discuss various algebraic, analytic and topological notions including *norms, metric spaces, compactness* and *continuous group actions.*

Chapter 2: The *exponential function* for matrices is introduced and *one-parameter subgroups* of matrix groups are studied. We show how these ideas can be used in the solution of certain types of differential equations.

Chapter 3: The idea of a *Lie algebra* is introduced and various algebraic properties are studied. *Tangent spaces* and Lie algebras of matrix groups are defined together with the *adjoint action.* The important special case of $\mathrm{SU}(2)$ and its relationship to $\mathrm{SO}(3)$ is studied in detail.

Chapters 4 and 5: *Finite dimensional algebras* over fields, especially $\mathbb{R}$ or $\mathbb{C}$, are defined and their units viewed as a source of matrix groups using the *reduced regular representation.* The *quaternions* and more generally the *real Clifford algebras* are defined and *spinor groups* constructed and shown to double cover the special orthogonal groups. The *quaternionic symplectic groups* $\mathrm{Sp}(n)$ are also defined, completing the list of compact connected classical groups and their universal covers. *Automorphism groups* of algebras are also shown to provide further examples of matrix groups.

Chapter 6: The geometry and linear algebra of *Lorentz groups* which are of importance in Relativity are studied. The relationship of $\mathrm{SL}_2(\mathbb{C})$ to the Lorentz group $\mathrm{Lor}(3,1)$ is discussed, extending the work on $\mathrm{SU}(2)$ and $\mathrm{SO}(3)$ in Chapter 3.

Chapter 7: The general notion of a *Lie group* is introduced and we show that all matrix groups are *Lie subgroups* of general linear groups. Along the way we introduce the basic ideas of *differentiable manifolds* and *smooth maps.* We show that not every Lie group can be realised as a matrix group by considering the simplest *Heisenberg group.*

Chapters 8 and 9: *Homogeneous spaces* of Lie groups are defined and we show how to recognise them as orbits of *smooth actions.* We discuss *connectivity* of Lie groups and use homogeneous spaces to prove that many familiar Lie groups are path connected. We also describe some important families of homogeneous spaces such as *projective spaces* and *Grassmannians,* as well as examples related to special factorisations of matrices such as *polar form.*

Chapters 10, 11 and 12: The basic theory of *compact connected Lie groups* and their *maximal tori* is studied and the relationship to some well-known matrix diagonalisation results highlighted. We continue this theme by describing the classification theory of compact connected *simple* Lie groups, showing how the families we meet in earlier chapters provide all but a finite number of the isomorphism types predicted. *Root systems, Weyl groups* and *Dynkin diagrams* are defined and many examples described.

Some suggestions for using this book

For an advanced undergraduate course of about 30 lectures to students already equipped with basic real and complex analysis, metric spaces, linear algebra, group and ring theory, the material of Chapters 1, 2, 3, 7 provide an introduction to matrix groups, while Chapters 4, 5, 6, 8, 9 supply extra material that might be quarried for further examples. A more ambitious course aimed at presenting the classical compact connected Lie groups might take in Chapters 4, 5 and perhaps lead on to some of the theory of compact connected Lie groups discussed in Chapters 10, 11, 12.

A reader (perhaps a graduate student) using the book on their own would find it useful to follow up some of the references [6, 8, 17, 18, 25, 29] to see more advanced approaches to the topics on differential geometry and topology covered in Chapters 7, 8, 9 and the classification theory of Chapters 10, 11, 12.

Each chapter has a set of Exercises of varying degrees of difficulty. Hints and solutions are provided for some of these, the more challenging questions being indicated by the symbols ⚠ or ⚠⚠ with the latter intended for readers wishing to pursue the material in greater depth.

Prerequisites and assumptions

The material in Chapters 1, 2, 3, 7 is intended to be accessible to a well-equipped advanced undergraduate, although many topics such as non-metric topological spaces, normed vector spaces and rings may be unfamiliar so we have given the relevant definitions. We do not assume much abstract algebra beyond standard notions of homomorphisms, subobjects, kernels and images and quotients; semi-direct products of groups are introduced, as are Lie algebras. A course on matrix groups is a good setting to learn algebra, and there are many significant algebraic topics in Chapters 4, 5, 11, 12. Good sources of background material are [5, 15, 16, 22, 28]

The more advanced parts of the theory which are described in Chapters 7, 8, 9, 10, 11, 12 should certainly challenge students and naturally point to more detailed studies of Differential Geometry and Lie Theory. Occasionally ideas from Algebraic Topology are touched upon (*e.g.*, the fundamental group and Lefschetz Fixed Point Theorem) and an interested reader might find it helpful to consult an introductory book on the subject such as [9, 20, 25].

Typesetting

This book was produced using LaTeX and the American Mathematical Society's **amsmath** package. Diagrams were produced using XY-pic. The symbol ⚠ was produced by my colleague J. Nimmo, who also provided other help with TeX and LaTeX.

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Remarks on the second printing

I have taken the opportunity to correct various errors found in the first printing and would like to thank R. Barraclough, R. Chapman, P. Eccles, S. Hendren, N. Pollock, R. Wickner and M. Yankelevitch for their helpful comments and error-spotting. The web page

`http://www.maths.gla.ac.uk/~ajb/MatrixGroups/`

contains an up to date list of known errors and corrections.

A reader wishing to pursue exceptional Lie groups and connections with Physics will find much of interest in the excellent recent survey paper of Baez [30], while representation theory is covered by Bröcker and tom Dieck [31] and Sternberg [27].

Contents

Part I. Basic Ideas and Examples

Part III. Compact Connected Lie Groups and their Classification

Part I

Basic Ideas and Examples