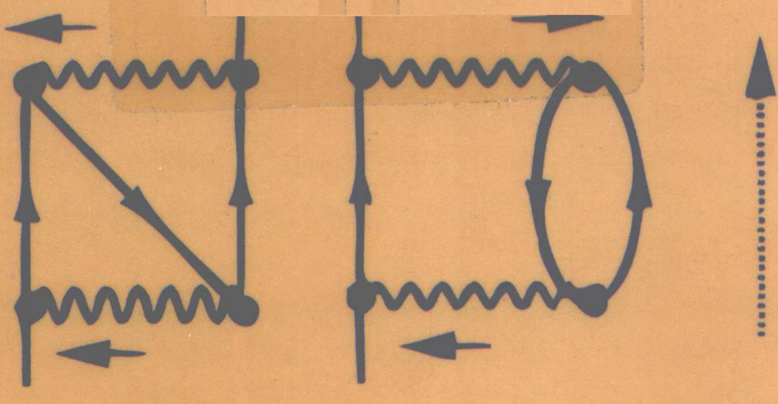
Michael Stone

The Physics of Quantum Fields

量子场物理学



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(continued following index)

Dedicated to the memory of my father, Thomas Alfred Stone.

Preface

This book is intended to provide a general introduction to the physics of quantized fields and many-body physics. It is based on a two-semester sequence of courses taught at the University of Illinois at Urbana-Champaign at various times between 1985 and 1997. The students taking all or part of the sequence had interests ranging from particle and nuclear theory through quantum optics to condensed matter physics experiment.

The book does not cover as much ground as some texts. This is because I have tried to concentrate on the basic conceptual issues that many students find difficult. For a computation-method oriented course an instructor would probably wish to suplement this book with a more comprehensive and specialized text such as Peskin and Schroeder *An Introduction to Quantum Field Theory*, which is intended for particle theorists, or perhaps the venerable *Quantum Theory of Many-Particle Systems* by Fetter and Walecka.

The most natural distribution of the material if the book is used for a two-semster course is as follows:

1st Semester: Chapters 1-11.

2nd semester: Chapters 12-18.

The material in the first 11 chapters is covered using traditional quantum mechanics operator language. This is because the text is intended for people with a wide range of interests. Were I writing for particle-theory students only, I would start with path integrals from chapter one. For a broader readership, it seems useful to maintain continuity with traditional hamiltonian quantum mechanics for as long as one as there is no penalty in ease of comprehension — and this is the case with the simple field theories discussed in the earlier chapters.

In the second half of the book the path integral comes into its own. It is seen as an efficient generator Feynman rules and Ward identities, and is, of course, indispensible for understanding the connection between renormalization and critical phenomena, as well as non-perturbative phemomena such as tunneling.

Although the book is not intended primarily for students of condensed matter physics, many of the examples discussed in the text are drawn from that field. This choice partly reflects my own interests, which have, over the years, wandered from high energy physics through lattice gauge theories to systems with real crystal lattices. There is, however, another reason: condensed matter systems can been seen and felt. I believe that it is far easier to acquire a visceral understanding of spontaneous symmetry breaking in a superfluid, than it is to grasp chiral symmetry breaking in QCD. Furthermore, condensed matter systems have mathematically well-defined hamiltonians and any field theory method applied to them has to reproduce the measured properties. This is usually not the case in relativistic systems, where additional principles have to be applied in order to decide which of several regularization-dependendent answers is correct. While there is nothing wrong with this, it is frequently disturbing to the beginner. Also it was only when Ken Wilson used field theory to address the concrete condensed matter problem of critical phenomena that the origin of the perturbation theory divergences was understood.

Michael Stone
Urbana, Illinois
October, 1999

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Discrete Systems

1.1 One-Dimensional Harmonic Crystal

We begin with the quantum mechanics of a vibrating crystal. To the naked eye the crystal appears to be a continuous elastic solid. We know, however, that, when viewed through a sufficiently powerful microscope it will be revealed to be composed of individual atoms held together by chemical bonds. For our purpose the atoms and bonds can be thought of as "balls and springs," and the crystal as an assembly of coupled harmonic oscillators. If you understand the quantum mechanics of harmonic oscillators, it will not be difficult to apply this understanding to study the effectively continuous crystal. This is our task in this chapter.

1.1.1 Normal Modes

To avoid the complexities of real crystals with their plethora of elastic constants and modes, we will consider a simple one-dimensional model of a crystal.

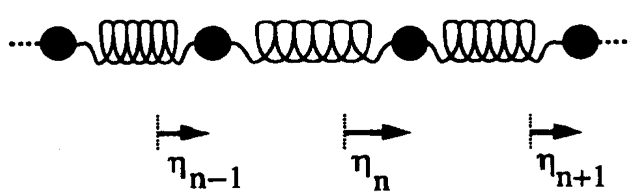

Fig 1. A one-dimensional crystal.

We will take a line of atoms of unit mass whose equilibrium positions are at a set of sites on the x axis labeled by the integer n, and separated by a distance a. We will assume the atoms are free to vibrate only in the x direction, so we are dealing with longitudinal waves, and denote the displacement of the atom at site n by η_n.

The quickest route to the dynamics uses the lagrangian. As always in mechanics this is the difference of the kinetic energy T and the potential energy V. For a *harmonic crystal* V is a sum of terms of the form $\frac{1}{2}\lambda(\eta_n - \eta_{n+1})^2$, where λ is the spring constant. Thus

$$L = T - V = \sum_n \left\{ \frac{1}{2}\dot{\eta}_n^2 - \frac{\lambda}{2}(\eta_n - \eta_{n+1})^2 \right\}. \tag{1.1}$$

From Lagrange's equations, one for each η_n,

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\eta}_n}\right) - \frac{\partial L}{\partial \eta_n} = 0, \tag{1.2}$$

we find the classical equations of motion

$$\ddot{\eta}_n = \lambda(\eta_{n+1} + \eta_{n-1} - 2\eta_n). \tag{1.3}$$

These have solutions in the form of complex traveling waves

$$\eta_n = e^{ikn - i\omega t}, \tag{1.4}$$

where

$$\omega^2 = 2\lambda(1 - \cos k). \tag{1.5}$$

In the long-wavelength limit $k \ll 1$, this dispersion relation reduces to

$$\omega^2 = \lambda k^2, \tag{1.6}$$

which means that the long-wavelength sound waves have velocity $\sqrt{\lambda}$.

In the next chapter we will have cause to consider an additional term in the lagrangian, which corresponds to a harmonic potential $\frac{1}{2}\Omega^2\eta_n^2$ pinning each of the particles to the vicinity of its initial location. Including this, L becomes

$$L = \sum_n \left\{ \frac{1}{2}\dot{\eta}_n^2 - \frac{\lambda}{2}(\eta_n - \eta_{n+1})^2 - \frac{1}{2}\Omega^2\eta_n^2 \right\}. \tag{1.7}$$

The dispersion relation is now

$$\omega^2 \to 2\lambda(1 - 1\cos k) + \Omega^2 \approx \lambda k^2 + \Omega^2. \tag{1.8}$$

The additional potential therefore creates a *gap* in the spectrum, so there are no solutions corresponding to any frequency below Ω.

To determine the normal modes we must impose boundary conditions. Suppose we take periodic boundary conditions by identifying atom $n + N$ with atom n. This means that η_n must equal η_{n+N}. Consequently, we require e^{ikN} to be unity and the allowed values of k are therefore

$$k_m = \frac{2\pi m}{N}, \quad m = 0, 1, \ldots, N - 1. \tag{1.9}$$

We can now write a normal-mode expansion

$$\eta_n(t) = \sum_{m=0}^{N-1} \left\{ A_m e^{ik_m n - i\omega_m t} + A_m^* e^{-ik_m n + i\omega_m t} \right\}. \tag{1.10}$$

Because the total displacement is a real number, we have added to each original complex exponential solution its complex conjugate.

From (1.1) we read off the momentum canonically conjugate to the dispacement η_n

$$\pi_n \stackrel{def}{=} \frac{\partial L}{\partial \dot{\eta}_n} = \dot{\eta}_n. \tag{1.11}$$

In quantum mechanics the displacement η_n and its canonical conjugate π_n become operators $\hat{\eta}_n$ and $\hat{\pi}_n$ with commutation relations

$$[\hat{\eta}_n, \hat{\pi}_m] = i\hbar\delta_{nm}. \tag{1.12}$$

From (1.10) we find that

$$\pi_n(t) = \dot{\eta}_n(t) = \sum_{m=0}^{N-1} \left\{ -i\omega_m A_m e^{ik_m n - i\omega_m t} + i\omega_m A_m^* e^{-ik_m n + i\omega_m t} \right\}. \tag{1.13}$$

We have a choice as to how to include time evolution in the quantum mechanics formalism. In the Schrödinger[1] picture we put the time dependence in the Hilbert-space states and leave the operators time independent. This is the customary approach in elementary quantum mechanics courses, and is what we usually have in mind when we write equations like (1.12). In field theory it turns out to be more convenient to use the *Heisenberg*[2] picture where the operators are explicitly time dependent. For any operator $\hat{O}$ we have

$$\hat{O}(t) = e^{\frac{i}{\hbar}\hat{H}t}\hat{O}(0)e^{-\frac{i}{\hbar}\hat{H}t}, \tag{1.14}$$

and

$$\frac{d\hat{O}}{dt} = \frac{i}{\hbar}[\hat{H}, \hat{O}]. \tag{1.15}$$

When we use the Heisenberg picture, we must specify the times at which the fields in the commutation relation are to be evaluated. To retain its simple form (1.12) must be replaced by an equal-time commutator

$$[\hat{\eta}_n(t), \hat{\eta}_m(t)] = i\hbar\delta_{nm}. \tag{1.16}$$

Finding the commutator with the operators evaluated at two different times requires solving the dynamics of the system.

---

[1]Erwin Schrödinger. Born August 12, 1887, Vienna. Died January 4, 1961, Vienna. Nobel Prize for Physics 1933.

[2]Werner Karl Heisenberg. Born December 5, 1901. Died February 1, 1976, Munich. Nobel Prize for Physics 1932.

1.1.2 Harmonic Oscillator

Let us recall how the Heisenberg picture works for the harmonic oscillator.

For a unit mass oscillator with angular frequency ω, the hamiltonian is

$$\hat{H} = \frac{1}{2}(\hat{p}^2 + \omega^2\hat{x}^2). \tag{1.17}$$

Here the operators $\hat{x}(t)$ and $\hat{p}(t)$ obey the equal-time commutation relation

$$[\hat{x}(t), \hat{p}(t)] = i\hbar. \tag{1.18}$$

The equations of motion are

$$\frac{d\hat{x}(t)}{dt} = \frac{i}{\hbar}[\hat{H}, \hat{x}] = \hat{p}(t), \tag{1.19}$$

$$\frac{d\hat{p}(t)}{dt} = \frac{i}{\hbar}[\hat{H}, \hat{p}] = -\omega^2\hat{x}(t). \tag{1.20}$$

Differentiating the first equation with repect to t, and substituting for $\frac{d\hat{x}}{dt}$ from the second shows that

$$\frac{d^2\hat{x}}{dt^2} + \omega^2\hat{x} = 0. \tag{1.21}$$

The Heisenberg operator $\hat{x}(t)$ therefore satisfies exactly the same equation of motion as the classical variable $x(t)$ it replaces.

We could write down the solution to (1.21) in terms of sines and cosines, but it is more productive to introduce the operators $\hat{a}(t)$ and $\hat{a}^\dagger(t)$ by writing

$$\hat{x}(t) = \sqrt{\frac{\hbar}{2\omega}}(\hat{a}(t) + \hat{a}^\dagger(t)) \tag{1.22}$$

$$\hat{p}(t) = \sqrt{\frac{\hbar}{2\omega}}(-i\omega\hat{a}(t) + i\omega\hat{a}^\dagger(t)). \tag{1.23}$$

Equivalently,

$$\hat{a}(t) = \sqrt{\frac{\omega}{2\hbar}}\left(\hat{x}(t) + i\frac{\hat{p}(t)}{\omega}\right), \tag{1.24}$$

$$\hat{a}^\dagger(t) = \sqrt{\frac{\omega}{2\hbar}}\left(\hat{x}(t) - i\frac{\hat{p}(t)}{\omega}\right). \tag{1.25}$$

Their equal-time commutation relations are found from those of $\hat{x}$, $\hat{p}$, to be

$$[\hat{a}(t), \hat{a}^\dagger(t)] = 1. \tag{1.26}$$

We also see that

$$\hat{H} = \hbar\omega(\hat{a}^\dagger(t)\hat{a}(t) + \frac{1}{2}). \tag{1.27}$$

So

$$\frac{d\hat{a}(t)}{dt} = \frac{i}{\hbar}[\hat{H}, \hat{a}(t)] = -i\omega\hat{a}(t) \quad \Rightarrow \hat{a}(t) = \hat{a}(0)e^{-i\omega t}, \tag{1.28}$$

$$\frac{d\hat{a}^\dagger(t)}{dt} = \frac{i}{\hbar}[\hat{H}, \hat{a}^\dagger(t)] = +i\omega\hat{a}^\dagger(t) \quad \Rightarrow \hat{a}^\dagger(t) = \hat{a}^\dagger(0)e^{+i\omega t}. \tag{1.29}$$

From now on we will write $\hat{a}$ for $\hat{a}(0)$, and similarly for $\hat{a}^\dagger(0)$. In field theory these are called the *annihilation* and *creation* operators, respectively.

The time dependence of $\hat{x}(t)$ and $\hat{p}(t)$ is now explicit:

$$\hat{x}(t) = \sqrt{\frac{\hbar}{2\omega}}(\hat{a}e^{-i\omega t} + \hat{a}^\dagger e^{+i\omega t}), \tag{1.30}$$

$$\hat{p}(t) = \sqrt{\frac{\hbar}{2\omega}}(-i\omega\hat{a}e^{-i\omega t} + i\omega\hat{a}^\dagger e^{+i\omega t}). \tag{1.31}$$

If we substitute these expressions into the hamiltonian, we find that it is time independent

$$\hat{H} = \hbar\omega(\hat{a}^\dagger\hat{a} + \frac{1}{2}), \tag{1.32}$$

just as it is in classical mechanics.

1.1.3 Annihilation and Creation Operators for Normal Modes

Inspired by the harmonic oscillator, let us try setting

$$\hat{\eta}_n(t) = \sum_{m=0}^{N-1} \sqrt{\frac{\hbar}{2\omega_m}} \frac{1}{\sqrt{N}} \left\{\hat{a}_m e^{ik_m n - i\omega_m t} + \hat{a}_m^\dagger e^{-ik_m n + i\omega_m t}\right\}, \tag{1.33}$$

$$\hat{\pi}_n(t) = \sum_{m=0}^{N-1} \sqrt{\frac{\hbar}{2\omega_m}} \frac{1}{\sqrt{N}} \left\{-i\omega_m\hat{a}_m e^{ik_m n - i\omega_m t} + i\omega_m\hat{a}_m^\dagger e^{-ik_m n + i\omega_m t}\right\}, \tag{1.34}$$

where $[\hat{a}_m, \hat{a}_n^\dagger] = \delta_{mn}$, and computing the equal-time commutator, $[\hat{\eta}_n(t), \hat{\pi}_m(t)]$, to see if it comes out right. We have some hope that this will work since the $\sqrt{\frac{\hbar}{2\omega}}$'s are suggested by the harmonic-oscillator case, and the $\frac{1}{\sqrt{N}}$'s serve to normalize the normal modes.

In dealing with these sorts of sums it is useful to remember the finite Fourier series identity

$$\sum_{m=0}^{N-1} e^{ik_m(n-n')} = N\delta_{nn'}, \tag{1.35}$$

which is easily proved from the formula for the sum of a geometric progression.