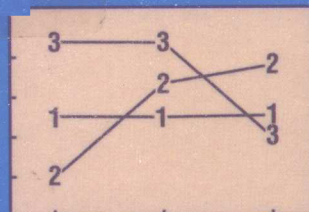
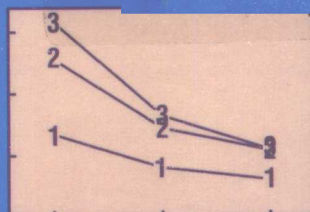


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Angela Dean
Daniel Voss

Design and Analysis of Experiments

实验设计和分析



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Preface

Our initial motivation for writing this book was the observation from various students that the subject of design and analysis of experiments can seem like “a bunch of miscellaneous topics.” We believe that the identification of the objectives of the experiment and the practical considerations governing the design form the heart of the subject matter and serve as the link between the various analytical techniques. We also believe that learning about design and analysis of experiments is best achieved by the planning, running, and analyzing of a simple experiment.

With these considerations in mind, we have included throughout the book the details of the planning stage of several experiments that were run in the course of teaching our classes. The experiments were run by students in statistics and the applied sciences and are sufficiently simple that it is possible to discuss the planning of the entire experiment in a few pages, and the procedures can be reproduced by readers of the book. In each of these experiments, we had access to the investigators’ actual report, including the difficulties they came across and how they decided on the treatment factors, the needed number of observations, and the layout of the design. In the later chapters, we have included details of a number of published experiments. The outlines of many other student and published experiments appear as exercises at the ends of the chapters.

Complementing the practical aspects of the design are the statistical aspects of the analysis. We have developed the theory of estimable functions and analysis of variance with some care, but at a low mathematical level. Formulae are provided for almost all analyses so that the statistical methods can be well understood, related design issues can be discussed, and computations can be done by hand in order to check computer output.

We recommend the use of a sophisticated statistical package in conjunction with the book. Use of software helps to focus attention on the statistical issues rather than on the calculation. Our particular preference is for the SAS software, and we have included the elementary use of this package at the end of most chapters. Many of the SAS program files and data sets used in the book can be found at www.springer-ny.com. However, the book can equally well be used with any other statistical package. Availability of statistical software has also helped shape the book in that we can discuss more complicated analyses—the analysis of unbalanced designs, for example.

The level of presentation of material is intended to make the book accessible to a wide audience. Standard linear models under normality are used for all analyses. We have avoided

using calculus, except in a few optional sections where least squares estimators are obtained. We have also avoided using linear algebra, except in an optional section on the canonical analysis of second-order response surface designs. Contrast coefficients are listed in the form of a vector, but these are interpreted merely as a list of coefficients.

This book reflects a number of personal preferences. First and foremost, we have not put side conditions on the parameters in our models. The reason for this is threefold. Firstly, when side conditions are added to the model, all the parameters appear to be estimable. Consequently, one loses the perspective that in factorial experiments, main effects can be interpreted only as averages over any interactions that happen to be present. Secondly, the side conditions that are the most useful for hand calculation do not coincide with those used by the SAS software. Thirdly, if one feeds a nonestimable parametric function into a computer program such as PROC GLM in SAS, the program will declare the function to be “nonestimable,” and the user needs to be able to interpret this statement. A consequence is that the traditional solutions to the normal equations do not arise naturally. Since the traditional solutions are for nonestimable parameters, we have tried to avoid giving these, and instead have focused on the estimation of functions of $E[Y]$, all of which are estimable.

We have concentrated on the use of prespecified models and preplanned analyses rather than exploratory data analysis. We have emphasized the experimentwise control of error rates and confidence levels rather than individual error rates and confidence levels.

We rely upon residual plots rather than formal tests to assess model assumptions. This is because of the additional information provided by residual plots. For example, plots to check homogeneity of variance also indicate when a variance-stabilizing transformation should be effective. Likewise, nonlinear patterns in a normal probability plot may indicate whether inferences under normality are likely to be liberal or conservative. Except for some tests for lack of fit, we have, in fact, omitted all details of formal testing for model assumptions, even though they are readily available in many computer packages.

The book starts with basic principles and techniques of experimental design and analysis of experiments. It provides a checklist for the planning of experiments, and covers analysis of variance, inferences for treatment contrasts, regression, and analysis of covariance. These basics are then applied in a wide variety of settings. Designs covered include completely randomized designs, complete and incomplete block designs, row-column designs, single replicate designs with confounding, fractional factorial designs, response surface designs, and designs involving nested factors and factors with random effects, including split-plot designs.

In the last few years, “Taguchi methods” have become very popular for industrial experimentation, and we have incorporated some of these ideas. Rather than separating Taguchi methods as special topics, we have interspersed them throughout the chapters via the notion of including “noise factors” in an experiment and analyzing the variability of the response as the noise factors vary.

We have introduced factorial experiments as early as Chapter 3, but analyzed them as one-way layouts (i.e., using a cell means model). The purpose is to avoid introducing factorial experiments halfway through the book as a totally new topic, and to emphasize that many factorial experiments are run as completely randomized designs. We have analyzed contrasts in a two-factor experiment both via the usual two-way analysis of variance model (where

the contrasts are in terms of the main effect and interaction parameters) and also via a cell-means model (where the contrasts are in terms of the treatment combination parameters). The purpose of this is to lay the groundwork for Chapters 13–15, where these contrasts are used in confounding and fractions. It is also the traditional notation used in conjunction with Taguchi methods.

The book is not all-inclusive. For example, we do not cover recovery of interblock information for incomplete block designs with random block effects. We do not provide extensive tables of incomplete block designs. Also, careful coverage of unbalanced models involving random effects is beyond our scope. Finally, inclusion of SAS graphics is limited to low-resolution plots.

The book has been classroom tested successfully over the past five years at The Ohio State University, Wright State University, and Kenyon College, for junior and senior undergraduate students majoring in a variety of fields, first-year graduate students in statistics, and senior graduate students in the applied sciences. These three institutions are somewhat different. The Ohio State University is a large land-grant university offering degrees through the Ph.D., Wright State University is a mid-sized university with few Ph.D. programs, and Kenyon College is a liberal arts undergraduate college. Below we describe typical syllabi that have been used.

At OSU, classes meet for five hours per week for ten weeks. A typical class is composed of 35 students, about a third of whom are graduate students in the applied statistics master's program. The remaining students are undergraduates in the mathematical sciences or graduate students in industrial engineering, biomedical engineering, and various applied sciences. The somewhat ambitious syllabus covers Chapters 1–7 and 10, Sections 11.1–11.4, and Chapters 13, 15, and 17. Students taking these classes plan, run, and analyze their own experiments, usually in a team of four or five students from several different departments. This project serves the function of giving statisticians the opportunity of working with scientists and seeing the experimental procedure firsthand, and gives the scientists access to colleagues with a broader statistical training. The experience is usually highly rated by the student participants.

Classes at WSU meet four hours per week for ten weeks. A typical class involves about 10 students who are either in the applied statistics master's degree program or who are undergraduates majoring in mathematics with a statistics concentration. Originally, two quarters (20 weeks) of probability and statistics formed the prerequisite, and the course covered much of Chapters 1–4, 6, 7, 10, 11, and 13, with Chapters 3 and 4 being primarily review material. Currently, students enter with two additional quarters in applied linear models, including regression, analysis of variance, and methods of multiple comparisons, and the course covers Chapters 1 and 2, Sections 3.2, 6.7, and 7.5, Chapters 10, 11, and 13, Sections 15.1–15.2, and perhaps Chapter 16. As at OSU, both of these syllabi are ambitious. During the second half of the course, the students plan, run, and analyze their own experiments, working in groups of one to three. The students provide written and oral reports on the projects, and the discussions during the oral reports are of mutual enjoyment and benefit. A leisurely topics course has also been offered as a sequel, covering the rest of Chapters 14–17.

At Kenyon College, classes meet for three hours a week for 15 weeks. A typical class is composed of about 10 junior and senior undergraduates majoring in various fields. The syllabus covers Chapters 1–7, 10, and 17.

For some areas of application, random effects, nested models, and split-plot designs, which are covered in Chapters 17–19, are important topics. It is possible to design a syllabus that reaches these chapters fairly rapidly by covering Chapters 1–4, 6, 7, 17, 18, 10, 19.

We owe a debt of gratitude to many. For reading of, and comments on, prior drafts, we thank Bradley Hartlaub, Jeffrey Nunemacher, Mark Irwin, an anonymous reviewer, and the many students who suffered through the early drafts. We thank Baoshe An, James Clark, Amy Ferketich, and Dionne Pratt for checking a large number of exercises, and Lisa Abrams, Paul Burte, Kathryn Collins, Yuming Deng, Joseph Mesaros, Dionne Pratt, Kari Rabe, Joseph Whitmore, and many others for catching numerous typing errors. We are grateful to Peg Steigerwald, Terry England, Dolores Wills, Jill McClane, and Brian J. Williams for supplying hours of typing skills. We extend our thanks to all the many students in classes at The Ohio State University, Wright State University, and the University of Wisconsin at Madison whose imagination and diligence produced so many wonderful experiments; also to Brian H. Williams and Bob Wardrop for supplying data sets; to Nathan Buurma, Colleen Brensinger, and James Colton for library searches; and to the publishers and journal editors who gave us permission to use data and descriptions of experiments. We are especially grateful to the SAS Institute for permission to reproduce portions of SAS programs and corresponding output, and to John Kimmel for his patience and encouragement throughout this endeavor.

This book has been ten years in the making. In the view of the authors, it is “a work in progress temporarily cast in stone”—or in print, as it were. We are wholly responsible for any errors and omissions, and we would be most grateful for comments, corrections, and suggestions from readers so that we can improve any future editions.

Finally, we extend our love and gratitude to Jeff, Nancy, Tommy, and Jimmy, often neglected during this endeavor, for their enduring patience, love, and support.

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