

ADVANCED TOPICS IN SCIENCE AND TECHNOLOGY IN CHINA

Wohua Zhang  
Yuanqiang Cai

# Continuum Damage Mechanics and Numerical Applications



ZHEJIANG UNIVERSITY PRESS  
浙江大学出版社



Springer

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With 425 figures



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Springer

## 图书在版编目 (CIP) 数据

连续损伤力学及其数值分析应用=Continuum Damage  
Mechanics and Numerical Applications: 英文 / 张我华, 蔡  
袁强著. —杭州: 浙江大学出版社, 2010.5

(中国科技进展丛书)

ISBN 978-7-308-06589-4

I. 连… II. ①张…②蔡… III. 连续性—损伤 (力学)—  
数值计算—研究—英文 IV. 0346.5

中国版本图书馆CIP数据核字 (2009) 第130797号

Not for sale outside Mainland of China  
此书仅限中国大陆地区销售

## 连续损伤力学及其数值分析应用

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责任编辑 杜希武  
封面设计 俞亚彤  
出版发行 浙江大学出版社  
网址: <http://www.zjupress.com>  
Springer-Verlag GmbH  
网址: <http://www.springer.com>  
排 版 浙江理想广告有限公司  
印 刷 杭州富春印务有限公司  
开 本 710mm×960mm 1/16  
印 张 59  
字 数 1699千  
版 印 次 2010年5月第1版 2010年5月第1次印刷  
书 号 ISBN 978-7-308-06589-4 (浙江大学出版社)  
ISBN 978-3-642-04707-7 (Springer-Verlag GmbH)  
定 价 379.00元

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浙江大学出版社发行部邮购电话 (0571) 88925591

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## Preface

The progress of failure in metals, under various loading conditions, is assumed to involve the degradation of a structure due to nucleation and growth of defects, such as microvoids and microcracks, and their coalescence into macrocracks. This process, generically termed damage, was first used to predict material failure and rupture in-service in an elevated condition. Although damage mechanics provides a measure of material degradation on a micromechanics scale, the damage variables are introduced to reflect average material degradation on a macromechanics scale and thus continuum damage mechanics (CDM) was developed. In the micro-cracking of materials under different stress conditions, damage is regarded as the progressive degradation. This material degradation is reflected in the non-linear behaviour of the structures. Non-linear analysis based on CDM provides conservative and realistic results. Since the pioneering work of Kachanov in 1958, continuum damage mechanics has been widely accepted to describe progressive failure due to material degradation. The reason for its popularity is as much the intrinsic simplicity and versatility of the approach, as well as its consistency based on the theory of the thermodynamics of irreversible processes. When the crack profiles are not known a priori, the continuum damage mechanics approaches are computationally very attractive. CDM is a very applicable and rapidly developing discipline. Now many papers are published and several international conferences, e.g., IUTAM-Symposia or EUROMECH-Colloquia, take place. Furthermore, a special International Journal of Damage Mechanics stresses the importance of this branch of solid mechanics.

Based on the concept of Kachanov, many constitutive equations have been developed to describe the phenomenological aspects of the damage process. In addition to rupture times, secondary and tertiary progressive failure behavior of materials can be well predicted using the phenomenological equations in which the material is treated as a continuum. Since the detailed process of degradation of the material is not easily examined, a theoretical description of the damage state in a continuum and its evolution can be rather complicated and some assumptions or postulates are made to describe the rate of damage

evolution. The usual assumptions have a certain generality, which allows the resulting equations to be fitted to different experimental data with a degree of success but they are not based on microstructural observations or physical reasoning. The material constants in these equations do not have clear physical meanings and the dominant damage mechanisms cannot be modeled using the equations. So experimental investigations of damage mechanism in this field are difficult, especially under multiaxial stress and non-proportional loading. Therefore, entitative experimental data are scarcely available, so that intrinsic comparisons between theory and the hypostatic experiment are often impossible. Material scientists studying damage are not content with this vague description of damage. The dissatisfaction is reinforced when attempts are made to model the growth of voids or cracks during degradation which can lead to equations that do not appear to resemble those of the continuum treatment. The weakness of the approach is further demonstrated by the obvious experimental fact that there are several mechanisms of complex damage, while the continuum equations appear to describe only one. Thus the research on different damages has been extended into the area of categorizing damage mechanisms. Mathematical representations of the corresponding damage mechanisms, damage evolution, and their effects on nonlinear deformation have been studied and developed. Based on the development of the understanding of the damage mechanisms, physically inspired, multivariable damage models have been proposed and used for the modeling of complex rupture of materials.

This book presents a systematic development of the theory of Continuum Damage Mechanics and its numerical engineering applications using a unified form of the mathematical formulations used in engineering for either anisotropic or isotropic damage models. The principles presented in this book include the latest progress in continuum damage mechanics and research in this area developed by the authors. The presentation is theoretical in nature emphasizing the detailed derivations of the various models and formulations. The advanced works of various active researchers in this area are also presented. The theoretical framework of this book is based on the thermodynamic theory of energy and material dissipation and is described by a set of fundamental formulations of constitutive equations of damaged materials, development equations of the damaged state and evolution equations of micro-structures. The theoretical framework of continuum damage mechanics presented in this book is constructed based on thermodynamics that deals with the theory of energy and material dissipation employing internal state variables and is described by a set of fundamental formulations of constitutive equations of damaged materials, development equations of the damaged state and evolution equations of micro-structures. According to concepts of damage-dissipation of the material state and effective evolution of material properties, all these advanced equations, which take the non-symmetrized effects of damage aspects into account, are developed and modified from the traditional general failure models so they are more easily applied and verified

in a wide range of engineering practices by experimental testing. A number of practical applications for continuum damage mechanics developed in this book are presented in different engineering topics analyzed by different numerical methods.

The book is divided into (1) an introduction; (2) review of damage mechanics; (3) basis of isotropic damage mechanics; (4) theory of isotropic elasto-plastic damage mechanics; (5) basis of anisotropic damage mechanics; (6) brittle damage mechanics of brittle materials; (7) theory of anisotropic elasto-plastic damage mechanics; (8) theory of elasto-visco-plastic damage mechanics; (9) dynamic damage problems of damaged materials.

Finally, the first author wishes to thank his hierophants Professor S. Valliapan of the University of New South Wales in Australia and Professor Xu Zhixin of Tongji University in China, without whose academic guidance this book would not have appeared. The authors would like to acknowledge the financial support for their research works provided by the National Natural Science Foundation of China.

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## Introduction

When engineering materials are subjected to unfavorable conditions such as cold and hot working processes, temperature variation, chemical action, radiation, mechanical loading or environmental conditions, microscopic defects and cracks may develop. It is generally accepted that a crack is induced or formed by nucleation of micro-cavities that are enclosed in a region of discontinuities or defects. The effects of these internal defects may reasonably be perceived by dividing them into a single finite number of discontinuities. The distributed defects in materials are responsible not only for the crack initiation and the final fracture, but also for the induced deterioration or damage, such as a reduction in strength, rigidity, toughness, stability, frequency, residual life or an increase in stress, strain, dynamic response and damping ratio.

The study of the behavior of microscopic defects and cracks within materials is of interest to both material scientists and researchers in the field of mechanics. For materials scientists, the major concern is the development process of the microscopic cracks and methods for improving the micro-structure of the material so as to improve the overall material performance whereas researchers in the field of mechanics tend to approach the effects of microscopic defects in materials by introducing an internal state variable within the framework of thermodynamics and continuum mechanics. This variable is termed the **damage variable** [1-1~1-4].

An emerging discipline called **Continuum Damage Mechanics** [1-4] has recently been receiving attention in an effort to systematically study the growth of micro-cavities and its effect on the engineering behavior of materials. This microscopic damage must somehow be quantified on a macroscopic level within the framework of continuum damage mechanics by representing the effects of distributed defects in terms of the internal state variables. A notion of damage tensor  $\Omega_{ij}$  was introduced to define the state of damage in the continuum.

Because of the significant influence of damage on the safety aspect of structures, a great deal of research has been directed to this field during the last twenty years. Kachanov [1-5] was the first to introduce such continuous vari-