Quantum Gravity

量子引力

CARLO ROVELLI

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Foreword

The problem of what happens to classical general relativity at the extreme short-distance Planck scale of 10^{-33} cm is clearly one of the most pressing in all of physics. It seems abundantly clear that profound modifications of existing theoretical structures will be mandatory by the time one reaches that distance scale. There exist several serious responses to this challenge. These include effective field theory, string theory, loop quantum gravity, thermogravity, holography, and emergent gravity. Effective field theory is to gravitation as chiral perturbation theory is to quantum chromodynamics – appropriate at large distances, and impotent at short. Its primary contribution is the recognition that the Einstein–Hilbert action is no doubt only the first term in an infinite series constructed out of higher powers of the curvature tensor. String theory emphasizes the possible roles of supersymmetry, extra dimensions, and the standard-model internal symmetries in shaping the form of the microscopic theory. Loop gravity most directly attacks the fundamental quantum issues, and features the construction of candidate wave-functionals which are background independent. Thermogravity explores the apparent deep connection of semiclassical gravity to thermodynamic concepts such as temperature and entropy. The closely related holographic ideas connect theories defined in bulk spacetimes to complementary descriptions residing on the boundaries. Finally, emergent gravity suggests that the time-tested symbiotic relationship between condensed matter theory and elementary particle theory should be extended to the gravitational and cosmological contexts as well, with more lessons yet to be learned.

In each of the approaches, difficult problems stand in the way of attaining a fully satisfactory solution to the basic issues. Each has its band of enthusiasts, the largest by far being the string community. Most of the approaches come with rather strong ideologies, especially apparent when they are popularized. The presence of these ideologies tends to isolate the communities from each other. In my opinion, this is extremely unfortunate, because it is probable that all these ideologies, including my own (which is distinct from the above listing), are dead wrong. The evidence is history: from the Greeks to Kepler to Newton to Einstein there has been no shortage of grand ideas regarding the Basic Questions. In the presence of new data available to us and not them, only fragments of those grand visions remain viable. The clutter of thirty-odd standard model parameters and the descriptive nature of modern cosmology suggests that we too have quite a way to go before ultimate simplicity is attained. This does not mean abandoning ideologies – they are absolutely essential in driving us all to work hard on the problems. But it does mean that an attitude of humility and of high sensitivity toward alternative approaches is essential.

This book is about only one approach to the subject – loop quantum gravity. It is a subject of considerable technical difficulty, and the literature devoted to it is a formidable one. This feature alone has hindered the cross-fertilization which is, as delineated above, so essential for progress. However, within these pages one will find a much more accessible description of the subject, put forward by one of its leading architects and deepest thinkers. The existence of such a fine book will allow this important subject, quite likely to contribute significantly to the unknown ultimate theory, to be assimilated by a much larger community of theorists. If this does indeed come to pass, its publication will become one of the most important developments in this very active subfield since its onset.

James Bjorken

Preface

A dream I have long held was to write a "treatise" on quantum gravity once the theory had been finally found and experimentally confirmed. We are not yet there. There is neither experimental support nor sufficient theoretical consensus. Still, a large amount of work has been developed over the last twenty years towards a quantum theory of spacetime. Many issues have been clarified, and a definite approach has crystallized. The approach, variously denoted,[1] is mostly known as "loop quantum gravity."

The problem of quantum gravity has many aspects. Ideas and results are scattered in the literature. In this book I have attempted to collect the main results and to present an overall perspective on quantum gravity, as developed during this twenty-year period. The point of view is personal and the choice of subjects is determined by my own interests. I apologize to friends and colleagues for what is missing; the reason so much is missing is due to my own limitations, for which I am the first to be sorry.

It is difficult to underestimate the vastitude of the problem of quantum gravity. The physics of the early twentieth century has modified the very foundation our understanding of the physical world, changing the meaning of the basic concepts we use to grasp it: matter, causality, space and time. We are not yet able to paint a consistent picture of the world in which these modifications taken together make sense. The problem of quantum gravity is nothing less than the problem of finding this novel consistent picture, finally bringing the twentieth century scientific revolution to an end.

Solving a problem of this sort is not just a matter of mathematical skill. As was the case with the birth of quantum mechanics, relativity, electromagnetism, and newtonian mechanics, there are conceptual and foundational problems to be addressed. We have to understand which (possibly

[1] See the notation section.

new) notions make sense and which old notions must be discarded, in order to describe spacetime in the quantum relativistic regime. What we need is not just a technique for computing, say, graviton–graviton scattering amplitudes (although we certainly want to be able to do so, eventually). We need to re-think the world in the light of what we have learned about it with quantum theory and general relativity.

General relativity, in particular, has modified our understanding of the spatio-temporal structure of reality in a way whose consequences have not yet been fully explored. A significant part of the research in quantum gravity explores foundational issues, and Part I of this book ("Relativistic foundations") is devoted to these basic issues. It is an exploration of how to rethink basic physics from scratch, after the general-relativistic conceptual revolution. Without this, we risk asking any tentative quantum theory of gravity the wrong kind of questions.

Part II of the book ("Loop quantum gravity") focuses on the loop approach. The loop theory, described in Part II, can be studied by itself, but its reason and interpretation are only clear in the light of the general framework studied in Part I. Although several aspects of this theory are still incomplete, the subject is mature enough to justify a book. A theory begins to be credible only when its original predictions are reasonably unique and are confirmed by new experiments. Loop quantum gravity is not yet credible in this sense. Nor is any other current tentative theory of quantum gravity. The interest of the loop theory, in my opinion, is that at present it is the only approach to quantum gravity leading to well-defined physical predictions (falsifiable, at least in principle) and, more importantly, it is the most determined effort for a genuine merging of quantum field theory with the world view that we have discovered with general relativity. The future will tell us more.

There are several other introductions to loop quantum gravity. Classic reports on the subject [1–8, in chronological order] illustrate various stages of the development of the theory. For a rapid orientation, and to appreciate different points of view, see the review papers [9–13]. Much useful material can be found in [14]. Good introductions to spinfoam theory are to be found in [9, 15–17]. This book is self-contained, but I have tried to avoid excessive duplications, referring to other books and review papers for nonessential topics well developed elsewhere. This book focuses on physical and conceptual aspects of loop quantum gravity. Thomas Thiemann's book [18], which is going to be completed soon, focuses on the mathematical foundation of the same theory. The two books are complementary: this book can almost be read as Volume 1 ("Introduction and conceptual framework") and Thiemann's book as Volume 2 ("Complete mathematical framework") of a general presentation of loop quantum gravity.

The book assumes that the reader has a basic knowledge of general relativity, quantum mechanics and quantum field theory. In particular, the aim of the chapters on general relativity (Chapter 2), classical mechanics (Chapter 3), hamiltonian general relativity (Chapter 4), and quantum theory (Chapter 5) is to offer the fresh perspective on these topics which is needed for quantum gravity to a reader already familiar with the conventional formulation of these theories.

Sections with comments and examples are printed in smaller fonts (see Section 1.3.1 for first such example). Sections that contain side or more complex topics and that can be skipped in a first reading without compromising the understanding of what follows are marked with a star (*) in the title. References in the text appear only when strictly needed for comprehension. Each chapter ends with a short bibliographical section, pointing out essential references for the reader who wants to go into more detail or to trace original works on a subject. I have given up the immense task of collecting a full bibliography on loop quantum gravity. On many topics I refer to specific review articles where ample bibliographic information can be found. An extensive bibliography on loop quantum gravity is given in [18].

I have written this book thinking of a researcher interested in working in quantum gravity, but also of a good Ph.D. student or an open-minded scholar, curious about this extraordinary open problem. I have found the journey towards general relativistic quantum physics, towards quantum spacetime, a fascinating adventure. I hope the reader will see the beauty I see, and that he or she will be capable of completing the journey. The landscape is magic, the trip is far from being over.

Acknowledgements

I am indebted to the many people that have sent suggestions and corrections to the draft of this book posted online. Special thanks in particular to Justin Malecki, Jacob Bourjaily and Leonard Cottrell.

My great gratitude goes to the friends with whom I have had the privilege of sharing this adventure:

To Lee Smolin, companion of adventures and friend. His unique creativity and intelligence, intellectual freedom and total honesty, are among the very best things I have found in life.

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To Laura Scodellari and Chris Isham, my first teachers who guided me into mathematics and quantum gravity.

To Ted Newman, who, with Sally, parented the little boy just arrived from the Empire's far provinces. I have shared with Ted a decade of intellectual joy. His humanity, generosity, honesty, passion and love for thinking, are the example against which I judge myself.

I would like to thank one by one all the friends working in this field, who have developed the ideas and results described in this book, but they are too many. I can only mention my direct collaborators, and a few friends outside this field: Luisa Doplicher, Simone Speziale, Thomas Schucker, Florian Conrady, Daniele Colosi, Etera Livine, Daniele Oriti, Florian Girelli, Roberto DePietri, Robert Oeckl, Merced Montesinos, Kirill Krasnov, Carlos Kozameh, Michael Reisenberger, Don Marolf, Berndt Brügmann, Junichi Iwasaki, Gianni Landi, Mauro Carfora, Jorma Louko, Marcus Gaul, Hugo Morales-Tecotl, Laurent Freidel, Renate Loll, Alejandro Perez, Giorgio Immirzi, Philippe Roche, Federico Laudisa, Jorge Pullin, Thomas Thiemann, Louis Crane, Jerzy Lewandowski, John

Baez, Ted Jacobson, Marco Toller, Jeremy Butterfield, John Norton, John Barrett, Jonathan Halliwell, Massimo Testa, David Finkelstein, Gary Horowitz, John Earman, Julian Barbour, John Stachel, Massimo Pauri, Jim Hartle, Roger Penrose, John Wheeler, and Alain Connes.

With all these friends I have had the joy of talking about physics which is far from problem-solving, from outsmarting each other, or from making weapons to make "us" stronger than "them." I think that physics is about escaping the prison of the received thoughts and searching for novel ways of thinking the world, about trying to clear a bit the misty lake of our insubstantial dreams, which reflect reality like the lake reflects the mountains.

Foremost, thanks to Bonnie – she knows why.

Terminology and notation

• In this book, "relativistic" means "*general* relativistic," unless otherwise specified. When referring to *special* relativity, I say so explicitly. Similarly, "nonrelativistic" and "prerelativistic" mean "non-*general*-relativistic" and "pre-*general*-relativistic." The choice is a bit unusual (special relativity, in this language, is "nonrelativistic"). One reason for it is simply to make language smoother: the book is about *general* relativistic physics, and repeating "*general*" every other line sounds too much like a Frenchman talking about de Gaulle. But there is a more substantial reason: the complete revolution in spacetime physics which truly deserves the name of relativity is general relativity, not special relativity. This opinion is not always shared today, but it was Einstein's opinion. Einstein has been criticized on this; but in my opinion the criticisms miss the full reach of Einstein's discovery about spacetime. One of the aims of this book is to defend in modern language Einstein's intuition that his gravitational theory is the full implementation of relativity in physics. This point is discussed at length in Chapter 2.

• I often indulge in the physicists' bad habit of mixing up function f and function values $f(x)$. Care is used when relevant. Similarly, I follow standard physicists' abuse of language in denoting a field such as the Maxwell potential as $A_\mu(x)$, $A(x)$, or A, where the three notations are treated as equivalent manners of denoting the field. Again, care is used where relevant.

• All fields are assumed to be smooth, unless otherwise specified. All statements about manifolds and functions are local unless otherwise specified; that is, they hold within a single coordinate patch. In general I do not specify the domain of definition of functions; clearly equations hold where functions are defined.

• Index notation follows the most common choice in the field: Greek indices from the middle of the alphabet $\mu, \nu, \ldots = 0, 1, 2, 3$ are 4d spacetime tangent indices. Upper case Latin indices from the middle of the alphabet $I, J, \ldots = 0, 1, 2, 3$ are 4d Lorentz tangent indices. (In the special relativistic context the two are used without distinction.) Lower case Latin indices from the beginning of the alphabet $a, b, \ldots = 1, 2, 3$ are 3d tangent indices. Lower case Latin indices from the middle of the alphabet $i, j, \ldots = 1, 2, 3$ are 3d indices in R^3. Coordinates of a 4d manifold are usually indicated as $x, y, \ldots$, while 3d manifold coordinates are usually indicated as $\vec{x}, \vec{y}$ (also as $\vec{\tau}$). Thus the components of a spacetime coordinate x are

$$x^\mu = (t, \vec{x}) = (x^0, x^a);$$

while the components of a Lorentz vector e are

$$e^I = (e^0, e^i).$$

• η_{IJ} is the Minkowski metric, with signature $[-, +, +, +]$. The indices $I, J, \ldots$ are raised and lowered with η_{IJ}. δ_{ij} is the Kronecker delta, or the R^3 metric. The indices $i, j, \ldots$ are raised and lowered with δ_{ij}.

• For reasons explained at the beginning of Chapter 2, I call "gravitational field" the tetrad field $e^I_\mu(x)$, instead of the metric tensor $g_{\mu\nu}(x) = \eta_{IJ}\, e^I_\mu(x) e^J_\nu(x)$.

• ϵ_{IJKL}, or $\epsilon_{\mu\nu\rho\sigma}$, is the completely antisymmetric object with $\epsilon_{0123} = 1$. Similarly for ϵ_{abc}, or ϵ_{ijk}, in 3d. The Hodge star is defined by

$$F^*_{IJ} = \epsilon_{IJKL}\, F^{KL}$$

in flat space, and by the same equation, where $F_{IJ} e^I_\mu e^J_\nu = F_{\mu\nu}$ and $F^*_{IJ} e^I_\mu e^J_\nu = F^*_{\mu\nu}$ in the presence of gravity. Equivalently,

$$F^*_{\mu\nu} = \sqrt{-\det g}\; \epsilon_{\mu\nu\rho\sigma}\, F^{\rho\sigma} = |\det e|\; \epsilon_{\mu\nu\rho\sigma}\, F^{\rho\sigma}.$$

• Symmetrization and antisymmetrization of indices is defined with a half: $A_{(ab)} = \frac{1}{2}(A_{ab} + A_{ba})$ and $A_{[ab]} = \frac{1}{2}(A_{ab} - A_{ba})$.

• I call "curve" on a manifold M, a map

$$\gamma : I \to M$$
$$s \mapsto \gamma^a(s),$$

where I is an interval of the real line R (possibly the entire R.) I call "path" an oriented unparametrized curve, namely an equivalence class of curves under change of parametrization $\gamma^a(s) \mapsto \gamma'^a(s) = \gamma^a(s'(s))$, with $ds'/ds > 0$.

• An orthonormal basis in the Lie algebras $su(2)$ and $so(3)$ is chosen once and for all and these algebras are identified with R^3. For $so(3)$, the basis vectors $(v_i)^j{}_k$ can be taken proportional to $\epsilon_i{}^j{}_k$; for $su(2)$, the basis vectors $(v_i)^A{}_B$ can be taken proportional to the Pauli matrices, see Appendix A1. Thus, an algebra element ω in $su(2) \sim so(3)$ has components ω^i.

• For any antisymmetric quantity v^{ij} with two 3d indices i, j, I use also the one-index notation

$$v^i = \frac{1}{2}\epsilon^i{}_{jk}\, v^{jk}, \qquad v^{ij} = \epsilon^{ij}{}_k\, v^k;$$

the one-index and the two-indices notation are considered as defining the same object. For instance the $SO(3)$ connections ω^{ij} and A^{ij} are equivalently denoted ω^i and A^i.

Symbols. Here is a list of symbols, with their name and the equation, chapter or section where they are introduced or defined.

A	area	Section 2.1.4
A	Yang–Mills connection	Equation (2.30)
$A, A^i_\mu(x)$	selfdual 4d gravitational connection	Equation (2.19)
$A, A^i_a(\vec{x})$	selfdual or real 3d gravitational connection	Sections 4.1.1, 4.2
$\mathcal{C}$	relativistic configuration space	Section 3.2.1
D_μ	covariant derivative	Equation (2.31)
*Diff**	extended diffeomorphism group	Section 6.2.2
$e^I_\mu(x)$	gravitational field	Equation (2.1)
e	determinant of e^I_μ	
e	edge (of spinfoam)	Section 9.1
$E, E^a_i(\vec{x})$	gravitational electric field	Section 4.1.1
f	face (of spinfoam)	Section 9.1
F	curvature two-form	Section 2.1.1
g or U	group element	
G	Newton constant	
$\mathcal{G}$	space of boundary data	Sections 3.2.5–3.3.3
h_γ	$U(A,\gamma)$	Section 7.1
H	relativistic hamiltonian	Section 3.2
H_0	nonrelativistic (conventional) hamiltonian	Section 3.2
$\mathcal{H}$	quantum state space	Chapter 5
$\mathcal{H}_0$	nonrelativistic quantum state space	Chapter 5

i_n	intertwiner on spin network node n	Section 6.3
i_e	intertwiner on spinfoam edge e	Chapter 9
j	irreducible representation (for $SU(2)$: spin)	
j_l	spin associated to spin network link l	Section 6.2.1
j_f	representation associated to spinfoam face f	Chapter 9
$\mathcal{K}$	kinematical quantum state space	Section 5.2
$\mathcal{K}_0$	$SU(2)$ invariant quantum state space	Section 6.2.3
$\mathcal{K}_{\text{diff}}$	diff-invariant quantum state space	Section 6.2.3
K	boundary quantum space	Sections 5.1.4, 5.3.5
l	link (of spin network)	Section 9.1
l_{P}	Planck length, $\sqrt{\hbar G c^{-3}}$	
$\mathbf{L}$	length	Section 2.1.4
M	spacetime manifold	
n	node (of spin network)	Section 9.1
p_a	relativistic momenta (including p_t)	Section 3.2
p_t	momentum conjugate to t	Section 3.2
P	the "projector" operator	Section 5.2
P_{G}	group G projector	Equation (9.117)
P_{H}	subgroup H projector	Equation (9.119)
$\mathcal{P}$	transition probability	Chapter 5
$\mathcal{P}$	path ordered	Equation (2.81)
q^a	partial observables	Section 3.2
$R^I{}_{J\mu\nu}(x)$	curvature	Equation (2.8)
$R^{(j)\alpha}{}_{\beta}(g)$	matrix of group element g in representation j	
$\mathcal{R}$	3d region	Section 2.1.4
s	s-knot: abstract spin network	Equation (6.4.1)
$\|s\rangle$	s-knot state	Equation (6.4.1)
S_{BH}	black-hole entropy	Section 8.2
S	embedded spin network	Section 6.3
$\|\mathrm{S}\rangle$	spin network state	Section 6.3.1
$\mathcal{S}$	2d surface	Section 2.1.4
$\mathcal{S}$	space of fast decrease functions	Chapter 5
$\mathcal{S}_0$	space of tempered distributions	Chapter 5
$S[\tilde{\gamma}]$	action functional	Section 3.2
$S(q^a)$	Hamilton–Jacobi function	Section 3.2.2
$S(q^a, q^a_0)$	Hamilton function	Section 3.2.5

t_ρ	thermal time	Sections 3.4, 5.5.1
T	target space of a field theory	Section 3.3.1
U or g	group element	
$U(A,\gamma)$	holonomy	Section 2.1.5
v	vertex (of spinfoam)	Section 9.1
$\mathbf{V}$	volume	Section 2.1.4
$W(q^a, q'^a)$	propagator	Chapter 5
W	transition amplitudes, propagator	Section 5.2
x	4d spacetime coordinates	
$\vec{x}$	3d coordinates	
Z	partition function	Chapter 9
α	loop, closed path	
β	inverse temperature	Section 3.4
γ	path	
γ	motion (in $\mathcal{C}$)	Section 3.2.1
γ	Immirzi parameter	Section 4.2.3
$\tilde{\gamma}$	motion in Ω	Section 3.2
Γ	relativistic phase space	Section 3.2.1
Γ	graph	Section 6.2
Γ	two-complex	Chapter 9
θ	Poincaré–Cartan form on Σ	Section 3.2.2
$\tilde{\theta}$	Poincaré form on Ω	Equation (3.9)
$\eta_{IJ}, \eta_{\mu\nu}$	Minkowski metric = diag[−1,1,1,1]	
λ	cosmological constant	Equation (2.11)
λ	gauge parameter	Section 2.1.3
ρ	statistical state	Sections 3.4, 5.5.1
Σ	constraint surface $H = 0$	Section 3.2.2
σ, Σ	3d boundary surface	Chapter 4
σ	spinfoam	Chapter 9
$\phi(x)$	scalar field	Equation (2.32)
$\psi(x)$	fermion field	Equation (2.35)
ω	presymplectic form on Σ	Section 3.2.2
$\omega^I_{\mu J}(x)$	spin connection	Equation (2.2)
$\tilde{\omega}$	symplectic form on Ω	Section 3.2.2
Ω	space of observables and momenta	Sections 3.2–3.3.2
$\{6j\}$	Wigner $6j$ symbol	Equation (9.33)
$\{10j\}$	Wigner $10j$ symbol	Equation (9.103)
$\{15j\}$	Wigner $15j$ symbol	Equation (9.56)
$\|0\rangle$	covariant vacuum in K	Sections 5.1.4, 5.3.5
$\|0_t\rangle$	dynamical vacuum in $\mathcal{K}_t$	Sections 5.1.4, 5.3.2
$\|0_{\mathrm{M}}\rangle$	Minkowski vacuum in $\mathcal{H}$	Sections 5.1.4, 5.3.1

• *The name of the theory.* Finally, a word about the name of the quantum theory of gravity described in this book. The theory is known as *"loop quantum gravity"* (LQG), or sometimes *"loop gravity"* for short. However, the theory is also designated in the literature using a variety of other names. I list here these other names, and the variations of their use, for the benefit of the disoriented reader.

– *"Quantum spin dynamics"* (QSD) is used as a synonym of LQG. Within LQG, it is sometimes used to designate in particular the dynamical aspects of the hamiltonian theory.

– *"Quantum geometry"* is also sometimes used as a synonym of LQG. Within LQG, it is used to designate in particular the kinematical aspects of the theory. The expression *"quantum geometry"* is generic: it is also widely used in other approaches to quantum spacetime, in particular dynamical triangulations [19] and noncommutative geometry.

– *"Nonperturbative quantum gravity," "canonical quantum gravity"* and *"quantum general relativity"* (QGR) are often used to designate LQG, although their proper meaning is wider.

– The expression *"Ashtekar approach"* was used in the past to designate LQG: it comes from the fact that a key ingredient of LQG is the reformulation of classical GR as a theory of connections, developed in particular by Abhay Ashtekar.

– In the past, LQG was also called *"the loop representation of quantum general relativity."* Today, *"loop representation"* and *"connection representation"* are used within LQG to designate the representations of the states of LQG as functionals of loops (or spin networks) and as functionals of the connection, respectively. The two are related in the same manner as the energy ($\psi_n = \langle n|\psi\rangle$) and position ($\psi(x) = \langle x|\psi\rangle$) representations of the harmonic oscillator states.

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