

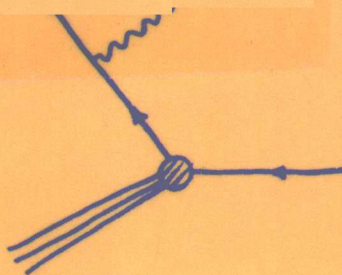
Rabindra N. Mohapatra

# Unification and Supersymmetry

The Frontiers of  
Quark-Lepton Physics

Third Edition

统一理论和超对称  
第3版



Springer

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Rabindra N. Mohapatra

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With 60 Illustrations



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To  
Manju  
Pramit and Sanjit

“... Where tireless striving stretches its arms towards perfection:

Where the clear stream of reason has not lost its way into the dreary  
desert sand of dead habit, ...”

GITANJALI, RABINDRA NATH TAGORE,  
NOBEL LAUREATE IN LITERATURE

# Preface to the Third Edition

The new millennium has brought new hope and vigor to particle physics. The menacing clouds of despair and discontent that enveloped the field following the collapse of SSC have all but vanished. The discovery of neutrino mass has brought the first light of new physics beyond the standard model. The LEP-SLC data has given strong hints of a light Higgs boson, which is widely hoped, will be discovered soon either at the Tevatron or LHC. LEP may quite possibly have missed it by a hair. Many neutrino experiments are either underway or are in the planning stages, and a rough outline of neutrino mixing is appearing on the horizon. There are discussions of pulling resources internationally to build a linear collider after the LHC. Many major breakthroughs in the sister discipline of cosmology have lightened up the sky. Even the job situation in the field is showing signs of improvement after a long plateau.

All this hope and optimism about a bright future for the field seem to be resting on two ideas: unification and supersymmetry. The first is based on the amazing success of the standard model, giving credence to the possibility that the final theory of particle physics could come from gauge theories and string theory, from which the gauge symmetries follow. The belief in supersymmetry arises not only from its beauty and elegance and its ability to truly unify matter and forces but also from the way it embraces gravity into the fold of particle physics. Its hold on the field is almost as pervasive as that of gauge theories. Even though there are many other competing ideas vying for the attention of theorists, the general direction seems to be largely set towards supersymmetry, supergravity, and superstrings. The possibility of extra dimensions playing a role in understanding

the new physics is being taken more seriously than ever. Still, there are many unsolved problems remaining: the origin of family replication, the origin of quark and lepton mixings, the origin of supersymmetry breaking, electroweak symmetry breaking, and the possibility of extra dimensions, to name some important ones. There is thus a lot to do, and the new century should be an exciting one.

With this background, I felt it necessary to update the Second Edition of the book by adding new materials that reflect recent developments and hopefully make it more useful to advanced graduate students as well as to researchers in the field. This edition uses the popular  $(+, -, -, -)$  metric instead of the Pauli metric used in the first two editions.

I would like to acknowledge the support of the National Science Foundation during the time this book was updated. I would also like to thank Xiang-Dong Ji for carefully reading the beginning chapters on supersymmetry and suggesting many improvements. Last, but not least, I would like to thank those colleagues who have used the book in their advanced particle physics courses at various universities and to those who have found some use for the book in their own intellectual pursuits.

RABINDRA MOHAPATRA  
College Park, Maryland  
July 10, 2002

# Preface to the Second Edition

Nearly six years have passed since the first edition of *Unification and Supersymmetry* was finished. Many new developments have occurred in both theoretical and experimental areas of particle physics. On the theoretical side, the most notable development was the rise of superstring theories as a candidate theory of everything. A great deal of euphoria swept the theory community that the “end” of theoretical physics was in sight. The years 1987 and 1988 saw the excitement peak. Limitations to the superstring approach have since led to a more sober reassessment of the prospects of string theories. While fewer people now believe the string theories to be the panacea for all the “ailments” of the standard model, it is regarded as a significant theoretical development. I have therefore added an extra chapter dealing with the salient features of string theories and the Calabi–Yau type compactification.

On the experimental side, parameters of the standard model are now much more precisely determined thanks to LEP experiments; the top quark, of course, still remains undiscovered, but it has now a higher, lower limit on its mass, leading to speculation that it may have a key role in understanding the electroweak symmetry breaking.

Finally, many great developments have taken place in neutrino physics—the two most outstanding ones being the apparent confirmation of the existence of a solar neutrino deficit and the possible existence of a 17 keV neutrino. Both of these results will be tested in several planned and ongoing experiments and are the subject of separate books. I have, therefore, only briefly touched on the first topic in Chapter 6. These may yet prove to be the first clue to new physics.

There is no direct evidence yet for supersymmetry, while the recent precise measurement of  $\sin^2 \theta_W$  has prompted some, prematurely, to conclude that it is evidence for supersymmetry. These physicists assume that coupling constants must eventually unify. (Although it is an appealing assumption, now there are fewer reasons than ever to require grand unification of gauge couplings, i.e., one can explain both charge quantization and baryon asymmetry without requiring grand unification.) Secondly, even if one assumed unification of couplings, any kind of intermediate scale [such as the ones in  $SO(10)$ ] would equally well predict  $\sin^2 \theta_W$  to match observations precisely.

Apart from several extensive changes in the text, I have also added exercises at the end of each chapter to stimulate further discussion of the issues. I hope that the key theoretical ideas discussed in this book will receive some confirmation in future experiments; but in any case, I will consider my efforts worthwhile if the book inspires even one more graduate student to choose a career as a particle physicist.

Finally, I wish to acknowledge several colleagues, especially C. Kalman and E. Golowich, who have pointed out several typographical errors in the first edition and to Rachel Needle for carefully typing the needed pages for the second edition. The support of the National Science Foundation during the period of the preparation of the second edition is gratefully acknowledged.

RABINDRA MOHAPATRA  
College Park, Maryland  
September 1, 1991

# Preface to the First Edition

The theoretical understanding of elementary particle interactions has undergone a revolutionary change during the past one and a half decades. The spontaneously broken gauge theories, which in the 1970s emerged as a prime candidate for the description of electroweak (as well as strong) interactions, have been confirmed by the discovery of neutral weak currents as well as the  $W$ - and  $Z$ -bosons. We now have a field theory of electroweak interactions at energy scales below 100 GeV—the Glashow–Weinberg–Salam theory. It is a renormalizable theory that enables us to do calculations without encountering unnecessary divergences. The burning question now is: What lies ahead at the next level of unification? As we head into the era of supercolliders and ultrahigh-energy machines to answer this question, many appealing possibilities exist: left–right symmetry, technicolor, compositeness, grand unification, supersymmetry, supergravity, Kaluza–Klein models, and, most recently, superstrings that even unify gravity along with other interactions. Experiments will decide if any one or any combination of these is to be relevant in the description of physics at the higher energies. As an outcome of our confidence in the possible scenarios for elementary particle physics, we have seen our understanding of the early universe improve significantly. Such questions as the origin of matter, the creation of galaxies, and the puzzle of the cosmic horizon are beginning to receive plausible answers in terms of new ideas in particle physics. Although a final solution is far from being at hand, reasonable theoretical frameworks for carrying out intelligent discussions have been constructed. Even such difficult questions as the “birth” and “death” of the universe have been discussed.

This book, based on advanced graduate courses offered by me at CCNY (City College of New York) and in Maryland, attempts to capture these exciting developments in a coherent chapter-by-chapter account, in the hope that the frontiers of our understanding (or lack of it) in this exciting field of science can be clearly defined for students as well as beginning researchers. The emphasis has been on physical, rather than technical and calculational, aspects, although some necessary techniques have been included at various points. Extensive references are provided to original works, which the readers are urged to consult in order to become more proficient in the techniques. The prerequisites for this book are an advanced course in quantum field theory (such as a knowledge of Feynman diagrams, the renormalization program, Callan-Symanzik equations, and so forth), group theory (Lie groups at the level of the books by Gilmore or Georgi), basic particle theory such as the quark model, weak interaction, and general symmetry principles (at the level of the books by Marshak, Riazuddin, and Ryan or by Commins), and familiarity with spontaneously broken gauge theories (at the level of the books by J. C. Taylor or Chris Quigg).

The book is divided into two parts: the first eight chapters deal with the introduction to gauge theories, and their applications to standard  $SU(2)_L \times U(1) \times SU(3)_C$  models and possible extensions involving left-right symmetry, technicolor, composite models, quarks and leptons, strong and weak  $CP$  violation, and grand unification. In the last eight chapters, we discuss global and local supersymmetry, its application to particle interactions, and the possibilities beyond  $N = 1$  supergravity. Interesting recent developments in the area of superstrings are only touched on; unfortunately, they could not be discussed as extensively as they ought to be.

I would like to acknowledge many graduate students at CCNY and in Maryland over the past seven years, as well as colleagues at both places who attended the lectures, and helped to sharpen the focus of presentation by their comments. I am grateful to many of my colleagues and collaborators for generously sharing with me their insight into physics. I wish to thank E. F. Redish for suggesting that I compile the lectures into book form, and C. S. Liu and the members of the editorial board of this lecture series for their support at many crucial stages in the production of this book. Finally, I wish to thank Mrs. Rachel Olexa for her careful and prompt typing of the manuscript, and J. Carr for reading several chapters and suggesting language improvements.

I wish to acknowledge the support of the U.S. National Science Foundation during the time this book was written.

RABINDRA MOHAPATRA  
College Park, Maryland  
March 1986

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