EVALUATION, INSPECTION AND MONITORING OF STRUCTURAL INTEGRITY

Fracture Mechanics and Applications

Editors: G.C. Sih, S.T. Tu and Z.D. Wang




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Preface

This book is the sixth volume of the proceedings for the symposium series on Fracture Mechanics, an annual conference devoted to the exchange of information among the institutes and research centers in China and abroad. FM2008 is held in Hangzhou, Zhejiang Province from October 31 to November 5, 2008. The first FM conference was held in 2003 at East China University of Science and Technology, Shanghai. Since then, the FM annual meetings have taken place at different cities and regions. This includes FM2004 in Huangshan, Anhui Province, FM2005 in Zhengzhou, Henan province, FM2006 in Nanjing, Jiangsu Province and FM2007 in Changsha, Hunan province. These annual events are indicative of the fact that the FM symposium series has played an important role in promoting information exchange, inspiring new ideas, integrating practical and research findings and breaking new grounds for the young generation. It cannot be denied that the efforts from the conference committee members and the local organizers have made the conference in the past five years a great success.

Even though different aspects of structural integrity have been dealt with in the past, it needs to be emphasized that the conventional approaches to evaluating structural integrity – rule-of-thumb guidelines, with excessive safety margins – leave much to be desired in today's highly competitive business climate. The trend of integrating inspection, monitoring techniques and assessment approaches has been advanced in relation to multiscaling. A key step in research lies in the translation of the low scale material parameters to the macroscopic scale where the designer can specify the material properties. The process can entail several orders of scale transitions, not to mention the ability to monitor minute changes in plant or infrastructure condition with a high degree of confidence. Such a demand has led to continuous advances in structural integrity technology and risk based assessment. The assessment of structural integrity may involve condition monitoring, intelligent computing, life cycle analysis, non-destructive evaluation, probabilistic methods for specimen and structure scales. It is a necessary concept to be used in design and manufacture, as well as the subsequent life of the product. These undertakings will be emphasized in FM2008 which aims to integrate new development from diverse areas of science and technology in relation to structural integrity. Besides the integration of theory and practice, science and technology, multidisciplinary exchange of ideas is encouraged.

The symposium (FM2008) is co-organized by Zhejiang University of Technology, East China University of Science and Technology, National Engineering Research Center of Pressure Vessels and Pipelines Safety Technology (General Machinery Research Institute), Nanjing University of Technology, Zhejiang University, Zhengzhou University, Changsha University of Science and Technology and Shandong University, and co-sponsored by the Chinese Pressure Vessel Institution, the High Temperature Strength of Materials Committee of Chinese Materials Institution, Natural Science Foundation of China and General Administration of Quality Supervision, Inspection and Quarantine of China.

On behalf of the organizing committee, we would like to thank the above co-organizers who made FM2008 possible. We also appreciate the efforts of the program committee members in reviewing and selecting the papers. We are grateful for Professor George C. Sih for his constant support to the symposium series. We are indebted to Professor Zhengdong Wang for his time to collect papers and edit this book. Special thanks will go to the authors whose contributions make FM2008 unique.

Zengliang Gao
Executive Chairman
Zhejiang University of Technology

Shan-Tung Tu
Symposium Series Chairman
East China University of Science & Technology

October, 2008

From the Editors

There is no concern that can be more important to the current melt down of the world economy that will start to alter the lives of each one of us and the outlook of educational, industrial and governmental institutions of the near future. The FM series and annual meetings will also be affected by this financial crisis. It is necessary to reorient, review and reinterpret the objectives of FM 2008 concerning *Evaluation, Inspection, and Monitoring of Structural Integrity* as the effective application of the above referenced theme clearly depends on the availability of financial support.

To begin with, profit motivated economy crumbles when credits turn into empty promises. The current profit oriented scientific and technological development infrastructure is now more transparent to the public at large and will be destined to collapse if physical based knowledge is continuously short changed for political and economical expediency at the expense of public well being. The harmful consequence of ill-directed research funds needs to be exposed and mitigated. It is no longer acceptable to foot the bill to the public and the future generations.

Data collection for the evaluation, inspection, and monitoring of structural integrity can be a complete waste of effort and resource if the methodology is not supported by the underlying physics. Worse yet when the methodology is disguised behind exotic and costly equipment. It is now becoming a known fact that even the basic building block of matter is in need of patching, less alone the monitoring of integrity for structures operating under harsh conditions such as high temperature and/or pressure. The indiscriminate use of nano reinforced materials is a case in point; it can have harmful effects instead of being beneficial. Scaling of the technology in proportion to the cost is a myth created by the perfume-product and name-brand marketing that is the *more expensive-the better*. Extension of this concept to the use of nano material for high temperature and pressure requires scrutiny. This is because all conventional approaches are based on using bulk properties. On the contrary, less expensive and more effective nano reinforcement should be applied using local properties. The demarcation line or *threshold* is where the physical laws change from those for the bulk to the local properties. The latter has escaped the attention of the development of the traditional 19^{th} century approach to mechanics. The application of the bulk property based on theories to nano reinforcement is in part responsible for the billions of USD that have fall short of the anticipated but undefined benefit.

In a nut shell, nano reinforcement delves into bodies with large surface to volume ratio the structural integrity of which must be translated from the nano scale to the macro scale because engineering design parameters are manipulated at the macroscopic scale while the gain in life longevity and structural integrity is administered at the nano scale. The need to understand multiscaling can be demonstrated by using the fatigue crack growth nano/micro and micro/macro dual scale models. The scaling in time and size is accomplished by breaking the continuum into discrete intervals, each differing in scale. This idea is similar to the replacement of the big-bang universe model in physics where the growth of the universe is analogous to climbing up the ladder of sizes; it is allowed to grow as it climbs. Referred to in the loop quantum gravity theory, spaced is filled with *spacetime atoms* much smaller than the regular atom of 10^{-8}cm. The diameter of the new atoms approaches the Planck distance 10^{-33}cm. The loop quantum gravity space would be more dense and can be packed with more energy. These concepts are also being explored in relation to enhance fatigue crack growth life and the increase of structural integrity by application of multiscaling and mesomechanics.

Even more challenging is the problems faced with the monitoring of existing operational systems. Accurate assessment of the history of nano/micro/macro structure degradation related signals is lacking. Establishment of the period between the discovery of damage and failure is unreliable or not available. Unwarranted shut down can be costly. The current health monitoring hardware and software are directed towards checking of design specifications for new structures. Assessment of the remaining life of used materials and structural members require a method that can evaluate the in situ integrity of the physical system. Hardware is definitely important. But hardware alone is not sufficient. Such a model has been explored for the fatigue of nano/micro/macro cracking of pre-cracked 2024-T3 and 7075-T6 panels where the predicted results can be checked with the test results. The work involves deriving the time dependent nano/micro/macro structure degradation properties, the validity of which is established by matching the predicted and the tested results. The critical structural components can thus be monitored and replaced before failure occurs.

In view of what has been said, the 21st century may initiate a new chapter in the advancement of science of technology where engineering and physics may merge to advance a more complete model based on spacetime atom where the engineers can further improve on the structural integrity. There is no reason why FM 2008 cannot take the first step to apply multiscaling and mesomechanics approaches to the following topics although there is no intention to exclude the others.

—Failure assessment theory to include multiscale damage and multiple damage mechanisms (e.g. thermal, corrosion, irradiation, cyclic loading effects).
—Testing of metallic and non-metallic materials ranging from room to high temperature.
—Modeling and simulation of damage and fracture process in materials and structure
—Determination of thermal-mechanical and/or chemo-mechanical coupling effects qualified in terms of the constitutive constants.
—Correlation between a measured physical/derived parameter and quantitative information on defects/stresses/microstructures.
—Real time structured integrity assessment
—Long term performance and reliability of sensor systems and instrumentation
—Uncertainty models for large system, which include environment, loads and materials/structure.

The editors are indebted to the authors of this volume and those who have spent their valuable time to review the most of the papers. The editors wish to take this opportunity to express their gratitude to the representatives of the organizing institutions who constitute the consortium of fracture mechanics in China. Thanks are due to Jinhua Zhang, Limin Dou, Jingjing Jia and Huimin Sun who assisted in typing, examining and editing manuscripts. In particular, thanks are also due to those who have assisted in taking care of meeting rooms, projectors, accommodations, local transportation and numerous tasks behind the scene that are often overlooked by the organizers.

Hangzhou, China
October, 2008

G. C. Sih
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Z. D. Wang

Contents

Conceptual and physical disparity inherent in the modeling of multiscale fatigue crack growth: nano-micro-macro

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Abstract

Use specific materials have led to countless failure criteria related to strength, toughness, fatigue life, etc. They all seem to fit the data for certain situations. The search for a single criterion that can account for different service conditions relies on a knowledge of the inherent unavoidable discrepancy between the conceptual and physical aspects of modeling, in addition to the ways with which the internal structure of the material are arranged. At the nano-micro-macro size and time scale, material behavior cannot be disassociated with physical damage, say by crack growth in fatigue, a frequently encountered mode of failure.

The current status of science and engineering has stymied the development of multiscale models. The representative specimen scheme in material testing assumes that the properties of the bulk can be obtained from a portion (specimen) of the whole (structural component) although the theory does not adhere to the same rule. It rlies on using a mathematically imagined element that can be summed continuously to obtained the bulk behavior. Incompatibility of the test procedure and analytical formulation becomes increasingly more seriously when the inhomogeneity of the specimen or the element differs drastically from the bulk. Such inconsistencies are revealed for the nano-micro-macro crack growth model in fatigue. Actual fatigue crack growth data for the 2024-T3 aluminum pre-cracked panels are used to show how damage can be assessed and connected at three different scales: the nano, micro and macro. This entails the evaluation of (a_{nano}, a_{micro} a_{macro}) and their corresponding crack growth rates.

Two sets of relative scale parameters $\mu^*_{m/\mathrm{m}}, d^*_{m/\mathrm{m}}, \sigma^*_{m/\mathrm{m}}$ and $\mu^*_{\mathrm{n}/m}, d^*_{\mathrm{n}/m}, \sigma^*_{\mathrm{n}/m}$ are used. The former refers to the micro and macro scales by the respective subscripts of m and m and the latter to the nano and micro scales by the respective subscripts of n and m. These parameters are time dependent and they express the degradation of the nano, micro and macro structure with time. It suffices to determine the bulk properties at the macroscopic scale while the local properties at the nano and micro scales are regarded as fictitious because they cannot be simulated by tests under the same conditions of material homogeneity. The objective therefore is not to simulate nature in its entity but only to model the conceptual portion that will nevertheless yield result anticipated by design. Prescription of the nano, micro, and macro structure of the material for a specific use will be used for demonstration. This entails a cracked 2024-T3 panel subjected to fatigue loading: σ_{m}=44.1MPa, σ_{a}=31.36MPa and R=0.169. Predicted is a total fatigue life of approximately 18 years out of which pre-nano and nano cracking prevailed for 8 years, microcracking for 4 years and macrocracking for 10 years. The individual time range of fatigue life can be altered by rearranging the construction of the nano, micro and macro material internal structure.

Keywords: Mesomechanics; Multiscaling; Nano; Micro; Macro; Interface; Scale transition; Segmentation; Fatigue; Creep; Mean stress; Stress amplitude; Crack growth; Discrete; Continuum; Microstructure; Nanostructure; Degradation; Size and time effect.

1. Introduction

A common mode of failure for structural components is that of repeated and/or reversal loading even though the stress amplitude may be well below that of the ultimate strength. This process may initiate from the growth of tiny defects of the order of nano meters or smaller that extend slowly to cracks of micro and macro meters in size. Depending on the size and shape of the component, one of the dominant cracks may reach catastrophic proportion causing the structure to collapse by fracture. Such phenomena are not uncommon in the 1990s and by in large can be avoided by design. Research of material science and structural engineering of the late 20$^{\mathrm{th}}$ and early 21$^{\mathrm{st}}$ century has been concerned with the more efficient use of material micro- and

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nano-structure. The chain of information transfer from the researcher to the material fabricator, model maker, designer and user has no doubt been stretched and made the dissemination of results more difficult. This has been brought to light in a recent review concerned with the anomalies due to the variance of interpreting fatigue data from the two-parameter crack growth rate relation [1] in fracture mechanics [2].

Successful use of scientific and technological results relies on consistent application of codes and standards established from cumulated experience and practice. Exceptions cannot be ruled out when encountering new technologies. Nano-engineering is a case in point. The reinforcement of materials by nanoparticles has been problematic because small bodies with large surface to volume ratio A/V obey different physical laws than those for large bodies that are governed by the bulk properties rather than the surface interface effects which have been regarded as a separate discipline up to now. Bodies with large A/V ratio are more susceptible to the chemical and physical properties of surface atoms that possess less cohesive energy for binding because they have fewer neighboring atoms in close proximity compared to atoms in the bulk of the solid. Since each bond of an atom shares with a neighboring atom provides cohesive energy, atoms with fewer bonds and neighboring atoms have lower cohesive energy. Nanoparticles exposed to more free surface and/or interface have lower cohesive energy. The cohesive energy of nanocrystals have been found to increase or decrease with A/V [3-5] depending on the service conditions that must be taken into consideration. Fatigue lives of nano-structures can be prolonged in principle by making sure that nano-scale damage is not by-passed for microscale and/or macroscale damage. The idea is to design the multiscale material such that energy dissipation would occur progressively in the order of nano-micro-macro or otherwise. The sequential order depends on the load history. Contrary to the strength of material concept of damage prevention, fracture mechanics relies on controlling the rate of energy dissipation to achieve the desired design life. To this end, the time dependent degradation of the nanostructure, microstructure and macrostructure of the material must not only be specified but also implemented into the fabrication process. This can be accomplished by the application of two dual scale fatigue crack growth models, referred to as micro/macro and nano/micro involving three physical parameters for each model. They are designated as $\mu^*_{m/\mathrm{m}}, d^*_{m/\mathrm{m}}, \sigma^*_{m/\mathrm{m}}$. The subscripts m and m stand, respectively, for micro and macro. Similarly, the set of relative scale parameters $\mu^*_{\mathrm{n}/m}$, $d^*_{\mathrm{n}/m}$, $\sigma^*_{\mathrm{n}/m}$ refers to the nano and micro scales by the respective subscripts of n and m. The two sets of parameters are connected via the micro-scale. The objective is to find the relative scale time dependent parameters ($\mu^*_{m/\mathrm{m}}, d^*_{m/\mathrm{m}}, \sigma^*_{m/\mathrm{m}}$) and ($\mu^*_{\mathrm{n}/m}, d^*_{\mathrm{n}/m}, \sigma^*_{\mathrm{n}/m}$) from which nano and micro properties can be derived from the static macroproperties at t=0. These results are translated into two dual scale fatigue crack growth da/dN models where the crack is modeled to undergo the transitions $a_{\mathrm{nano}} \rightarrow a_{\mathrm{micro}} \rightarrow a_{\mathrm{macro}}$ and eventually reaching the critical state of global instability designated by a_{cr}. This procedure is applied to the fatigue crack growth data of pre-cracked 2024-T3 aluminum panel [6] where arbitrary lives corresponding to a_{nano}, a_{micro} and a_{macro} are assigned to illustrate the method of solution.

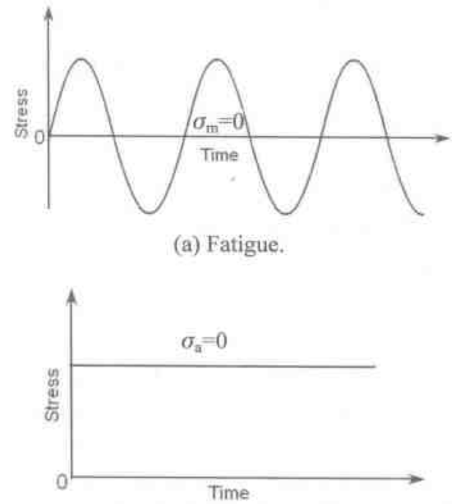
Toughening and weakening of ceramics doped with nano-particles have been contemplated on three fronts. They consists of: crack pinning, interface-matrix stiffening, and particle clustering. Crack pinning impedes possible crack skirting around or pull-out of the nano-particle while nano-particle reinforcement changes the relative stiffness of the matrix and interface grain such that a more uniform pattern of cracking can be obtained in the polycrystal. Nano-particle clustering is undesirable as it enhances non- uniformity and additional passage for cracks to pass through. The foregoing remarks imply that nano-particle reinforcement can be beneficial or detrimental. Although the conditions associated with these opposing effects are known by empirical means, the results are not conclusive [7-10].

2. Nano-micro-macro fatigue crack growth

The concept of multiscaling can be deceiving at first sight as it implicates the involvement of several scales at the same time. A closer examination shows that it suffices to consider only two scales at one time such that the full range of scaling may be obtained by connecting segments of the overlapping dual scale models such as atomic-nano, nano-micro, and micro-macro to arrive at the atomic-nano- micro-macro model with well define range of accuracy between any two tandem scales. Refinement of interconnecting scales may be achieved by the introduction of meso-scales. Refer to the works in [11-14] for details.

2.1. Consistency requirement

Consistency requirement can be put to good use for reducing the choice of model selection. To begin with, consider the classical definition of fatigue and creep as illustrated in Figs. 1(a) and 1(b), respectively. The fatigue of smooth specimens as related to the construction of SN diagrams is concerned with the application of alternating tension and compression loading with zero mean stress σ_m=0. The time axis in Fig. 1(a) is usually replaced by the number of cycles. The conversion can be made by considering the frequency of the cyclic load dN/dt. Creep loading on the other hand entails the application of a constant load with the absence of stress amplitude or σ_a=0. These definitions are by in large still observed. The advent of fracture mechanics has not only added an initial crack to the test specimen but it has altered the ways that test data are taken. As a rule, both the mean stress σ_m and stress amplitude σ_a are present even for the case of constant amplitude and frequency loading as indicated in Fig. 2. Hence, it becomes mandatory to include both σ_m and σ_a as the primary load variables in any fatigue crack growth models. On fundamental grounds, the use of ΔK alone for estimating the crack growth rate da/dN would not be adequate. The inclusion of both σ_m and σ_a leads to the dependency of da/dN on ΔS that is related to the energy density function [15]. Hence, the task is not just one of selecting an effective ΔK_{eff} or equivalent ΔK_{eq}. to match the test data, but rather to implement the physical mechanisms of fatigue crack growth. Of particular concern is the transitional phenomena of nano-, micro- and macro-cracking that can be modeled by two dual stage da/dN models referred to as micro/macro and nano/micro as illustrated schematically in Fig.3. Each stage consists of the sigmoidal curve, the meaning of which will differ from the conventional crack growth relation [1]. The transitional regions can be refined to approximate a straight line relation similar to the transformation used in the form invariant approach [16].



(b) Creep.

Fig. 1. Classical definition of fatigue and creep.

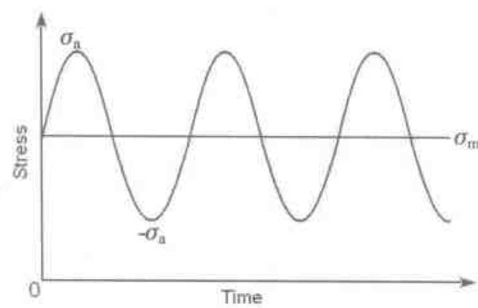


Fig. 2. Fatigue in fracture mechanics as practiced.

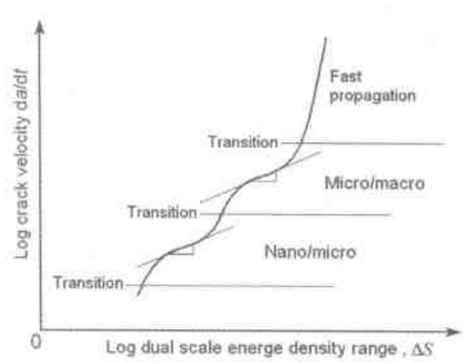


Fig. 3. Two dual stage nano-, micro- and macro-cracking in fatigue.

2.2. Micro/macro dual scale crack growth

Cross linking dual scale fatigue crack growth models has been discussed in [17] where $a_{nano} \rightarrow a_{micro} \rightarrow a_{macro}$ were predicted from a knowledge of ΔS_{micro}^{macro} and ΔS_{nano}^{micro}. A fundamental departure from the conventional approach is that a double singularity stress field [8] were used to couple the microcrack and macrocrack behavior. The latter has been known to be governed by the $1/r^{0.5}$ singularity with r being the radial distance from the crack tip. The microcrack were found [8] to possess a weak singularity of $1/r^{0.15}$ and a strong singularity of $1/r^{0.75}$. Note that the order 0.5 is in between 0.15 and 0.75 as illustrated in Fig. 4. In other words, the micro- and macro-stress fields are interconnected, a solution derived from the use of the theory of elasticity differing in physical interpretation [18]. Keep in mind that the formalism of the classical theory of elasticity is immune to scaling because size effect has been deliberately neglected when invoking the limit $dV/dA \rightarrow 0$. As long as the constants in the linear theory are kept in short range and interpreted with the appropriate physical scaling size, any non-linear phenomenon can be approximated in segments. Non-linearity can be avoided by reducing scale size. Dislocation is a linear theory and yet it was developed to support the nonlinear theory of plasticity. Linearity also applies to quantum mechanics that will not shy away from explaining non-linear physical phenomenon. The inconsistency of hoping to associate linear dislocation theory with nonlinear continuum plasticity has been pointed to be a euphoria [20] that is inconsistent with the development of modern physics. In retrospect, scrutiny of the continuum theory of plasticity is long overdue.

For the sake of continuity, the two-parameter micro/macro fatigue crack growth model [11-13] will be referenced:

$$\frac{da}{dt} = \Psi_{micro}^{macro} [\Delta S_{micro}^{macro}]^{\psi} \tag{1}$$

in which Ψ_{micro}^{macro} accounts for the scale transitional behavior while ψ represents the slope of the da/dt versus ΔS_{micro}^{macro} curve when referred to a log-log plot. The specific form of ΔS_{micro}^{macro} is given by [11-13]:

$$\Delta S_{micro}^{macro} = \frac{2(1-2\nu_{micro})(1-\nu_{macro})^2 a\sigma_a\sigma_m}{\mu_{macro}} \times \frac{\mu_{micro}}{\mu_{macro}} \left(1-\frac{\sigma_o^{micro}}{\sigma_\infty^{macro}}\right)^2 \sqrt{\frac{d_{micro}}{r}} \tag{2}$$

Before putting Eq. (2) into (1) to yield da/dt, it should be noted that the near tip radial distance r is a micro/macro transitional parameter. To accentuate this property, the normalized quantity $d_{n/m}^*$ will be introduced by using the factor d_{macro}^{micro} such that $(d_{micro}/r)^{0.5}$ in Eq. (3) can be written as $(d_{n/m}^* d_{macro}^{micro}/r)^{0.5}$ to bring out the scale transitional character of ΔS_{micro}^{macro}. It follows that

$$\frac{da}{dt} = \Psi_{micro}^{macro} \left[\frac{2(1-2\nu_{micro})(1-\nu_{macro})^2 a\sigma_a\sigma_m}{\mu_{macro}} \times \mu_{m/m}^* (1-\sigma_{m/m}^*)^2 \sqrt{d_{m/m}^*} \sqrt{\frac{d_{macro}^{micro}}{r}}\right]^{\psi} \tag{3}$$

The dual scale physical parameters are

$$\begin{cases} \mu_{m/m}^* = \dfrac{\mu_{micro}}{\mu_{macro}} \\ \sigma_{m/m}^* = \dfrac{\sigma_o^{micro}}{\sigma_\infty^{macro}} \\ d_{m/m}^* = \dfrac{d_{micro}}{d_{macro}^{micro}} \end{cases} \tag{4}$$

The subscript and superscript notation in Eqs. (4) is self-explanatory where m stands for micro while m stands for macro. It should now be clear that r in Eq. (3) is a scale transitional parameter may be understood as $r_{m/m}$. Approximating da/dt by $\Delta a/\Delta t$, the i^{th} increment of crack growth Δa_i can be found as

$$\Delta a_i = \Psi_{micro}^{macro} \left[\frac{2(1-2\nu_{micro})(1-\nu_{macro})^2 \sigma_a\sigma_m}{\mu_{macro}} \times \mu_{m/m}^* (1-\sigma_{m/m}^*)^2 \sqrt{d_{m/m}^*} \sqrt{\frac{d_{macro}^{micro}}{r}}\right]^{\psi} a_{i-1}^{\psi} \Delta t \tag{5}$$

which can be inserted into