Eberhard Zeidler

Nonlinear Functional Analysis and its Applications II/B

Nonlinear Monotone Operators

非线性泛函分析及其应用

第2B卷

Springer

世界图书出版公司
www.wpcbj.com.cn

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II/B: Nonlinear Monotone Operators

Translated by the Author and by Leo F. Boron†

With 74 Illustrations

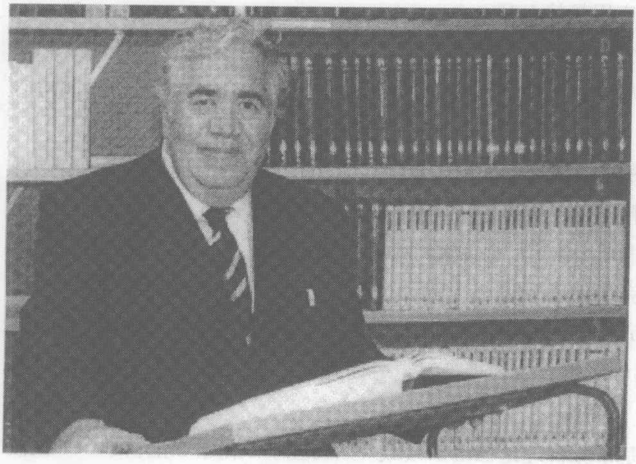
图书在版编目（CIP）数据

非线性泛函分析及其应用．第2B卷，非线性单调算子
= Nonlinear Functional Analysis & It's Applications Vol. 2/B
Nonlinear Monotone Operators：英文/（德）宰德勒著．
—北京：世界图书出版公司北京公司，2009. 8
ISBN 978-7-5100-0521-3

Ⅰ．非… Ⅱ．宰… Ⅲ．①非线性—泛函分析—英文
②非线性算子：单调算子—英文 Ⅳ．O177.91

中国版本图书馆 CIP 数据核字（2009）第 126347 号

书　　名：	Nonlinear Functional Analysis & Its Applications Vol. 2/B
作　　者：	E. Zeidler
中 译 名：	非线性泛函分析及其应用 第2B卷
责任编辑：	高蓉
出 版 者：	世界图书出版公司北京公司
印 刷 者：	三河国英印务有限公司
发　　行：	世界图书出版公司北京公司（北京朝内大街 137 号　100010）
联系电话：	010-64021602，010-64015659
电子信箱：	kjb@ wpcbj. com. cn
开　　本：	24 开
印　　张：	31.5
版　　次：	2009 年 08 月
版权登记：	图字：01-2009-1090
书　　号：	978-7-5100-0521-3/O · 737　　**定　　价：** 89.00 元



Eberhard Zeidler
德国 Max Planck 研究所数学领头教授，Leipzig 大学数学系荣誉教授。

影印版前言

自 1932 年，波兰数学家 Banach 发表第一部泛函分析专著"Théorie des operations linéaires"以来，这一学科取得了巨大的发展，它在其他领域的应用也是相当成功。如今，数学的很多领域没有了泛函分析恐怕寸步难行，不仅仅在数学方面，在理论物理方面的作用也具有同样的意义，M. Reed 和 B. Simon 的"Methods of Modern Mathematical Physics"在前言中指出："自 1926 年以来，物理学的前沿已与日俱增集中于量子力学，以及奠定于量子理论的分支：原子物理、核物理固体物理、基本粒子物理等，而这些分支的中心数学框架就是泛函分析。"所以，讲述泛函分析的文献已浩如烟海。而每个时代，都有这个领域的代表性作品。例如上世纪 50 年代，F. Riesz 和 Sz.-Nagy 的《泛函分析讲义》(中译版，科学出版社，1985)，就是那个时代的一部具有代表性的著作；而 60 年代，N. Dunford 和 J. Schwartz 的三大卷"Linear Operators"则是泛函分析学发展到那个时代的主要成果和应用的一个较全面的总结。泛函分析一经产生，它的发展就受到量子力学的强有力的推动，上世纪 70 年代，M. Reed 和 B. Simon 的 4 卷"Methods of Modern Mathematical Physics"是泛函分析对于量子力学应用的一个很好的总结。

呈现在我们眼前这部 5 大卷鸿篇巨制——E. Zeidler 的"Nonlinear Functional Analysis and its Applications"是非线性泛函分析到了上世纪 80 年代的主要成果和最典型应用的一个全面的论述，是一部百科全书。该书写作思想是：

(1) 讲述什么样的概念是基本的具有支配地位的概念，它们之间的关系是什么？

(2) 上述思想与经典分析以及线性泛函分析已有结果的关系是什么？

(3)　最典型的应用是什么?

一般的泛函分析书往往注重抽象理论的阐述，写应用常常不够详尽。而 Zeidler 这部书大为不同，其最大的特点是，书中讲了大量的各方面的应用，而且讲得非常清楚深入。

首先，这部书讲清楚了泛函分析理论对数学其他领域的应用。例如，第 2A 卷讲述线性单调算子。他从椭圆型方程的边值问题出发，讲问题的古典解，由于具体物理背景的需要，问题须作进一步推广，而需要讨论问题的广义解。这种方法背后的分析原理是什么？其实就是完备化思想的一个应用！将古典问题所依赖的连续函数空间，完备化成为 Sobolev 空间，则可讨论问题的广义解。在这种讨论中间，我们可以看到 Hilbert 空间的作用。书中不仅有这种理论讨论，而且还讲了怎样计算问题的近似解（Ritz 方法）。

其次，这部书讲清楚了分析理论在诸多领域（如物理学、化学、生物学、工程技术和经济学等等）的广泛应用。例如，第 3 卷讲解变分方法和优化，它从函数极值问题开始，讲到变分问题及其对于 Euler 微分方程和 Hammerstein 积分方程的应用；讲到优化理论及其对于控制问题（如庞特里亚金极大值原理）、统计优化、博弈论、参数识别、逼近论的应用；讲了凸优化理论及应用；讲了极值的各种近似计算方法。比如第 4 卷，讲物理应用，写作原理是：由物理事实到数学模型；由数学模型到数学结果；再由数学结果到数学结果的物理解释；最后再回到物理事实。

再次，该书由浅入深地讲透了基本理论的发展历程及走向，它既讲清楚了所涉及学科的具体问题，也讲清楚了其背后的数学原理及其作用。数学理论讲得也非常深入，例如，不动点理论，就从 Banach 不动点定理讲到 Schauder 不动点定理，以及 Bourbaki-Kneser 不动点定理等等。

这套书的写作起点很低，具备本科数学水平就可以读；应用都是从最简单情形入手，应用领域的读者也可以读；全书材料自足，各部分又尽可能保持独立；书后附有极其丰富的参考文献及一些文献评述；该书文字优美，引用了许多大师的格言，读之你会深受启发。这套书的优点不胜枚举，每个与数理学科相关的人，搞理论的，搞应用的，搞研究的，搞教学的，都可读该书，哪怕只是翻一翻，都不会空手而返！

全书共有 4 卷（5 本）：

第 1 卷　不动点定理　　　第 2 卷　A. 线性单调算子，B. 非线性单调算子

第 3 卷　变分方法与优化　　第 4 卷　数学物理的应用

Zeidler 教授著述很多，他后来于 90 年代又写了两本“Applied Functional Analysis”（Springer-Verlag, Applied Mathematical Sciences, 108,109），篇幅虽然比眼前这套书小了很多，但特点没有变。近期，他又在写 6 大卷“Quantum Field Theory”，第 1 卷“Basics in Mathematics and Physics, a Bridge between Mathematicians and Physicists”，已经由 Springer-Verlag 出版社出版。

非常感谢刘景麟对本文建议。

南京理工大学 黄振友

To the memory of my parents

Preface to Part II/B

The present book is part of a comprehensive exposition of the main principles of nonlinear functional analysis and its numerous applications to the natural sciences and mathematical economics. The presentation is self-contained and accessible to a broader audience of mathematicians, natural scientists, and engineers. The material is organized as follows:

Part I: Fixed-point theorems.
Part II: Monotone operators.
Part III: Variational methods and optimization.
Parts IV/V: Applications to mathematical physics.

Here, Part II is divided into two subvolumes:

Part II/A: Linear monotone operators.
Part II/B: Nonlinear monotone operators.

These two subvolumes form a *unit* equipped with a uniform pagination. The contents of Parts II/A and II/B and the basic strategies of our presentation have been discussed in detail in the Preface to Part II/A. The present volume contains the complete index material for Parts II/A and II/B.

For valuable hints I would like to thank Ina Letzel, Frank Benkert, Werner Berndt, Günther Berger, Hans-Peter Gittel, Matthias Günther, Jürgen Herrler, and Rainer Schumann. I would also like to thank Professor Stefan Hildebrandt for his generous hospitality at the SFB in Bonn during several visits in the last few years. In conclusion, I would like to thank Springer-Verlag for a harmonious collaboration.

Leipzig
Summer 1989

Eberhard Zeidler

Contents (Part II/B)

GENERALIZATION TO NONLINEAR STATIONARY PROBLEMS

When the answers to a mathematical problem cannot be found, then the reason is frequently the fact that we have not recognized the general idea, from which the given problem appears only as a single link in a chain of related problems.
David Hilbert (1900)

In the preceding chapters we studied linear monotone problems. In the following chapters we want to generalize these results to nonlinear monotone problems.

(i) In Chapters 25 through 29 we investigate stationary problems, i.e., we study operator equations of the form

$$Au = b, \qquad u \in X,$$

together with applications to quasi-linear elliptic differential equations and to Hammerstein integral equations.

In this connection we consider the following cases:

Lipschitz continuous, strongly monotone operators on H-spaces (Chapter 25);
monotone coercive operators on B-spaces (Chapter 26);
pseudomonotone operators (Chapter 27);
maximal monotone operators (Chapter 32).

For example, strongly continuous perturbations of monotone continuous operators are pseudomonotone. In Part I we considered the two fundamental fixed-point principles of Banach and Schauder. Figure 25.1 shows that these two principles also play a crucial role in the theory of monotone operators.

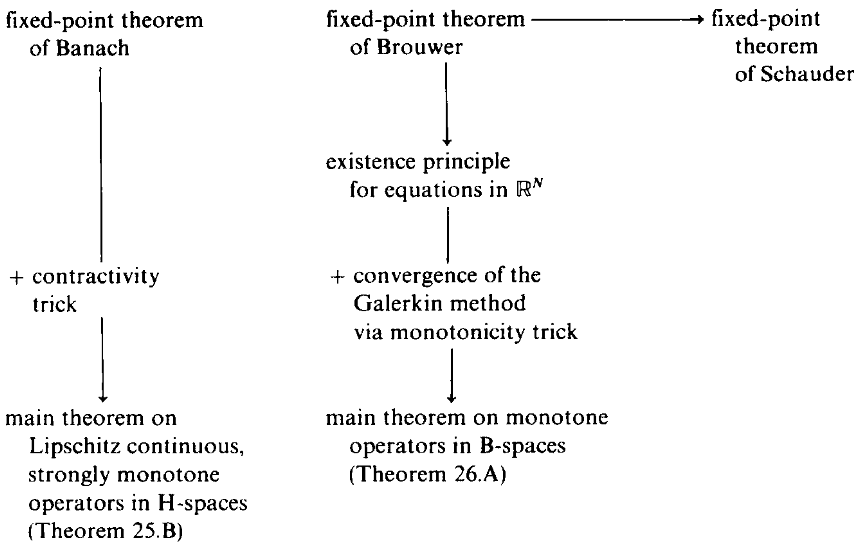


Figure 25.1

The special case of variational methods will be considered in Chapter 25. In this connection, the solutions of the convex minimum problem

$$f(u) = \min!, \qquad u \in X,$$

are solutions of the monotone operator equation

$$f'(u) = b, \qquad u \in X,$$

which generalizes the classical Euler equation.

(ii) In Chapters 30 through 33 we consider nonstationary problems, i.e., we study evolution equations of first and second order

$$u^{(n)} + Au = b, \qquad n = 1, 2,$$

together with applications to quasi-linear parabolic and hyperbolic equations.

In (i) and (ii), the operator A is monotone or, more generally, pseudomonotone. Here, the notion of *maximal monotone operators* plays a fundamental role.

(iii) In Chapters 34 through 36 we investigate a general theory of discretization methods together with applications to Galerkin methods (inner approximation schemes) and difference methods (external approximation schemes).

In this connection, the notion of *A-proper operators* is crucial.

Basic Ideas of the Theory of Monotone Operators

> Riemann has shown us that proofs are better achieved through ideas than through long calculations.
>
> David Hilbert (1897)

> One can understand a mathematical statement, if one
>
> (i) can use it,
> (ii) has completely understood the proof, or
> (iii) one can independently find the proof again at any time.
>
> Only when one reaches the third step can one speak of understanding in a real sense.
>
> Aleksander Ostrowski (1951)

> Male lovers sometimes lack experience before their fortieth year, and later on, there is often a lack of opportunity. In the same way, younger mathematicians often lack the knowledge, and older ones lack the ideas.
>
> Wilhelm Blaschke (1942)

The theory of nonlinear monotone operators is based on only a few tricks. For the convenience of the reader, we summarize these tricks here. This way, we want to make the proofs of the main results as transparent as possible. In the following Chapters 26 through 36, the items (1), (2), etc. below, will be quoted as (25.1), (25.2), etc.

The theory of nonlinear monotone operators generalizes the following elementary result. We consider the real equation

$$\text{(E)} \qquad F(u) = b, \qquad u \in \mathbb{R},$$

and assume that:

(i) The function $F: \mathbb{R} \to \mathbb{R}$ is monotone.
(ii) F is continuous.
(iii) $F(u) \to \pm\infty$ as $u \to \pm\infty$.

Then, for each $b \in \mathbb{R}$, equation (E) has a solution. If F is strictly monotone, then the solution is unique (cf. Fig. 25.2).

This classical existence theorem follows from the intermediate value theorem of Bolzano, whereas the uniqueness statement is obvious.

In particular, if $f: \mathbb{R} \to \mathbb{R}$ is convex and C^1, then $F = f'$ is monotone, and (E) with $b = 0$ is the Euler equation to the minimum problem

$$\text{(M)} \qquad f(u) = \min!, \qquad u \in \mathbb{R}.$$

This observation is the key to the application of variational methods in the theory of monotone operators. However, note that the general theory of monotone operators concerns operator equations which are not necessarily the Euler equations of extremal problems.

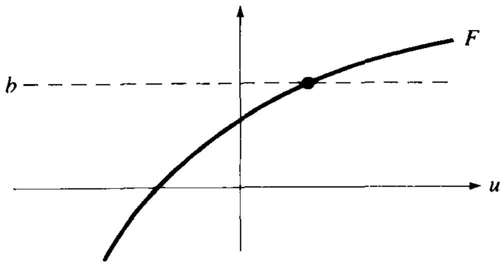


Figure 25.2

We now want to generalize the result above to monotone operator equations of the form

(E*) $$Au = b, \qquad u \in X.$$

Suppose that:

(i*) The operator $A: X \to X^*$ is monotone on the real reflexive B-space X, i.e.,

$$\langle Au - Av, u - v \rangle \geq 0 \qquad \text{for all} \quad u, v \in X.$$

(ii*) A is hemicontinuous, i.e., the map

$$t \mapsto \langle A(u + tv), w \rangle$$

is continuous on $[0, 1]$ for all $u, v, w \in X$.

(iii*) A is coercive, i.e.,

$$\lim_{\|u\| \to \infty} \frac{\langle Au, u \rangle}{\|u\|} = +\infty.$$

Then the main theorem on monotone operators (Theorem 26.A) tells us the following: For each $b \in X^*$, equation (E*) has a solution.

By Theorem 32.H, this fundamental result remains true if we replace (iii*) with the weaker condition that A is weakly coercive, i.e.,

$$\lim_{\|u\| \to \infty} \|Au\| = \infty.$$

If, in addition, A is strictly monotone, i.e.,

$$\langle Au - Av, u - v \rangle > 0 \qquad \text{for all} \quad u, v \in X \quad \text{with } u \neq v,$$

then the solution of (E*) is unique.

The result for equation (E) above is a special case of this theorem. In this connection, set $X = \mathbb{R}$ and

$$F(u) = Au.$$

Note that $X^* = \mathbb{R}$ and

$$\langle Au - Av, u - v \rangle = (F(u) - F(v))(u - v)$$