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Clifford. V. Johnson

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To my siblings Carol and Robert, my father Victor Reginald, and especially my mother Delia, who have all shaped me.

Preface

In view of the exciting developments in our understanding of those particular aspects of fundamental physics that string theory seems to capture, it seems appropriate to collect together some of the key tools and ideas which helped move things forward. The developments included a true revolution, since the physical perspective changed so radically that it undermined the long-standing status of strings as the basic fundamental objects, and instead the idea has arisen that a string theory description is simply a special (albeit rather novel and beautiful) corner of a larger theory called 'M-theory'. This book is not an attempt at a history of the revolution, as we are (arguably) still in the midst of it, especially since we are in the awkward position of not knowing even one satisfactory intrinsic definition of M-theory, and have implicit knowledge of it only through interconnections of its various limits.

All revolutions are supposed to have a collection of characters who played a crucial role in it, 'heroes' if you will. Hence, one would be expected to proceed to list here the names of various individuals. While I was lucky to be in a position to observe a lot of the activity at first hand and collect many wonderful anecdotes about how some things came to be, I will decline to start listing names at this juncture. It is too easy to yield to the temptation to emphasise a few personalities in a short space (such as this preface), and the result can sometimes be at the expense of others, a practice which happens all too often elsewhere. This seems to me to be especially inappropriate in a field where the most striking characteristic of the contributions has been the *collective* effort of hundreds of thinkers all over the planet, often linked by e-mail and the web, often never having met each other in person.

There were marvellous weeks, back in 1995 and 1996 especially, where there was one key paper after another, from all over the world, driven by the fact that new ideas were pouring in from conversations everyone was excitedly having at blackboards, in the sand, over lunch, *via* e-mail, on the back of an envelope, *etc.* However, when one is speculating about aspects of fundamental physics which are not yet in the directly testable realm it should be noted that ideas – even radical ones – are cheap. Computational tools are needed to test them, and to provide access to the new regimes to which the ideas beckon. The collection of tools which filled this crucial role in this context was built around 'D-branes', and it was the change of perspective and computational power that they brought that unlocked that steady flow of marvellous papers. In my mind, they can indisputably be placed high on list of characters cast as heroes of the revolution. Indeed, many will speak of the feeling that often arose after working with them for a while in those exciting days, that the D-branes simply had a life and character of their own. They shaped the ideas and language of the field in a way that was directed by no single personality, and – most importantly – were a wonderful and sharp tool for investigating in detail the nature of the many bold conjectures which were made.

D-branes were discovered well before the revolution, of course, but in the Summer of 1995 it was shown by Joseph Polchinksi that they were relevant to strongly coupled string theory. I arrived as a postdoctoral researcher at the Institute for Theoretical Physics (Santa Barbara, California) in the following Autumn, and by then it was already clear that there were many people, both young and old, who could benefit from a refresher course on issues outside the realm of heterotic string theory (on which much of the focus had been up to then, with an eye on phenomenology) and an introduction to D-branes. Furthermore, there was some need for an agreement about language and conventions, since there had not been much in the way of texts or other notes which focused on the relevant aspects. (Polchinski's modern textbook[1] was still only partially written, and the manuscript had been seen only by a privileged few.)

Some of us begged Joe to give us some lectures at the ITP, and I (and probably others) quickly had the idea for a written set of notes that could be circulated to the world at large, as a basic toolbox. I suggested this to him, and he eventually agreed. During the lectures, I took such notes as I could and then together with Shyamoli Chaudhuri, we produced some notes with Joe, which we released[2] with his name listed as first author – breaking the strict alphabetical convention in this field – as it seemed to me highly inappropriate, given our roles as scribes, that his name might come last. Happily, the 'D-notes' (as I liked to call them) seemed to be well received by very many, and proved to be useful in forming a common point of departure for almost everyone working in the field.

I was fortunate enough to be asked to give introductory lectures on D-branes over the following months and years, and this led me to write more notes to embellish the D-notes, finding new ways of explaining things, sometimes making illustrative links between different aspects, depending on the theme of the lecture series in which I was participating.

This book grew out of such lecture notes[3, 4], and contains my own biased perspective on what aspects of D-branes ought to be included in an introductory text. Pressures of space mean that I have omitted a large number of remarkably interesting and useful material, and my choices will no doubt not suit everyone. I have made many efforts for it to be a stand alone handbook. It is intended that the person who knows little or no string theory (but with some background in quantum field theory and relativity) can open this book, and upon working through it, learn many things about string theory, and become adept at computing with D-branes, making no reference to another string theory text. Perhaps as a bonus, they will even learn various aspects of advanced topics in relativity, geometry and quantum gravity and quantum field theory since those are the meat and drink of D-brane physics. However, if they want a deeper knowledge of many aspects of string theory which are only sketched here due to lack of space, then they can consult the excellent text of Polchinski[1], and also that of Green, Schwarz and Witten[5], which is still an excellent text for many aspects of the subject. There are also many other sources, on the web (*e.g.*, www.arXiv.org) and elsewhere, of detailed reviews of various specialised topics, even other string theory books[6].

So, this is not intended to be a string theory textbook. It is instead a handbook or toolbox for concepts concerning branes in string theory, with emphasis on D-branes. However, since many of the applications are in what I like to call 'extreme string theory' – taking limits like strong coupling, low energy, large N, etc. – the reader will also learn important physics of those regimes and others, which are not covered in any other text at this time.

Over the years I have had the great benefit of lengthy conversations about string theory and D-branes with many people, out of which my intuition for these matters developed, and I would like to thank them all. Chief among these are Robert Myers, Joseph Polchinski, and Edward Witten, all of whose patience (and refreshing open-mindedness in the early days) is much appreciated. I also thank all of the people with whom I have collaborated in very many exciting research projects, and from whom I learned a great deal. Aspects of some of that work will appear in this text, and I would like it made clear that any inaccuracies in presenting the results are my own.

Parts of this book were written (or sometimes day-dreamt about) in many inspiring places, not all of which I can recall, but I should thank especially a number of institutions for providing facilities: The New York Public Library's Rose Reading Room, and Columbia University's Butler Library (New York, NY, USA), the Bodlean Library (Oxford, England), The Aspen Centre for Physics (Aspen, Co., USA), The Park City Mathematical Institute 2001 (Park City, Utah, USA), El Centro de Estudious Científicos (Valdivia, Chile), The Physics Department at Stellenbosch University, and the Stellenbosch Institute for Advanced Study (Stellenbosch, South Africa), The Perimeter Institute for Fundamental Research (Waterloo, Canada), The Village Vanguard (New York, USA), Broadway (New York, USA), and various United Airlines lounges worldwide.

Thanks to Ian Davies, James Gregory, Laur Järv, Ken Lovis, Rob Myers and David Page for reading and commenting on parts of the manuscript, and many people around the world for their useful remarks upon earlier notes which were absorbed into this book. Thanks also go to Jim Gates, Brian Greene, David Gross, Ted Jacobson and Lenny Susskind for their thoughts on a late title change, and on other important matters concerning the book, logistical and otherwise†. I'd like to thank all of my colleagues at the Department of Mathematical Sciences, University of Durham, for providing such a friendly and supportive working environment, and Carol, Delia and Robert Johnson for their encouragement. Thanks also to Elizabeth and Nich Butler for much appreciated culinary provision and other matters of hospitality over Christmas 2001.

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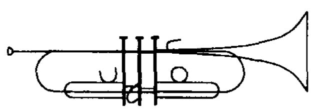

† A conversation with Brian led to a flirtation with the slightly irreverent idea of giving this book the simple title 'Volume III'. I abandoned this after a while, since it would produce confusion amongst those not aware of the affection held for (or existence of) the two-volume texts in references [1] and [5].

Contents

List of inserts *xviii*

Preface *xx*

1 **Overview and overture** 1
1.1 The classical dynamics of geometry 1
1.2 Gravitons and photons 7
1.3 Beyond classical gravity: perturbative strings 11
1.4 Beyond perturbative strings: branes 15
1.5 The quantum dynamics of geometry 19
1.6 Things to do in the meantime 20
1.7 On with the show 22

2 **Relativistic strings** **24**
2.1 Motion of classical point particles 24
2.1.1 Two actions 24
2.1.2 Symmetries 26
2.2 Classical bosonic strings 27
2.2.1 Two actions 27
2.2.2 Symmetries 29
2.2.3 String equations of motion 30
2.2.4 Further aspects of the two dimensional perspective 31
2.2.5 The stress tensor 35
2.2.6 Gauge fixing 35
2.2.7 The mode decomposition 37
2.2.8 Conformal invariance as a residual symmetry 37
2.2.9 Some Hamiltonian dynamics 38

2.3 Quantised bosonic strings 40
2.3.1 The constraints and physical states 41
2.3.2 The intercept and critical dimensions 42
2.3.3 A glance at more sophisticated techniques 45
2.4 The sphere, the plane and the vertex operator 47
2.4.1 States and operators 48
2.5 Chan–Paton factors 51
2.6 Unoriented strings 52
2.6.1 Unoriented open strings 52
2.6.2 Unoriented closed strings 54
2.6.3 World-sheet diagrams 55
2.7 Strings in curved backgrounds 56
2.8 A quick look at geometry 61
2.8.1 Working with the local tangent frames 61
2.8.2 Differential forms 63
2.8.3 Coordinate vs. orthonormal bases 65
2.8.4 The Lorentz group as a gauge group 67
2.8.5 Fermions in curved spacetime 68
2.8.6 Comparison to differential geometry 68

3 A closer look at the world-sheet 70
3.1 Conformal invariance 70
3.1.1 Diverse dimensions 70
3.1.2 The special case of two dimensions 73
3.1.3 States and operators 74
3.1.4 The operator product expansion 75
3.1.5 The stress tensor and the Virasoro algebra 76
3.2 Revisiting the relativistic string 80
3.3 Fixing the conformal gauge 85
3.3.1 Conformal ghosts 85
3.3.2 The critical dimension 86
3.4 The closed string partition function 87

4 Strings on circles and T-duality 94
4.1 Fields and strings on a circle 94
4.1.1 The Kaluza–Klein reduction 95
4.1.2 Closed strings on a circle 96
4.2 T-duality for closed strings 99
4.3 A special radius: enhanced gauge symmetry 100
4.4 The circle partition function 103
4.5 Toriodal compactifications 104

4.6 More on enhanced gauge symmetry 108
4.6.1 Lie algebras and groups 108
4.6.2 The classical Lie algebras 111
4.6.3 Physical realisations with vertex operators 113
4.7 Another special radius: bosonisation 113
4.8 String theory on an orbifold 117
4.9 T-duality for open strings: D-branes 119
4.9.1 Chan–Paton factors and Wilson lines 121
4.10 D-brane collective coordinates 123
4.11 T-duality for unoriented strings: orientifolds 125

5 Background fields and world-volume actions 129
5.1 T-duality in background fields 129
5.2 A first look at the D-brane world-volume action 131
5.2.1 World-volume actions from tilted D-branes 133
5.3 The Dirac–Born–Infeld action 135
5.4 The action of T-duality 136
5.5 Non-Abelian extensions 136
5.6 D-branes and gauge theory 138
5.7 BPS lumps on the world-volume 138

6 D-brane tension and boundary states 141
6.1 The D-brane tension 142
6.1.1 An open string partition function 142
6.1.2 A background field computation 145
6.2 The orientifold tension 148
6.2.1 Another open string partition function 148
6.3 The boundary state formalism 150

7 Supersymmetric strings 155
7.1 The three basic superstring theories 155
7.1.1 Open superstrings: type I 155
7.1.2 Closed superstrings: type II 160
7.1.3 Type I from type IIB, the prototype orientifold 165
7.1.4 The Green–Schwarz mechanism 166
7.2 The two basic heterotic string theories 169
7.2.1 $SO(32)$ and $E_8 \times E_8$ from self-dual lattices 171
7.2.2 The massless spectrum 172
7.3 The ten dimensional supergravities 174
7.4 Heterotic toroidal compactifications 176
7.5 Superstring toroidal compactification 178
7.6 A superstring orbifold: discovering the K3 manifold 179

7.6.1 The orbifold spectrum 180
7.6.2 Another miraculous anomaly cancellation 183
7.6.3 The K3 manifold 184
7.6.4 Blowing up the orbifold 185
7.6.5 Some other K3 orbifolds 189
7.6.6 Anticipating D-manifolds 191

8 Supersymmetric strings and T-duality 192
8.1 T-duality of supersymmetric strings 192
8.1.1 T-duality of type II superstrings 192
8.1.2 T-duality of type I superstrings 193
8.1.3 T-duality for the heterotic strings 194
8.2 D-branes as BPS solitons 195
8.3 The D-brane charge and tension 197
8.4 The orientifold charge and tension 200
8.5 Type I from type IIB, revisited 201
8.6 Dirac charge quantisation 201
8.7 D-branes in type I 202

9 World-volume curvature couplings 205
9.1 Tilted D-branes and branes within branes 205
9.2 Anomalous gauge couplings 206
9.3 Characteristic classes and invariant polynomials 210
9.4 Anomalous curvature couplings 216
9.5 A relation to anomalies 218
9.6 D-branes and K-theory 220
9.7 Further non-Abelian extensions 221
9.8 Further curvature couplings 222

10 The geometry of D-branes 224
10.1 A look at black holes in four dimensions 224
10.1.1 A brief study of the Einstein–Maxwell system 224
10.1.2 Basic properties of Schwarzschild 225
10.1.3 Basic properties of Reissner–Nordstrom 228
10.1.4 Extremality, supersymmetry, and the BPS condition 228
10.1.5 Multiple black holes and multicentre solutions 232
10.1.6 Near horizon geometry and an infinite throat 233
10.1.7 Cosmological constant; de Sitter and anti-de Sitter 233
10.1.8 de-Sitter spacetime and the sphere 234
10.1.9 Anti-de Sitter in various coordinate systems 235

10.1.10 Anti-de Sitter as a hyperbolic slice 236
10.1.11 Revisiting the extremal solution 237
10.2 The geometry of D-branes 238
10.2.1 A family of 'p-brane' solutions 238
10.2.2 The boost form of solution 239
10.2.3 The extremal limit and coincident D-branes 240
10.3 Probing p-brane geometry with Dp-branes 243
10.3.1 Thought experiment: building p with Dp 243
10.3.2 Effective Lagrangian from the world-volume action 244
10.3.3 A metric on moduli space 245
10.4 T-duality and supergravity solutions 246
10.4.1 D$(p+1)$ from Dp 246
10.4.2 D$(p-1)$ from Dp 248

11 Multiple D-branes and bound states 249
11.1 Dp and Dp' from boundary conditions 249
11.2 The BPS bound for the Dp–Dp' system 252
11.3 Bound states of fundamental strings and D-strings 254
11.4 The three-string junction 255
11.5 Aspects of D-brane bound states 258
11.5.1 0–0 bound states 258
11.5.2 0–2 bound states 258
11.5.3 0–4 bound states 259
11.5.4 0–6 bound states 260
11.5.5 0–8 bound states 260

12 Strong coupling and string duality 261
12.1 Type IIB/type IIB duality 261
12.1.1 D1-brane collective coordinates 261
12.1.2 S-duality and $SL(2,\mathbb{Z})$ 263
12.2 $SO(32)$ Type I/heterotic duality 264
12.2.1 D1-brane collective coordinates 264
12.3 Dual branes from 10D string–string duality 265
12.3.1 The heterotic NS-fivebrane 267
12.3.2 The type IIA and type IIB NS5-brane 268
12.4 Type IIA/M-theory duality 271
12.4.1 A closer look at D0-branes 271
12.4.2 Eleven dimensional supergravity 271
12.5 $E_8 \times E_8$ heterotic string/M-theory duality 273
12.6 M2-branes and M5-branes 276
12.6.1 Supergravity solutions 276

12.6.2 From D-branes and NS5-branes to M-branes and back 277
12.7 U-duality 278
12.7.1 Type II strings on T^5 and $E_{6(6)}$ 278
12.7.2 U-duality and bound states 279

13 D-branes and geometry I **282**
13.1 D-branes as probes of ALE spaces 282
13.1.1 Basic setup and a quiver gauge theory 282
13.1.2 The moduli space of vacua 285
13.1.3 ALE space as metric on moduli space 286
13.1.4 D-branes and the hyper-Kähler quotient 289
13.2 Fractional D-branes and wrapped D-branes 291
13.2.1 Fractional branes 291
13.2.2 Wrapped branes 292
13.3 Wrapped, fractional and stretched branes 294
13.3.1 NS5-branes from ALE spaces 295
13.3.2 Dual realisations of quivers 296
13.4 D-branes as instantons 300
13.4.1 Seeing the instanton with a probe 301
13.4.2 Small instantons 305
13.5 D-branes as monopoles 306
13.5.1 Adjoint Higgs and monopoles 309
13.5.2 BPS monopole solution from Nahm data 311
13.6 The D-brane dielectric effect 314
13.6.1 Non-Abelian world-volume interactions 314
13.6.2 Stable fuzzy spherical D-branes 316
13.6.3 Stable smooth spherical D-branes 318

14 K3 orientifolds and compactification **322**
14.1 $\mathbb{Z}_N$ orientifolds and Chan–Paton factors 322
14.2 Loops and tadpoles for ALE $\mathbb{Z}_M$ singularities 324
14.2.1 One-loop diagrams and tadpoles 324
14.2.2 Computing the one-loop diagrams 325
14.2.3 Extracting the tadpoles 330
14.3 Solving the tadpole equations 333
14.3.1 T-duality relations 333
14.3.2 Explicit solutions 334
14.4 Closed string spectra 336
14.5 Open string spectra 339
14.6 Anomalies for $\mathcal{N} = 1$ in six dimensions 341

15 D-branes and geometry II **345**
15.1 Probing p with $D(p-4)$ 345
15.2 Probing six-branes: Kaluza–Klein monopoles and M-theory 346
15.3 The moduli space of 3D supersymmetric gauge theory 348
15.4 Wrapped branes and the enhançon mechanism 352
15.4.1 Wrapping D6-branes 353
15.4.2 The repulson geometry 354
15.4.3 Probing with a wrapped D6-brane 356
15.5 The consistency of excision in supergravity 360
15.6 The moduli space of pure glue in 3D 362
15.6.1 Multi-monopole moduli space 363

16 Towards M- and F-theory **367**
16.1 The type IIB string and F-theory 367
16.1.1 $SL(2,\mathbb{Z})$ duality 368
16.1.2 The (p,q) strings 369
16.1.3 String networks 371
16.1.4 The self-duality of D3-branes 373
16.1.5 (p,q) Fivebranes 375
16.1.6 $SL(2,\mathbb{Z})$ and D7-branes 376
16.1.7 Some algebraic geometry 379
16.1.8 F-theory, and a dual heterotic description 383
16.1.9 (p,q) Sevenbranes 384
16.1.10 Enhanced gauge symmetry and singularities of K3 386
16.1.11 F-theory at constant coupling 387
16.1.12 The moduli space of $\mathcal{N}=2$ $SU(N)$ with $N_\mathrm{f}=4$ 392
16.2 M-theory origins of F-theory 394
16.2.1 M-branes and odd D-branes 396
16.2.2 M-theory on K3 and heterotic on T^3 399
16.2.3 Type IIA on K3 and heterotic on T^4 400
16.3 Matrix theory 400
16.3.1 Another look at D0-branes 401
16.3.2 The infinite momentum frame 402
16.3.3 Matrix string theory 404

17 D-branes and black holes **409**
17.1 Black hole thermodynamics 409
17.1.1 The path integral and the Euclidean calculus 409
17.1.2 The semiclassical approximation 411
17.1.3 The temperature of black holes 412

17.2 The Euclidean action calculus 414
17.2.1 The action for Schwarzschild 414
17.2.2 The action for Reissner–Nordström 416
17.2.3 The laws of thermodynamics 417
17.3 $D = 5$ Reissner–Nordström black holes 418
17.3.1 Making the black hole 420
17.3.2 Microscopic entropy and a 2D field theory 425
17.3.3 Non-extremality and a 2D dilute gas limit 427
17.4 Near horizon geometry 429
17.5 Replacing T^4 with K3 432
17.5.1 The geometry 432
17.5.2 The microscopic entropy 433
17.5.3 Probing the black hole with branes 434
17.5.4 The enhançon and the second law 437

18 D-branes, gravity and gauge theory 440
18.1 The AdS/CFT correspondence 441
18.1.1 Branes and the decoupling limit 441
18.1.2 Sphere reduction and gauged supergravity 443
18.1.3 Extracts from the dictionary 446
18.1.4 The action, counterterms, and the stress tensor 449
18.2 The correspondence at finite temperature 452
18.2.1 Limits of the non-extremal D3-brane 452
18.2.2 The AdS–Schwarzschild black hole in global coordinates 453
18.3 The correspondence with a chemical potential 455
18.3.1 Spinning D3-branes and charged AdS black holes 455
18.3.2 The AdS–Reissner–Nordström black hole 459
18.3.3 Thermodynamic phase structure 459
18.4 The holographic principle 464

19 The holographic renormalisation group 467
19.1 Renormalisation group flows from gravity 467
19.1.1 A BPS domain wall and supersymmetry 469
19.2 Flowing on the Coulomb branch 472
19.2.1 A five dimensional solution 472
19.2.2 A ten dimensional solution 475
19.2.3 Probing the geometry 475
19.2.4 Brane distributions 478
19.3 An $\mathcal{N} = 1$ gauge dual RG flow 480
19.3.1 The five dimensional solution 482