Springer 大学数学图书——影印版

The Heritage of Thales

泰勒斯的遗产

W. S. Anglin　J. Lambek　著

TSINGHUA UNIVERSITY PRESS　Springer

Springer 大学数学图书——影印版

The Heritage of Thales

泰勒斯的遗产

TSINGHUA UNIVERSITY PRESS
北京

Springer

内容提要

本书以专题方式讲述数学的历史和数学的哲学（非史论型著作），每个专题相对独立。全书以数学历史为线索，以数学为内容主体，以数学哲学为引申，易读、易懂，是本科生学习数学过程中非常好的课外读物。

W. S. Anglin, J. Lambek
The Heritage of Thales
EISBN: 0-387-94544-X

图书在版编目（CIP）数据

泰勒斯的遗产 = The Heritage of Thales: 英文 /（美）安格林（Anglin, W. S.），（美）兰贝克（Lambek, J.）著. —影印本. —北京：清华大学出版社，2009.11
（Springer 大学数学图书）

ISBN 978-7-302-21483-0

Ⅰ. ①泰… Ⅱ. ①安… ②兰… Ⅲ. ①数学史－高等学校－教材－英文 ②数学哲学－高等学校－教材－英文 Ⅳ. ①O11 ②O1-0

中国版本图书馆 CIP 数据核字（2009）第 200092 号

责任编辑：陈朝晖
责任印制：孟凡玉
出版发行：清华大学出版社 **地　址**：北京清华大学学研大厦 A 座
http://www.tup.com.cn **邮　编**：100084
社 总 机：010-62770175 **邮　购**：010-62786544
投稿与读者服务：010-62776969，c-service@tup.tsinghua.edu.cn
质 量 反 馈：010-62772015，zhiliang@tup.tsinghua.edu.cn
印 刷 者：清华大学印刷厂
装 订 者：三河市金元印装有限公司
经　销：全国新华书店
开　本：175×245 **印　张**：21.75
版　次：2009 年 11 月第 1 版 **印　次**：2009 年 11 月第 1 次印刷
印　数：1~4000
定　价：45.00 元

本书如存在文字不清、漏印、缺页、倒页、脱页等印装质量问题，请与清华大学出版社出版部联系调换。
联系电话：010-62770177 转 3103 产品编号：023898-01

Springer 大学数学图书——影印版

- Information and Coding Theory（信息和编码理论）
  Jones, Gareth A., Jones, J.Mary
- Symmetries（对称）
  Johnson, D.L.
- Matrix Groups: An Introduction to Lie Group Theory（矩阵群：李群理论基础）
  Baker, Andrew
- Essential Mathematical Biology（生物数学引论）
  Britton, Nicholas F.
- Special Relativity（狭义相对论）
  Woodhouse, N.M.J.
- Second Year Calculus（高等微积分）
  Bressoud, David M.
- Linear Functional Analysis（线性泛函分析）
  Rynne, Bryan P., Youngson, Martin A.
- A First Course in Discrete Mathematics（离散数学引论）
  Anderson, Ian
- Discrete Mathematics（离散数学）
  Lovász, L., Pelikán, J., Vesztergombi, K.
- Reading, Writing, and Proving（数学的读写和证明）
  Daepp Ulrich, Gorkin Pamela
- Basic Stochastic Processes（随机过程基础）
  Brzeźniak Zdzisław, Zastawniak Tomasz
- The Fundamental Theorem of Algebra（代数基本定理）
  Fine Benjamin, Rosenberger Gerhard
- The Heritage of Thales（泰勒斯的遗产）
  Anglin W. S., Lambek J.

Springer 大学数学图书——翻译版

- ○ 信息和编码理论（Information and Coding Theory）
  Jones, Gareth A., Jones, J.Mary
- ○ 对称（Symmetries）
  Johnson, D.L.
- ○ 矩阵群：李群理论基础（Matrix Groups: An Introduction to Lie Group Theory）
  Baker, Andrew
- ○ 生物数学引论（Essential Mathematical Biology）
  Britton, Nicholas F.
- ○ 狭义相对论（Special Relativity）
  Woodhouse, N.M.J.
- ○ 高等微积分（Second Year Calculus）
  Bressoud, David M.
- ○ 线性泛函分析（Linear Functional Analysis）
  Rynne, Bryan P., Youngson, Martin A.
- ○ 离散数学引论（A First Course in Discrete Mathematics）
  Anderson, Ian
- ○ 离散数学（Discrete Mathematics）
  Lovász, L., Pelikán, J., Vesztergombi, K.
- ○ 数学的读写和证明（Reading, Writing, and Proving）
  Daepp Ulrich, Gorkin Pamela
- ○ 随机过程基础（Basic Stochastic Processes）
  Brzeźniak Zdzisław, Zastawniak Tomasz
- ○ 代数基本定理（The Fundamental Theorem of Algebra）
  Fine Benjamin, Rosenberger Gerhard
- ○ 泰勒斯的遗产（The Heritage of Thales）
  Anglin W. S., Lambek J

序　　言

在学校教书多年，当学生（特别是本科生）问有什么好的参考书时，我们所能推荐的似乎除了教材还是教材，而且不同教材之间的差别并不明显、特色也不鲜明。所以多年前我们就开始酝酿，希望为本科学生引进一些好的参考书，为此清华大学数学科学系的许多教授与清华大学出版社共同付出了很多心血。

这里首批推出的十余本图书，是从 Springer 出版社的多个系列丛书中精心挑选出来的。在丛书的筹划过程中，我们挑选图书最重要的标准并不是完美，而是有特色并包容各个学派（有些书甚至有争议，比如从数学上看也许不够严格），其出发点是希望我们的学生能够吸纳百家之长；同时，在价格方面，我们也做了很多工作，以使得本系列丛书的价格能让更多学校和学生接受，使得更多学生能够从中受益。

本系列图书按其定位，大体有如下四种类型（一本书可以属于多类，但这里限于篇幅不能一一介绍）。

一、适用面比较广、有特色并可以用作教材或参考书的图书。例如：

● Lovász et al.: Discrete Mathematics. 2003

该书是离散数学的入门类型教材。与现有的教材（包括国外的教材）相比，它涵盖了离散数学新颖而又前沿的研究课题，同时还涉及信息科学方面既基本又有趣的应用；在着力打好数学基础的同时，也强调了数学与信息科学的关联。姚期智先生倡导和主持的清华大学计算机科学试验班，已经选择该书作为离散数学课程的教材。

二、在目前国内的数学教育中，课程主要以学科的纵向发展为主线，而对数学不同学科之间的联系讨论很少。学生缺乏把不同学科视为一个数学整体的训练，这方面的读物尤其欠缺。这是本丛书一个重要的着力点。最典型的是：

● Fine/Rosenberger: The Fundamental Theorem of Algebra. 1997

该书对数学中最重要的定理——代数基本定理给出了六种证明，方法涉及到分析、代数与拓扑；附录中还给出了 Gauss 的证明和 Cauchy 的证明。全书以一个数学问题为主线展开，纵横数学的核心领域；结构严谨、文笔流畅、浅显易懂、引人入胜，是一本少见的能够让读者入迷的好读物，用它来引导学生欣赏和领会“数学的美”绝对不会落于空谈。该书适于自学、讨论，也是极好的短学期课程教材。

● Baker: Matrix Groups. 2001

就内容而言，本书并不超出我国大学线性代数、抽象代数和一般拓扑学课程的内容，但是本书所讲的是李群和李代数的基础理论——这是现代数学和物理学非常重要的工具。各种矩阵群和矩阵代数是李群和李代数最典型和

最重要的例子，同时也能帮助学生建立数学不同学科之间的联系。从矩阵出发，既能把握李群和李代数的实质，又能学会计算和运用，所以这是一本不可多得的好书。

三、科学与技术的发展不断为数学提出新的研究课题，因此在数学学科的发展过程中，来自其他学科的推动力是不能忽视的。本系列中第三种类型的读物就是强调数学与其他学科的联系。例如：

● Woodhouse: Special Relativity. 2003

该书将物理与数学有机结合，体现了物理学家伽利略的名言："大自然是一部用数学语言写成的巨著。" 不仅如此，本书作者还通过对线性代数、微积分、场论等数学的运用进一步强调并贯穿这样的观点：数学的真谛和发展存在并产生于物理或自然规律及其发现中。精读此书有助于理解物理学和数学的整体关系。

● Britton: Essential Mathematical Biology. 2003

生命科学在本世纪一定会有很大发展，其对数学的需求和推动是可以预见的。因此生物数学在应用数学中占有日益重要的地位，数学系培养的学生至少一部分人应当对这个领域有所了解。随着生命科学的迅速发展，生物数学也发展很快。本书由浅入深，从经典的问题入手，最后走向学科前沿和近年的热点问题。该书至少可以消除学生对生物学的神秘感。

四、最后一类是适合本科学生的课外读物。这类图书对激发和引导学生学习数学的兴趣会非常有帮助，而且目前国内也急需这样的图书。例如：

● Daepp/Gorkin: Reading, Writing and Proving. 2003

该书对初学高等数学的读者来说特别有意义。它的基本出发点是引导读者以研究的心态去学习，让读者养成独立思考的习惯，并进而成为研究型的学习者。该书将一个学习数学的过程在某种意义下程序化，努力让学习者养成一个好的学习习惯，以及学会如何应对问题。该书特色鲜明，类似的图书确实很少。

● Brzezniak/Zastawniak: Basic Stochastic Processes. 1998

随机过程理论在数学、科学和工程中有越来越广泛的应用，本书适合国内的需要。其主要特点是：书中配有的习题是巩固和延伸学习内容的基本手段，而且有十分完整的解答，非常适合自学和作为教学参考书。这是一本难得的好书，它 1999 年出版，到 2000 年已经是第 3 次印刷，到 2003 年则第 6 次重印。

● Anglin/Lambek: The Heritage of Thales. 1995

该书的基本内容是数学的历史和数学的哲学。数学历史是该书的线索，数学是内容的主体，引申到的是数学哲学。它不是一本史论型的著作，而是采用专题式编写方式，每个专题相对独立，所以比较易读、易懂，是本科生学习数学过程中非常好的课外读物。

本系列丛书中的大部分图书还将翻译为中文出版，以适应更多读者的需要。丛书筹划过程中，冯克勤、郑志勇、卢旭光、郑建华、王殿军、杨利军、叶俊、扈志明等很多清华大学的教授都投入了大量精力。他们之中很多人也将是后面中文版的译者。此外，我们今后还将不断努力丰富引进丛书的种类，同时也会将选书的范围在可能情况下进一步扩大到其他高水平的出版机构。

教育是科学和技术发展的基石，数学教育更是基石的基础。因为是基础所以它重要；也因为是基础所以它显示度不高，容易不被重视。只有将人才培养放到更高的地位上，中国成为创新型国家的目标才会成为可能。

本系列丛书的正式推出，圆了一个我们多年的梦，但这无疑仅仅是开始。

白峰杉

2006 年 6 月于清华园

Preface

This is intended as a textbook on the history, philosophy and foundations of mathematics, primarily for students specializing in mathematics, but we also wish to welcome interested students from the sciences, humanities and education. We have attempted to give approximately equal treatment to the three subjects: history, philosophy and mathematics.

History
We must emphasize that this is not a scholarly account of the history of mathematics, but rather an attempt to teach some good mathematics in a historical context. Since neither of the authors is a professional historian, we have made liberal use of secondary sources. We have tried to give references for cited facts and opinions. However, considering that this text developed by repeated revisions from lecture notes of two courses given by one of us over a 25 year period, some attributions may have been lost. We could not resist retelling some amusing anecdotes, even when we suspect that they have no proven historical basis. As to the mathematicians listed in our account, we admit to being colour and gender blind; we have not attempted a balanced distribution of the mathematicians listed to meet today's standards of political correctness.

Philosophy
Both authors having wide philosophical interests, this text contains perhaps more philosophical asides than other books on the history of mathematics. For example, we discuss the relevance to mathematics of the pre-Socratic philosophers and of Plato, Aristotle, Leibniz and Russell. We also have

presented some original insights. However, on some points our opinions diverge; so, in a spirit of compromise, we have agreed to excise some of our more extreme views. Some of these divergent opinions have been expressed in Anglin [1994] and Lambek [1994].

Mathematics

One of the challenges one faces in offering a course on the history and philosophy of mathematics is to persuade one's colleagues that the course contains some genuine mathematics. For this reason, we have included some mathematical topics, usually not treated in standard courses, for example, the renaissance method for solving cubic equations and an elementary proof of the impossibility of trisecting arbitrary angles by ruler and compass constructions. We have taken the liberty of presenting many mathematical ideas in modern garb, with the hindsight inspired by more recent developments, since a presentation faithful to the original sources, while catering to the serious scholar, would bore most students to tears.

In Part I we deal essentially with the history of mathematics up to about 1800. This is because thereafter mathematics tends to become more specialized and too advanced for the students we have in mind. However, we make occasional excursions into more modern mathematics, partly to relieve the tedium associated with a strictly chronological development and partly to present modern answers to some ancient questions, whenever this can be done without overly taxing the students' ability.

In Part II we deal with some selected topics from the nineteenth and twentieth centuries. In that period, mathematics became rather specialized and made spectacular progress in different directions, but we confine attention to questions in the foundations and philosophy of mathematics.

The more universal aspects of mathematics are sketched briefly in the last five sections. We introduce the language of category theory, which attempts a kind of unification of different branches of mathematics, albeit at a very basic and abstract level.

Acknowledgements

The authors wish to acknowledge partial support from the Social Sciences and Humanities Research Council of Canada, from the Natural Sciences and Engineering Council of Canada and from the Quebec Department of Education.

We wish to express our sincerest thanks to Matthew Egan for his undaunted dedication in typing and editing and to Mira Bhargava, Henri Darmon and Ramona Behravan for their conscientious reading and criticism of the manuscript.

W. S. Anglin and J. Lambek

Contents

0

Introduction

Remarks on prehistory

Long before written records were kept, people were concerned with the seasons, important in agriculture, and the sky, which permitted them to read off the passage of time. Everyone knows that the *year* is the time it takes the sun to complete its orbit about the earth. (Copernicus notwithstanding, mathematical readers will see nothing wrong with placing the origin of the coordinate system at the center of the earth.) Also, a *month* is supposed to be the time it takes the moon to go around the earth; at least, this was the case before the lengths of the months were laid down by law. But what about the *week*? Theological explanations aside, it is the smallest period, longer than a day, that can be easily observed by looking at the sky: the time it takes the moon to pass from one phase to another, from new moon to half moon, from half moon to full moon, etc.

The days of the week are named after the sun, the moon and the five planets visible to the naked eye: Mars (French *mardi*), Mercury (French *mercredi*), Jupiter (French *jeudi*), Venus (French *vendredi*) and Saturn (English *Saturday*). The English *Tuesday, Wednesday, Thursday* and *Friday* are named after the Teutonic deities which supposedly correspond to the Roman gods after whom the planets were named.

In Hindu astronomy there are nine planetary deities, the *graha*. In addition to the seven associated with the days of the week, there are two others, *rahu* and *kebu*, alleged to be associated with the so-called 'nodes'. These are the points where the orbits of the sun and the moon, when traced out on the firmament, intersect. (See Freed and Freed [1980].) The importance of the nodes is that an eclipse of the sun or the moon can only occur when both sun and moon fall on the nodes, to within 10°; according to an ancient rule of thumb, this was supposed to happen once in about 18.6 years.

At Stonehenge in England there is an imposing prehistoric monument, dating from about 2,500 BC. The huge standing stones of the monument were presumably used to sight the points on the horizon where the sun and the moon, and perhaps Venus, rise and set at certain dates (Hawkins [1965]). They are surrounded by a circle of 56 holes in the ground, and Fred Hoyle [1977] has proposed the ingenious hypothesis that these were used as a calendar and to calculate the dates of possible eclipses.

According to Hoyle, the idea was to move a *sun marker* two holes in 13 days, a *moon marker* two holes each day, and two *nodal stones* three holes per year. The sun marker would thus complete an orbit in 364 days; the discrepancy could be fixed by appropriate adjustments at midsummer and midwinter. The moon marker would complete an orbit in 28 days, that is, four weeks. Of course, this should really be 29.5 days, so adjustments might have to be made each full moon and each new moon. The nodal stones would take 56/3 years to perform a complete orbit. On those occasions when both sun marker and moon marker were about to catch up with the nodal stones, the presiding priest could risk predicting an eclipse.

Foreword on history

Even so-called 'primitive' societies may be engaged in some fairly sophisticated mathematical activities, for example, the calculations involved in kinship descriptions. (How many students can tell on the spot what exactly is a *second cousin three times removed*?) For interested readers, we recommend two recent books: *Africa Counts* by Claudia Zaslovsky and *Ethnomathematics* by Marcia Ascher.

Mathematics, as usually conceived, begins with the development of agriculture in the river valleys of Egypt, Iraq, India and China. If we pay more attention to the Near East than to the Far East, this is because the former has provided us with more accessible records and because modern mathematics can be traced back directly to it. We possess written records concerning the state of mathematics in Egypt and Mesopotamia (Iraq) from as early as about 2000 BC. Around 500 BC, mathematical knowledge spread to the Greek world. This included not only modern Greece, but also the coast of Asia Minor (modern Turkey) and Magna Grecia (southern Italy and Sicily). About 300 BC, the center of mathematics moved from Athens to Alexandria in Egypt, where it was to remain for the next 800 years; for it was in Alexandria that all the books were kept.

Around 500 AD, mediterranean civilization finally came to a stop, perhaps because of the repeated impact of epidemic diseases. About 800 AD, mathematics in the Alexandrian tradition resurfaced in India, which had a long mathematical tradition of its own. The Arabs, aided by translations of Greek texts, developed and transmitted mathematical knowledge from India back to the mediterranean area and ultimately to Europe. During the so-called 'renaissance', mathematics flourished in Italy and, aided by the Chinese invention of printing, spread to Western and Central Europe. Of course, today mathematics is being pursued in all the industrial countries of the world.

Introduction to the number system

The historical and pedagogical development of the number system goes somewhat like this:

$$\mathbf{N}^+ \to \mathbf{Q}^+ \to \mathbf{R}^+ \to \mathbf{R} \to \mathbf{C} \to \mathbf{H}\ .$$

Here $\mathbf{N}^+$ is the set of positive integers, the *numbers* used for counting, known to all societies. $\mathbf{Q}^+$ is the set of positive rationals, namely, *quotients* of positive integers, surely known to all agricultural civilizations. At one time, they were believed to exhaust all the numbers, until the Pythagoreans discovered that the diagonal of a square was not a rational multiple of its side. We use $\mathbf{R}^+$ to denote the positive *reals*; these were certainly used effectively by Thales, though the Greeks originally tended to regard them as ratios of geometric quantities. A formal treatment, anticipating the nineteenth century definition by Dedekind, was first given by Eudoxus in Athens. The transition from $\mathbf{R}^+$ to $\mathbf{R}$, the set of all reals, positive, zero and negative, took place in India and may be ascribed to Brahmagupta. The set $\mathbf{C}$ of *complex* numbers was first considered by Cardano to describe the intermediate steps in solving a cubic equation with real coefficients and three real solutions. The set $\mathbf{H}$ of *quaternions* is named after their inventor William Hamilton, who may have been preceded by Olinde Rodrigues and perhaps even by Carl Friedrich Gauss.

Most of the advances in the development of the number system may have been motivated by the desire to solve equations. Thus, the equations $2x = 3$, $x^2 = 2$, $x + 1 = 0$ and $x^2 + 1 = 0$ led to the successive introduction of $\mathbf{Q}^+, \mathbf{R}^+, \mathbf{R}$ and $\mathbf{C}$, respectively. However, all polynomial equations with complex coefficients do have complex solutions, so the introduction of quaternions requires a different justification. They were motivated by the desire to pass from the plane, describable by complex numbers, to three or four dimensions.