Michio Kaku

Introduction to Superstrings and M-Theory

Second Edition

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To my parents

Preface

We are all agreed that your theory is crazy. The question which divides us is whether it is crazy enough.

—Niels Bohr

Superstring theory (and its latest formulation, M-theory) has emerged as the most promising candidate for a quantum theory of all known interactions. Superstrings apparently solve a problem that has defied solution for the past 50 years, namely the unification of the two great fundamental physical theories of the century, quantum field theory and general relativity. Superstring theory introduces an entirely new physical picture into theoretical physics and a new mathematics that has startled even the mathematicians.

Ironically, although superstring theory is supposed to provide a unified field theory of the Universe, the theory itself often seems like a confused jumble of folklore, random rules of thumb, and intuitions. This is because the development of superstring theory has been unlike that of any other theory, such as general relativity, which began with a geometry and an action and later evolved into a quantum theory. Superstring theory, by contrast, has been evolving backward for the past 30 years. It has a bizarre history, beginning with the purely accidental discovery of the quantum theory in 1968 by G. Veneziano and M. Suzuki.

Thumbing through old mathematics books, they stumbled by chance on the Beta function, written down in the last century by mathematician Leonhard Euler. To their amazement, they discovered that the Beta function satisfied almost all the stringent requirements of the scattering matrix describing particle interactions. Never in the history of physics has an important scientific discovery been made in quite this random fashion.

Because of this accident of history, physicists have ever since been trying to work backward to fathom the physical principles and symmetries that underlie the theory. Unlike Einstein's theory of general relativity, which began with a geometric principle, the equivalence principle, from which the action could be derived, the fundamental physical and geometric principles that lie at the foundation of superstring theory are still unknown.

To reduce the amount of hand-waving and confusion this has caused, three themes have been stressed throughout this book. To provide the student with a solid foundation in superstring theory, we have first stressed the method of *Feynman path integrals*, which provides by far the most powerful formalism in which to discuss the model. Path integrals have become an indispensable tool for theoretical physicists, especially when quantizing gauge theories. Therefore, we have devoted Chapter 1 of this book to introducing the student to the methods of path integrals for point particles.

The second theme of this book is the method of *second quantization*. Although traditionally field theory is formulated as a second quantized theory, the bulk of superstring theory is formulated as a first quantized theory, presenting numerous conceptual problems for the beginner. Unlike the method of second quantization, where all the rules can be derived from a single action, the method of first quantization must be supplemented with numerous other rules and conventions. The hope is that the second quantized theory will reveal the underlying geometry on which the entire model is based.

The third theme of this book is *duality*, which has revolutionized the way we formulate string theory. Duality, which was first discovered in Maxwell's equations, allows us to determine the equivalence of two seemingly different theories. Using duality, we can show that the five different superstring theories in 10 dimensions are actually unified into a single theory in 11 dimensions, a still mysterious theory called M-theory, which reduces to 11-dimensional supergravity in the low-energy limit. Since duality allows us to show the equivalence of a weak coupling theory to a strong coupling theory, it has allowed us to probe, for the first time, the nonperturbative region of string theory, where we find a host of new objects, such as membranes, M-branes, and D-branes. Although the complete action of M-theory is still unknown, already it has yielded a flood of new information concerning the strong coupling behavior of string theory.

We now know that strings coexist with membranes of various dimensions; ultimately, the entire theory may be formulated in terms of a master theory in 11 dimensions, perhaps in terms of membranes of some sort. Ironically, although the action for M-theory is not known, physicists believe that the theory exists because of the vast web of consistency checks given by duality. (However, because M-theory itself is such an obscure theory, we will still refer to the theory by the name superstring theory.)

In addition to providing the student with a firm foundation in path integrals and field theory, the other purpose of this book is to introduce students to the latest developments in superstring theory, that is, to acquaint them with the fast-paced areas that are currently the most active in theoretical research, such as:

String field theory;
Conformal field theory;
Kac–Moody algebras;
Multiloop amplitudes and Teichmüller spaces;
Calabi–Yau phenomenology; and
Orbifolds and four-dimensional superstrings.

The goal of this book is to provide students with an overview by which to evaluate the research areas of string theory and perhaps even engage in original research. The only prerequisite for this book is a familiarity with advanced quantum mechanics. However, the mathematics of superstring theory has soared to dizzying heights. In order to provide an introduction to more advanced mathematical concepts, such as Lie groups, general relativity, supersymmetry, and supergravity, we have included a short introduction to them in the Appendix, which we hope will fill the gaps that may exist in the students' preparation.

For the student, we should mention how to approach this book. Chapters 1–5 represent Part I, the results of first quantization. They form an essential foundation for the next chapters and cannot be skipped. Chapter 1, however, may be skipped by one who is relatively fluent in the methods of ordinary quantum field theory, such as gauge invariance and Faddeev–Popov quantization. (But we emphasize that the method of path integrals forms the foundation for this book, and hence even an advanced student may profit from reviewing Chapter 1.)

Chapters 2 and 3 form the heart of an elementary introduction to string theory. Chapter 4, however, can be omitted by one who only wants an overview of string theory. With the exception of the fermion vertex function and ghosts, most of the results of string theory can be developed using Chapters 2 and 3 without conformal field theory, and hence a beginner may overlook this chapter. (However, we emphasize that most modern approaches to first quantized string theory use the results of conformal field theory because it is the most versatile. A serious student of string theory, therefore, should be thoroughly familiar with the results of Chapter 4.)

Chapter 5 is essential to understand the miraculous cancellation of divergences of the theory, which separates string theory from all other field theories. Because the theory of automorphic functions gets increasingly difficult as one describes multiloop amplitudes, the beginner may skip the discussion of higher loops. The serious student, though, will find that multiloop amplitudes form an area of active research.

Part II begins a discussion of the field theory of strings, and Part III examines phenomenology. The order of these two parts can be interchanged without difficulty. Each part was written to be relatively independent of the other, so the more phenomenologically inclined student may skip directly to Part III without suffering any loss.

Chapters 6 and 7 in Part II present the evolution of several approaches to string theory. Chapter 6 discusses the original light cone theory and how to quantize multiloop theories based on strings. However, Chapter 7 was written in a relatively self-contained fashion, so the serious student may skip Chapter 6 and delve directly into the covariant theory.

In Part III, the beginner may skip Chapter 8. The discussion of anomalies is rather technical and mainly based on point particles, and overlaps the discussions found in other books. Chapter 9 cannot be omitted, as it represents one of the most promising of the various superstring theories. Likewise, Chapter 10 forms an essential part of our understanding of how the superstring theory may eventually make contact with experimental data.

In Part IV, we finally have a presentation of duality and M-theory. In Chapter 11, we have a basic presentation of M-theory, and how the five known superstings can all be expressed in terms of a single theory. In Chapter 12, we explore the duality relations that exist in $D = 8$, 6 and 4 dimensions, giving us, for the first time, a look at the nonperturbative region of string theory. In Chapter 13, we explore more advanced topics, such as D-brane physics, matrix models, and applications of D-branes to black hole physics.

The author hopes that this will help both the beginner and the more advanced student to decide how to approach this book.

Michio Kaku

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Part I

First Quantization and Path Integrals