MATHEMATICS FOR ECONOMICS AND FINANCE

METHODS AND MODELLING

经济数学和金融数学

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Mathematics for economics and finance

Methods and modelling

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For Colleen and my parents, M.A.

For Christine and Juliet, N.B.

Preface

This book is an introduction to calculus and linear algebra for students of disciplines such as economics, finance, business, management, and accounting. It is intended for readers who may have already encountered some differential calculus, and it will also be appropriate for those with less experience, possibly used in conjunction with one of the many more elementary texts on basic mathematics.

Parts of this book arise from a lecture course given by the authors to students of economics, management, accounting and finance, and management sciences at the London School of Economics.

We thank Duncan Anthony, Reza Arabsheibani, Juliet Biggs and, particularly, Graham Brightwell for their invaluable comments on various drafts of the book. The final draft was read by Dr Stephen Siklos of Cambridge University, and his pertinent comments resulted in a number of improvements. We are also grateful to Roger Astley of Cambridge University Press for his efficient handling of the project, and to Alison Adcock for her help in preparing the manuscript.

London, October 1995.

Contents

1. Mathematical models in economics

1.1 Introduction

In this book we use the language of mathematics to describe situations which occur in economics. The motivation for doing this is that mathematical arguments are logical and exact, and they enable us to work out in precise detail the consequences of economic hypotheses. For this reason, mathematical modelling has become an indispensable tool in economics, finance, business and management. It is not always simple to use mathematics, but its language and its techniques enable us to frame and solve problems that cannot be attacked effectively in other ways. Furthermore, mathematics leads not only to numerical (or *quantitative*) results but, as we shall see, to *qualitative* results as well.

1.2 A model of the market

One of the simplest and most useful models is the description of *supply and demand* in the market for a single good. This model is concerned with the relationships between two things: the *price* per unit of the good (usually denoted by p), and the *quantity* of it on the market (usually denoted by q). The 'mathematical model' of the situation is based on the simple idea of representing a pair of numbers as a point in a diagram, by means of coordinates with respect to a pair of axes. In economics it is customary to take the horizontal axis as the q-axis, and the vertical axis as the p-axis. Thus, for example, the point with coordinates $(2000, 7)$ represents the situation when 2000 units are available at a price of \$7 per unit.

How do we describe *demand* in such a diagram? The idea is to look at those pairs (q, p) which are related in the following way: if p were the selling price, q would be the demand, that is the quantity which would be sold to consumers at that price. If we fill in on a diagram all the pairs (q, p) related in this way, we get something like Figure 1.1.

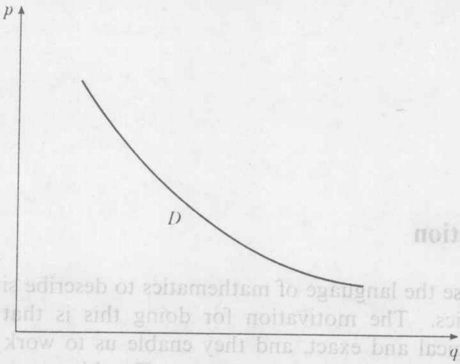

Figure 1.1: The demand set

We shall refer to this as the *demand set D* for the particular good. In economics you will learn reasons why it ought to look rather like it does in our diagram, a smooth, downward sloping curve.

Suppose the demand set D contains the point $(30, 5)$. This means that when the price $p = 5$ is given, then the corresponding demand will be for $q = 30$ units. In general, provided D has the 'right' shape, as in Figure 1.1, then for each value of p there will be a uniquely determined value of q. In this situation we say that D determines a *demand function*, q^D. The value written $q^D(p)$ is the quantity which would be sold if the price were p, so that $q^D(5) = 30$, for example.

Example Suppose the demand set D consists of the points (q, p) on the straight line $6q + 8p = 125$. Then for a given value of p we can determine the corresponding q; we simply rearrange the equation of the line in the form $q = (125 - 8p)/6$. So here the demand function is

$$q^D(p) = \frac{125 - 8p}{6}.$$

For any given value of p we find the corresponding q by substituting in this formula. For example, if $p = 4$ we get

$$q = q^D(4) = (125 - 8 \times 4)/6 = 93/6.$$

□

There is another way of looking at the relationship between q and p. If we suppose that the quantity q is given, then the value of p for which (q, p) is in the demand set D is the price that consumers would be prepared to pay if q is the quantity available. From this viewpoint we are expressing p in terms of q, instead of the other way round. We write $p^D(q)$ for the value of p corresponding to a given q, and we call p^D the *inverse demand function*.

Example (*continued*) Taking the same set D as before, we can now rearrange the equation of the line in the form $p = (125 - 6q)/8$. So the inverse demand function is

$$p^D(q) = \frac{125 - 6q}{8}.$$

□

Next we turn to the supply side. We assume that there is a *supply set* S consisting of those pairs (q, p) for which q would be the amount supplied to the market if the price were p. There are good economic reasons for supposing that S has the general form shown in Figure 1.2.

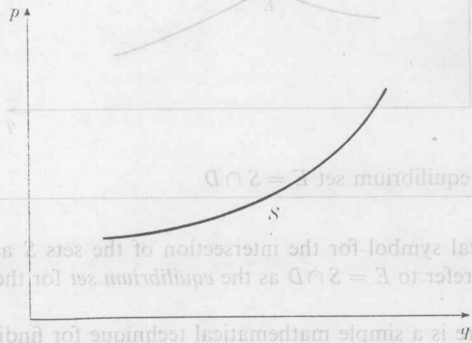

Figure 1.2: The supply set

If we know the supply set S we can construct the *supply function* q^S and the *inverse supply function* p^S in the same way as we did for the demand function and its inverse. For example, if S is the set of points on the line $2q - 5p = -12$, then solving the equation for q and for p we get

$$q^S(p) = \frac{5p - 12}{2}, \quad p^S(q) = \frac{2q + 12}{5}.$$