Scaling and Renormalization in Statistical Physics

统计物理学中的定标和重正化

CAMBRIDGE LECTURE NOTES IN PHYSICS

5

JOHN CARDY

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To the memory of my father

This book provides an introduction to the concepts which underlie the modern understanding of the behaviour of complicated physical systems which exhibit the property of scale invariance or self-similarity. This is most clearly illustrated in materials, such as magnets or fluids, in the vicinity of a second order phase transition. The theoretical framework for understanding these phenomena, known as the renormalization group, first arose in the late 1960s and has evolved into a common language used by workers in such diverse fields as particle physics, cosmology, neural networks and biophysics, as well as the more conventional aspects of condensed matter physics.

Beginning with a brief review of phase transitions in simple systems and of mean field theory, the text then goes on to introduce the core ideas of the renormalization group. Following chapters cover phase diagrams, fixed points, cross-over behaviour, finite-size scaling, perturbative renormalization methods, low-dimensional systems, surface critical behaviour, random systems, percolation, polymer statistics, critical dynamics and conformal symmetry. The book closes with an appendix on Gaussian integration, a selected bibliography, and a detailed index. Many problems are included.

The emphasis throughout is on providing an elementary and intuitive approach. In particular, the perturbative method introduced leads, among other applications, to a simple derivation of the epsilon expansion in which all the actual calculations (at least to lowest order) reduce to simple counting, avoiding the need for Feynman diagrams.

Preface

Scaling concepts play a central role in the analysis of the ever more complex systems which nowadays are the focus of much attention in the physical sciences. Whether these problems relate to the very large scale structure of the universe, to the complicated forms of everyday macroscopic objects, or to the behaviour of the interactions between the fundamental constituents of matter at very short distances, they all have the common feature of possessing large numbers of degrees of freedom which interact with each other in a complicated and highly non-linear fashion, often according to laws which are only poorly understood. Yet it is often possible to make progress in understanding such problems by isolating a few relevant variables which characterise the behaviour of these systems on a particular length or time scale, and postulating simple *scaling relations* between them. These may serve to unify sets of experimental and numerical data taken under widely differing conditions, a phenomenon called *universality.* When there is only a single independent variable, these relations often take the form of power laws, with exponents which do not appear to be simple rational numbers yet are, once again, universal.

The existence of such scaling behaviour may often be explained through a framework of theoretical ideas loosely grouped under the term *renormalization.* Roughly speaking, this describes how the parameters specifying the system must be adjusted, under putative changes of the underlying dynamics, in such a way as not to modify the measurable properties on the length or time scales of interest. The simple postulate of the existence of a fixed point of these renormalization flows is then sufficient to explain qualitatively the appearance of universal scaling laws. Unfortunately, for most examples of complex systems, such a renormalization approach has not, as yet, been put on a systematic basis starting from the underlying microscopic dynamics. In trying to understand scaling arguments applied to such problems it is often difficult, especially for newcomers, to understand why certain variables should be neglected while others are retained in such scaling descriptions, and why in some cases power law relations should hold while they fail in others.

Fortunately, there is a class of physical problems within which the concepts of scaling and renormalization may be derived systematically, and which therefore have become a paradigm for the whole approach. These concern equilibrium critical behaviour. The systems which exhibit such behaviour are governed by the simple and well understood laws of statistical mechanics. Indeed, along with the high energy behaviour of quantum field theories, this was the area of physics in which the concepts of renormalization were first formulated. Although the subject of equilibrium critical behaviour is, apart from a few unsolved problems, no longer of the greatest topical theoretical or experimental interest, its study is nonetheless important in providing a solid grounding to anyone who wishes to go on to attempt to understand scaling and renormalization in more esoteric systems. Yet the typical student in condensed matter theory faces a problem in trying to accomplish this. Historically, the subjects of renormalization in quantum field theory (as applied to particle physics) and in equilibrium critical behaviour have developed in parallel. This is no coincidence – the two sets of problems have, mathematically, a great deal in common, and, indeed, the most systematic formulation of the subject relies heavily on the property of renormalizability in quantum field theory. However, much of the qualitative structure of renormalization may be introduced through the alternative real space methods which are both simple and appealing. But students who learn this approach, and then wish to go further in existing accounts of the subject, must make a complete change of gears to momentum space methods which require a great deal of investment of time and effort in digesting the whole formalism of Feynman diagrams and renormalization theory. As a result, the study of the subject rapidly becomes overladen with formalism, and the student, if he or she is lucky, has just about time to learn how to calculate the critical exponents of the Ising model in $4-\epsilon$ dimensions before the course comes to an end. The average student thereby often misses out on any account of the tremendously wide range of problems, even within critical behaviour, on which these methods may be brought to bear.

In my opinion, these field theoretic details are appropriate only for the relatively small number of students who wish to go on and apply these methods to particle physics, or for those who really need to compute critical exponents to $O(\epsilon^2)$ and higher. For the majority, whose goal is to understand how scaling and renormalization ideas might be applied to the rich variety of complex phenomena apparent in many other branches of the physical sciences, the main object is to learn the concepts, and the best way to do this is by covering as many examples as possible. This small book was written with this goal in mind. It is, in fact, based on a set of lectures which were given, in various incarnations, to physics graduate students at Santa Barbara and Oxford. A significant fraction of the audience consisted of students planning to do experimental rather than theoretical research.

I have assumed that the reader has already had a basic course in statistical mechanics, and, indeed, has had some exposure to critical phenomena, a subject which is, nowadays, often discussed in such courses. However, for completeness, the basic phenomena and some simple models are recalled in the first chapter. Next comes a discussion of the 'classical' approach to critical behaviour through mean field theory, before the renormalization group idea is introduced. As mentioned above, the simplest conceptual route to renormalization concepts is through real space methods, and I have chosen this approach. At this level, all of the qualitative properties of scaling and universality may then be discussed.

However, it is also important that the student understand how quantitative methods, such as the ϵ-expansion, come about. In this book I describe an approach, which is certainly not new but deserves to become better known, by which at least the first order perturbative renormalization group equations may be derived from a simple continuum real space approach, thus linking up directly with the earlier more intuitive considerations. It relies on the operator product expansion of field theory, an impressive sounding name for something which is basically very intuitive and simple, and easy to calculate with in lowest order. As a result, the $O(\epsilon)$ results for the critical exponents emerge as a consequence of elementary combinatorics, with no Feynman diagrams required at all! This approach also lays stress on the modern idea of scaling 'operators' and their associated scaling dimensions as being the central objects of attention, rather than derived quantities like the traditionally defined critical exponents.

Although I have deliberately tried to avoid couching the discussion in the language of quantum field theory, a few of its elementary results, particularly those of Gaussian integration and the combinatorial version of Wick's theorem, are nonetheless required. The details of these are summarised in a brief Appendix, for readers unfamiliar with these simple formulae.

After this, the book embarks on a tour of many of the important applications of renormalization group to critical phenomena. After a few simple generalisations of the Ising magnet in $4-\epsilon$ dimensions, we descend to the neighbourhood of two dimensions and show how the same perturbative renormalization group methods which gave us the ϵ-expansion may be applied to the famous examples of the XY model and other systems with continuous symmetries. Then come accounts of the application of similar methods to critical behaviour near surfaces, to systems with quenched random impurities, and to the configurational statistics of large polymers in solution. These are all problems in equilibrium critical behaviour, but the next chapter brings in the dynamics. This is a tremendously rich subject, indeed one which deserves a whole book in itself at this level, and it is therefore impossible to do it justice in a single chapter. However, I have tried to include a number of examples apart from the standard ones, including in particular directed percolation, an example from the rapidly expanding subject of dynamic critical behaviour in systems far from equilibrium. Finally, the tour ends with an elementary account of some of the recent developments in the application of the ideas of conformal symmetry to equilibrium critical behaviour. The non-mathematical reader may find this section slightly harder going than the earlier chapters, although all that is in fact required is a basic knowledge of tensor calculus and complex analysis.

Unfortunately, many important examples of scaling in statistical physics have been omitted in this survey, due to reasons of lack of space and/or expertise on the part of the author. In particular, I would have liked to have spent time on the problems of fluctuating interfaces and of spin glasses, where the concepts of scaling are amply illustrated, but these subjects are too complicated for inclusion in a single course. Similarly, the modern approach to localisation of waves and electrons in random systems is replete with scaling arguments, but it too requires too extensive an introduction. The dynamics of phase ordering following a rapid temperature quench is another fascinating related subject where scaling arguments play a central role, but for which, as yet, no systematic renormalization approach has been formulated.

A more profound apology is required for the lack of any detailed reference to comparison with experimental data. I hope that this does not create the wrong impression. The subject of critical phenomena is one which is, ultimately, driven by observation and experiment, and it is important that all theorists continue to bear this in mind. However, the basic experiments which established the phenomena of scaling and universality in critical behaviour were performed some time ago, and their results are by now adequately summarised in a number of standard references. It is not the purpose of this book to make detailed comparison with experimental results on particular systems, but rather to emphasise the generality of the principles involved. In this sense the current status of the theory is akin to that of quantum mechanics, where, in similarly introductory texts, it is considered adequate nowadays to illustrate the theoretical principles with applications to simple and rather idealised systems, rather than by comparison with detailed experimental data.

Since this is an introductory account, I have not included bibliographic references in the text. Rather, I have provided a list of selected sources of further reading at the end. I have also included a number of exercises, the aim of which is to lead the inquisitive reader into further examples and extensions of the ideas discussed in the text.

I thank my graduate students and colleagues at Santa Barbara and Oxford who have helped me formulate the material of this book over the years. I am particularly grateful to Benjamin Lee for a careful reading of the manuscript, and to Reinhard Noack for helping produce the Ising model pictures in Chapter 3.

Contents

1
Phase transitions in simple systems

Take a large piece of material and measure some of its macroscopic properties, for example its density, compressibility or magnetisation. Now divide it into two roughly equal halves, keeping the external variables like pressure and temperature the same. The macroscopic properties of each piece will then be the same as those of the whole. The same holds true if the process is repeated. But eventually, after many iterations, something different must happen, because we know that matter is made up of atoms and molecules whose individual properties are quite different from those of the matter which they constitute. The length scale at which the overall properties of the pieces begin to differ markedly from those of the original gives a measure of what is termed the *correlation length* of the material. It is the distance over which the fluctuations of the microscopic degrees of freedom (the positions of the atoms and suchlike) are significantly correlated with each other. The fluctuations in two parts of the material much further apart than the correlation length are effectively disconnected from each other. Therefore it makes no appreciable difference to the macroscopic properties if the connection is completely severed.

Usually the correlation length is of the order of a few interatomic spacings. This means that we may consider really quite small collections of atoms to get a very good idea of the macroscopic behaviour of the material. (This statement needs qualification. In reality, small clusters of atoms will exhibit very strong surface effects which may be quite different from, and dominate, the bulk behaviour. However, since this is only a thought experiment, we may imagine employing the theoretician's device of periodic boundary conditions, thereby eliminating them.) However, the actual value of the correlation length depends on the external conditions determining the state of the system, such as the temp-