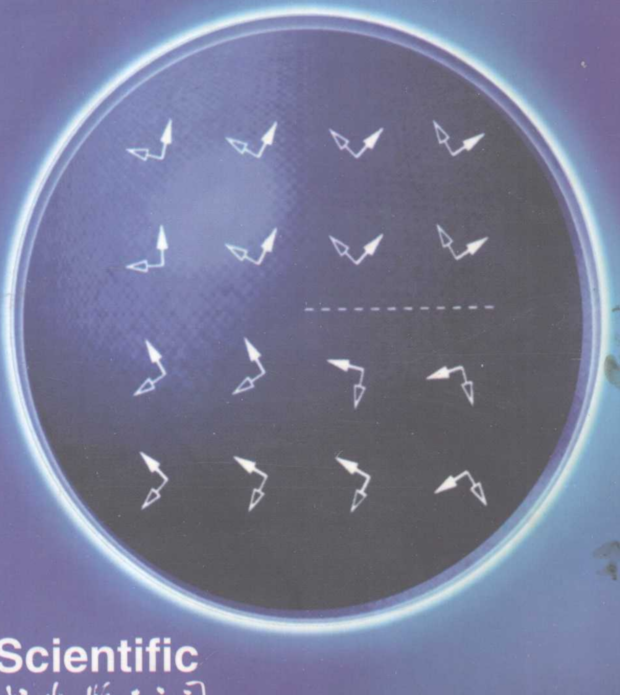
David J. Thouless

Topological Quantum Numbers in Nonrelativistic Physics

非相对论物理学中的拓扑量子数



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Topological Quantum Numbers in Nonrelativistic Physics

Preface

Topological quantum numbers crept up on the physics community before the community was aware of them. I did not think in these terms until I started working on the topological aspects of long range order in the early 1970s, although I had been working on aspects of superfluidity that are now regarded as topological for several years before that. I should have known earlier of the importance of topology, as I was then a colleague of Tony Skyrme, whose pioneering work on topological quantum numbers is now so well known. It was around this time that there began to be a wide awarenesss of the importance of topology both amongst elementary particle theorists and field theorists, and amongst people who worked on superfluids and liquid crystals. The issue was brought sharply into focus for me in 1980, when Hans Dehmelt asked me about how the quantum Hall effect could possibly be used to determine the fine-structure constant when so little was known about the details of the devices used and so little understood about the theory.

Dehmelt's question is one of the unifying themes of this book, particularly in Chapters 2 to 5 and in Chapter 7. The answer is not entirely simple, since, although topological quantum numbers can provide a correspondence between countable integer quantities and physical observables, this correspondence is not usually exact, and corrections may be more or less important.

A second theme, provided by the work on liquid crystals, and on the A phase of superfluid ^{3}He, is the use of topological quantum numbers to classify defects, in situations where the relevant group is finite, rather than isomorphic to the infinite group of integers.

The third theme, covered in the last chapter, is the importance of topological concepts in the theory of phase transitions in two dimensions.

I have tried in this book to give enough background material to make it accessible to people whose knowledge of quantum mechanics and statistical mechanics is at the level expected in the second year of a U.S. graduate program in physics. For Chapters 6 and 8 a little knowledge of the theory of finite groups is also necessary. I have not assumed any previous knowledge of topology.

This book developed from a series of lectures given at the University of Washington in the winter of 1994, and I am particularly grateful to the people who attended those lectures and provided lively discussion of the material presented. Among the people who have particularly helped to sharpen my views of this material are my colleagues and former colleagues Ian Aitchison, Ping Ao, Michael Geller, Qian Niu and Boris Spivak, and my students Junghoon Han, Kiril Tsemekhman, Vadim Tsemekhman and Carlos Wexler. Carlos Wexler and Kiril Tsemekhman also helped by turning many of my figures into postscript files, and all of my students have provided me with instruction in the intelligent use of Latex.

I supplemented my lectures by providing copies of classic papers on the subject, and I have done the same in this book. There is a selection of relevant papers, some very old and a few quite new, in the second part of this book. I am grateful to the publishers and authors who have given permission for the reprinting of these papers, and particularly grateful to those authors who have supplied the Publisher with reprints of their papers, which reproduce much better than photocopies of bound periodicals. I am also grateful to members of the Theory of Condensed Matter group at the Cavendish Laboratory for allowing me to copy from their extensive collection of unbound periodicals.

The book has been written slowly because I have been much involved in other things during the past four years. Some of these things, concerned with the properties of quantized vortices, have made their appearance in Chapter 3. I am grateful for the help that I have had from my colleagues at the University of Washington, and for the hospitality I have received from those institutions to which I have escaped from my normal responsibilities, the Institute for Theoretical Physics at Santa Barbara, the Aspen Center for Physics, and the Isaac Newton Institute at Cambridge. The National Science Foundation has encouraged this activity by financial support provided through grant number DMR-9528345. Finally I wish to thank my wife Margaret for her support throughout the years, and for tolerating my absences listed above.

David Thouless
December 1997

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Reprinted Papers

1 Introduction

1.1 Whole numbers in physics

The relevance of integer quantities in physics has been known for at least 25 centuries, and goes back to the work of Pythagoras on the normal modes of a stretched string and the theory of musical harmony, and to the ideas of Democritus on the atomic constitution of matter. In many ways a physics based on integers and rational numbers is more natural than a continuum theory, and it was a shock to Greek mathematicians to discover that the ratio of the diagonal of a square to its side could not be expressed as a ratio of integers.

In modern physics the initial developments were in terms of continuum theories, although discrete aspects of physics were always in people's minds — Kepler's attempt to relate radii of planetary orbits to the regular solids, and Newton's corpuscular theory of light and theory of surface tension are examples. Integers really came into their own again at the beginning of the nineteenth century, with the work of Dalton and of Avogadro on chemical proportions, naturally interpreted in terms of atoms, and with Faraday's establishment of an analogous quantization of electric charge. There was a hundred years of rearguard resistance to atomic theory, which persisted until the beginning of the twentieth century. The measurements within a few years of one another of the Boltzmann constant, of the electron charge, of single events in radioactive decay, and of the atomic spacing in crystals put a stop to the idea that laws of proportion might exist in the absence of a fundamental atomic scale.

The idea that fundamental quantized entities might be related to one another through a continuum theory never really died, and the first successful theory of this sort was derived by Dirac [1], in the paper reprinted here as Reprint 2.1. This paper was the ancestor of all later work on topological quantum numbers, and showed that magnetic monopoles can be allowed in quantum theory only if their charge is related to the electric charge of the electron and of the proton in a definite way. Later work by Heisenberg, Skyrme, Witten and many others tried to build up quantized particles as topological singularities of a field. That is not what this book is primarily about, but it is rather concerned with aspects of nonrelativistic physics that display topological quantization. Some topological quantum numbers turn out to be very important in high precision measurements, while others, while useful for classification, are much less directly related to readily measurable quantities.

High precision work generally depends on two ingredients. These are reproducibility, and the reduction of a measurement to a counting procedure. A ruler is a device for comparing a length with the number of marks along the ruler, and a vernier allows interpolation between marks on the main scale also to be done by counting. A pendulum clock and its successors are devices for comparing a time interval with the number of ticks that occur in the interval. Such devices are not completely reproducible, and may vary when conditions change. The earth's rotational and orbital motion provide time standards that can be used for calibration, but they are difficult to measure with very high precision, and we know that the rotational motion is subject to random as well as to systematic changes. Cesium atoms and ammonia molecules are reproducible, and they can form the basis for length measurements in which interference fringes are counted, or as time standards by driving the system in resonance with with a standard atomic or molecular transition and counting beats against some uncalibrated frequency. The measurement of g for the electron to parts in 10^{11} is achieved, in part, by measuring the frequency difference between the spin and twice the orbital resonance frequency in a magnetic field [2].

Counting can be made very precise because, although any particular counter may make mistakes, comparison between the outputs of several independent counters can reduce the error rate to an extremely low value.

Over the past 30 years several devices have been developed for use in high precision work where the devices themselves are manifestly not reproducible, but nevertheless give fantastically reproducible results. Among such devices are the SQUID magnetometer, which compares magnetic flux in a superconducting ring with the quantum of flux $h/2e$ for a superconductor, the Josephson voltmeter, which compares the frequency of a microwave device with the frequency $2eV/h$ generated by a voltage V across a superconducting weak link, and a quantum Hall conductance standard, which compares electrical conductance with the quanta e^2/h for conductance in a quantum Hall device. In none of these cases does the fabrication of the device have to be very tightly controlled, but there are good theoretical reasons and very strong experiments to show that different devices, even those made of different materials, give measurements that are essentially identical. This insensitivity to details is a characteristic of topological quantum numbers that is one of the themes of this book.

1.2 Quantum numbers due to symmetry and topological quantum numbers

The quantum numbers we know from elementary quantum mechanics are, for the most part, related to symmetry operations. The classic case of such a set of numbers is provided by the theory of angular momentum, which is based on rotation invariance. The usual development of the theory uses the Lie algebra of the three generators of the rotation group,

$$[\hat{J}_i, \hat{J}_j] = -i\hbar \sum_{k=1}^{3} \epsilon_{ijk} \hat{J}_k \ , \tag{1.1}$$

where ϵ_{ijk} is zero if two of the indices i, j, k are equal, is $+1$ if they are an even permutation of $1, 2, 3$, and -1 if they are an odd permutation. The textbooks show that the Casimir operator

$$\hat{J}^2 = \sum_i \hat{J}_i^2 \tag{1.2}$$

commutes with each of the components $\hat{J}_i$, and that the operators $\hat{J}_1 \pm i\hat{J}_2$ are raising and lowering operators for the eigenvalues of $\hat{J}_3$. Taken together with the boundedness of $\hat{J}^2$ and the positivity of $\hat{J}_i^2$ for each value of i this is enough to show that $\hat{J}^2$ has eigenvalues $\hbar^2 J(J+1)$, where J is an integer or half integer, while $\hat{J}_3$ has a simultaneous eigenvalue $\hbar M_J$, where M_J is an integer or half integer in the range $-J \leq M_J \leq J$.

This structure is closely related to the existence of the rotational symmetry of the Hamiltonian, and once there are terms in the Hamiltonian that violate this symmetry the stationary states of the system lose this structure. Measurements of the angular momentum components still yield integer values, but stationary states are characterized by continuously variable expectation values of the components if the symmetry is broken.

Similar algebraic structure comes up in many other systems. The $SU(2)$ symmetry of a two-dimensional harmonic oscillator or the $SU(3)$ symmetry of a three-dimensional oscillator are examples, and this symmetry is broken by the anharmonicity of the potential. The same feature of a potential inversely proportional to distance that leads to closed orbits in classical mechanics gives the degeneracy of the hydrogen atomic spectrum that is described by the four-dimensional rotation group $O(4)$; again, deviations of the potential from this form lead to breaking of this symmetry, such as one sees in the spectra of the alkali metal atoms. The isospin symmetry of strong interactions, postulated in 1936 by Cassen and Condon [3], is another famous example, and this is broken by the electromagnetic interactions of the baryons and mesons.

Topological quantum numbers can behave very differently, preserving their identity under relatively strong perturbations. For example, as is discussed in more detail in Chapter 3, the circulation of superfluid ^{4}He around a wire is quantized in multiples of h/m. Although this circulation can be related, under conditions of ideal axial symmetry about the wire, to the average angular momentum per atom in the superfluid, this is quite irrelevant. Quantized circulation in this case is related to the winding number of a condensate wave function, the change of phase of the wave function along a circuit around the wire, and is quite insensitive to the geometrical details of the wire and the enclosure. The angular momentum per atom, by contrast, is rather sensitive to the detailed geometry of the system. The story about flux quantization in superconducting rings, discussed in Chapter 4, is very similar, with the number of flux quanta again equal to the winding number of a condensate wave function. There are no special requirements on the geometrical or material properties of the ring, just that it should not be grossly inhomogeneous or irregular in shape. In this case it is thought that the measured flux differs from the ideal quantized value by an amount that can be made exponentially small by a suitable choice of parameters, and experimental measurements confirm that different devices made with different materials display the same quantum of flux with very high accuracy.

Not all topological quantum numbers work in this way. Just because there is a mathematical quantity with well-defined integer values does not mean that there is a corresponding measurable physical quantity which has to be integer-valued. The superfluid phases of liquid ^{3}He, which are discussed in Chapter 6, illustrate this point. Whereas circulation in the B phase is quantized in much the same way as the circulation in ^{4}He, there is no quantized circulation in the A phase. There is a topological quantum number corresponding to the one which measures superfluid circulation in ^{4}He and B–^{3}He, but this quantum number has only the values 0 and 1, and does not correspond in any direct way to the circulation of the fluid. Such quantum numbers are still useful for classifying defects, and for determining whether two apparently different states of the system can actually be continuously changed from one to the other.

1.3 Topics covered in this book

In the remainder of this chapter some important concepts are introduced and discussed. In Sec. 1.4 the concept of an order parameter in a macroscopic system is discussed. This concept has been of great importance in the theory of phase transitions since the early work of Landau[4], and it underlies all the discussion of topological quantum numbers in macroscopic materials in this book. Closely related to