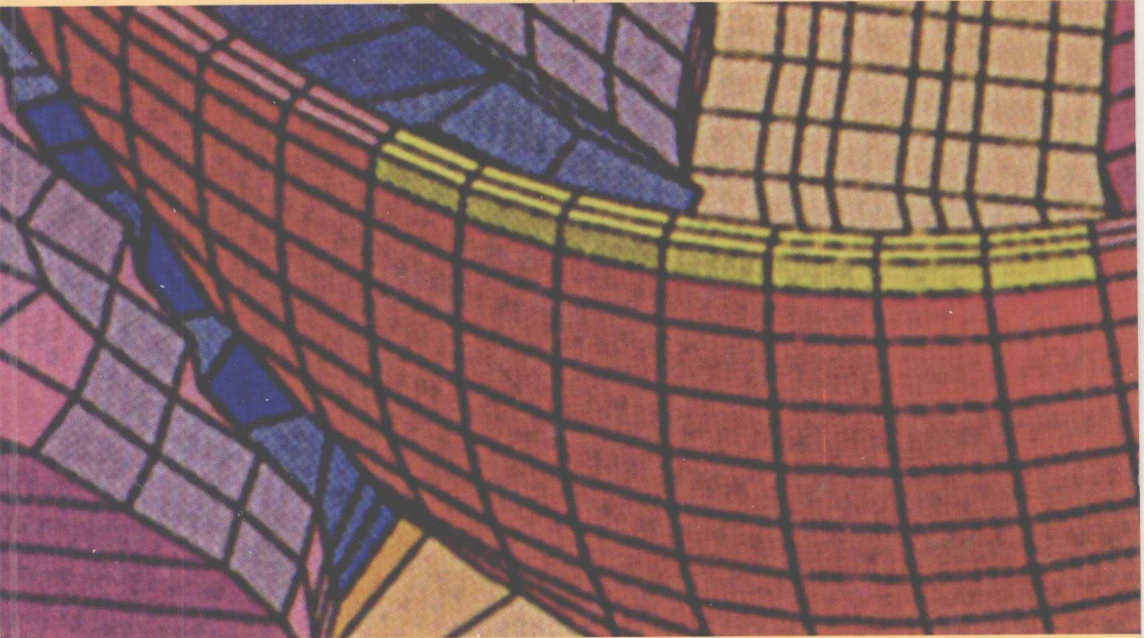


有限元方法
固体力学和结构力学

第6版

THE FINITE ELEMENT METHOD
FOR SOLID AND
STRUCTURAL MECHANICS



O.C. ZIENKIEWICZ, R.L. TAYLOR



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The Finite Element Method for Solid and Structural Mechanics

Sixth edition

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O. C. Zienkiewicz & R. L. Taylor

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Dedication

This book is dedicated to our wives Helen and Mary Lou and our families for their support and patience during the preparation of this book, and also to all of our students and colleagues who over the years have contributed to our knowledge of the finite element method. In particular we would like to mention Professor Eugenio Oñate and his group at CIMNE for their help, encouragement and support during the preparation process.

Preface

It is thirty-eight years since the *The Finite Element Method in Structural and Continuum Mechanics* was first published. This book, which was the first dealing with the finite element method, provided the basis from which many further developments occurred. The expanding research and field of application of finite elements led to the second edition in 1971, the third in 1977, the fourth as two volumes in 1989 and 1991 and the fifth as three volumes in 2000. The size of each of these editions expanded geometrically (from 272 pages in 1967 to the fifth edition of 1482 pages). This was necessary to do justice to a rapidly expanding field of professional application and research. Even so, much filtering of the contents was necessary to keep these editions within reasonable bounds.

In the present edition we retain the three volume format of the fifth edition but have decided not to pursue having three contiguous volumes – rather we treat the whole work as an assembly of three separate works. Each one is capable of being used without the others and each one appeals perhaps to a different audience. Though naturally we recommend the use of the whole ensemble to people wishing to devote much of their time and study to the finite element method.

The first volume is renamed *The Finite Element Method: Its Basis and Fundamentals*. This volume covers the topic starting from a physical approach to solve problems in linear elasticity. The volume then presents a mathematical framework from which general problems may be formulated and solved using variational and Galerkin methods. The general topic of shape functions is also presented for situations in which the approximating functions are C^0 continuous. The two- and three-dimensional problems of linear elasticity are then presented in a unified manner using higher order shape functions. This is followed by consideration of quasi-harmonic problems governed by Laplace and Poisson differential equations. The patch test is introduced and used as a means to guarantee convergence of the method. We also cover in some depth solution forms using mixed methods with special consideration given to problems in which incompressibility can occur. The solution of transient problems is presented using semi-discrete formulations and finite element in time concepts. The volume concludes with a presentation of coupled problems.

In this volume we consider more advanced problems in solid and structural mechanics while in a third volume we consider applications in fluid dynamics. It is our intent that the present volume can be used by investigators familiar with the finite element method

at the level presented in the first volume or any other basic textbook on the subject. However, the volume has been prepared such that it can stand alone. The volume has been reorganized from the previous edition to cover consecutively two main subject areas. In the first part we consider non-linear problems in solid mechanics and in the second part linear and non-linear problems in structural mechanics.

In Chapters 1 to 9 we consider non-linear problems in solid mechanics. In these chapters the special problems of solving non-linear equation systems are addressed. We begin by restricting our attention to non-linear behaviour of materials while retaining the assumptions on small strain. This serves as a bridge to more advanced studies later in which geometric effects form large displacements and deformations are presented. Indeed, non-linear applications are of great importance today and of practical interest in most areas of engineering and physics. By starting our study first using a small strain approach we believe the reader can more easily comprehend the various aspects which need to be understood to master the subject matter. We cover in some detail formulations of material models for viscoelasticity, plasticity and viscoplasticity which should serve as a basis for applications to other material models. In our study of finite deformation problems we present a series of approaches which may be used to solve problems including extensions for treatment of constraints such as near incompressibility, rigid and multi-body motions and discrete element forms. The chapter on discrete element methods was prepared by Professor Nenad Bićanić of the University of Glasgow, UK.

In the second part of the volume we consider problems in structural mechanics. In this class of applications the dimension of the problem is reduced using basic kinematic assumptions. We begin the presentation in a new chapter that considers rod problems where two of the dimensions of the structure are small compared to the third. This class of problems is a combination of beam bending, axial extension and torsion. Again we begin from a small strain assumption and introduce alternative forms of approximation for the Euler–Bernoulli and the Timoshenko theory. In the former theory it is necessary now to use C^1 interpolation (i.e. continuous displacement and slope) to model the bending behaviour, whereas in the latter theory use of C^0 interpolation is permitted when special means are included to avoid ‘locking’ in the transverse shear response. Based upon the study of rods we then present a detailed study of problems in which only one dimension is small compared to the other two. Building on the results from rods we present a coverage for thin plates (Kirchhoff theory), thick plates (Reissner–Mindlin theory) and their corresponding forms for shells. We then consider the problem of large strains and present forms for buckling and large displacements.

The volume includes a new chapter on multi-scale effects. This is a recent area of much research and the chapter presents a summary of some notable recent results. We are indebted to Professor Bernardo Schrefler of the University of Padova, Italy, for preparing this timely contribution.

The volume concludes with a short chapter on computational methods that describes a companion computer program that can be used to solve several of the problem classes described in this volume.

We emphasize here the fact that all three of our volumes stress the importance of considering the finite element method as a unique and whole basis of approach and that it contains many of the other numerical analysis methods as special cases. Thus, imagination and knowledge should be combined by the readers in their endeavours.

The authors are particularly indebted to the International Centre of Numerical Methods in Engineering (CIMNE) in Barcelona who have allowed their pre- and post-processing code (GiD) to be accessed from the web site. This allows such difficult tasks as mesh generation and graphic output to be dealt with efficiently. The authors are also grateful to Professor Eric Kasper for his careful scrutiny of the entire text. We also acknowledge the assistance of Matt Salvesson who also helped in proofreading the text.

Resources to accompany this book

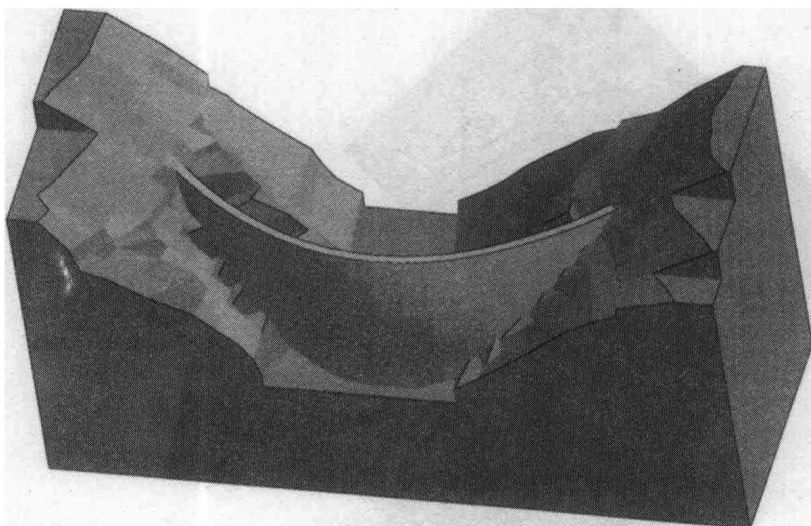
Complete source code and user manual for program *FEAPpv* may be obtained at no cost from the publisher's web page: <http://books.elsevier.com/companions/> or from the author's web page: <http://www.ce.berkeley.edu/~rlt>

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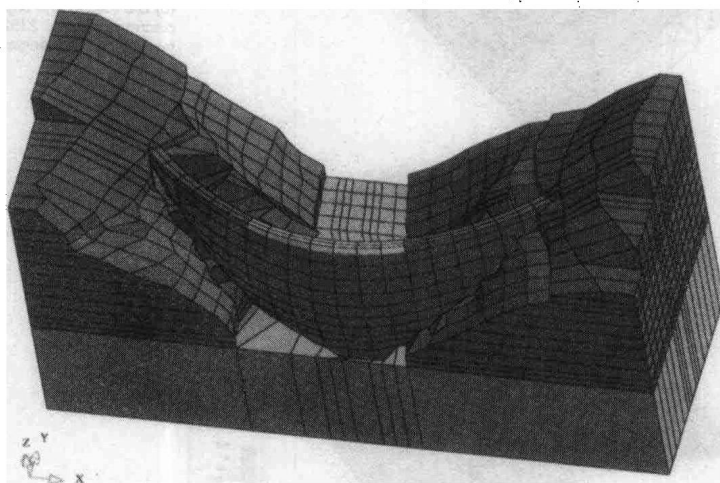
**The Finite Element Method for
Solid and Structural Mechanics
Sixth edition**

Professor O.C. Zienkiewicz, CBE, FRS, FREng is Professor Emeritus at the Civil and Computational Engineering Centre, University of Wales Swansea and previously Director of the Institute for Numerical Methods in Engineering at the University of Wales Swansea, UK. He holds the UNESCO Chair of Numerical Methods in Engineering at the Technical University of Catalunya, Barcelona, Spain. He was the head of the Civil Engineering Department at the University of Wales Swansea between 1961 and 1989. He established that department as one of the primary centres of finite element research. In 1968 he became the Founder Editor of the *International Journal for Numerical Methods in Engineering* which still remains today the major journal in this field. The recipient of 27 honorary degrees and many medals, Professor Zienkiewicz is also a member of five academies – an honour he has received for his many contributions to the fundamental developments of the finite element method. In 1978, he became a Fellow of the Royal Society and the Royal Academy of Engineering. This was followed by his election as a foreign member to the US Academy of Engineering (1981), the Polish Academy of Science (1985), the Chinese Academy of Sciences (1998), and the National Academy of Science, Italy (Accademia dei Lincei) (1999). He published the first edition of this book in 1967 and it remained the only book on the subject until 1971.

Professor R.L. Taylor has more than 40 years' experience in the modelling and simulation of structures and solid continua including two years in industry. He is Professor in the Graduate School and the Emeritus T.Y. and Margaret Lin Professor of Engineering at the University of California at Berkeley. In 1991 he was elected to membership in the US National Academy of Engineering in recognition of his educational and research contributions to the field of computational mechanics. Professor Taylor is a Fellow of the US Association of Computational Mechanics – USACM (1996) and a Fellow of the International Association of Computational Mechanics – IACM (1998). He has received numerous awards including the Berkeley Citation, the highest honour awarded by the University of California at Berkeley, the USACM John von Neumann Medal, the IACM Gauss–Newton Congress Medal and a Dr.-Ingenieur ehrenhalber awarded by the Technical University of Hannover, Germany. Professor Taylor has written several computer programs for finite element analysis of structural and non-structural systems, one of which, *FEAP*, is used world-wide in education and research environments. A personal version, *FEAPPv*, available from the publisher's website, is incorporated into the book.



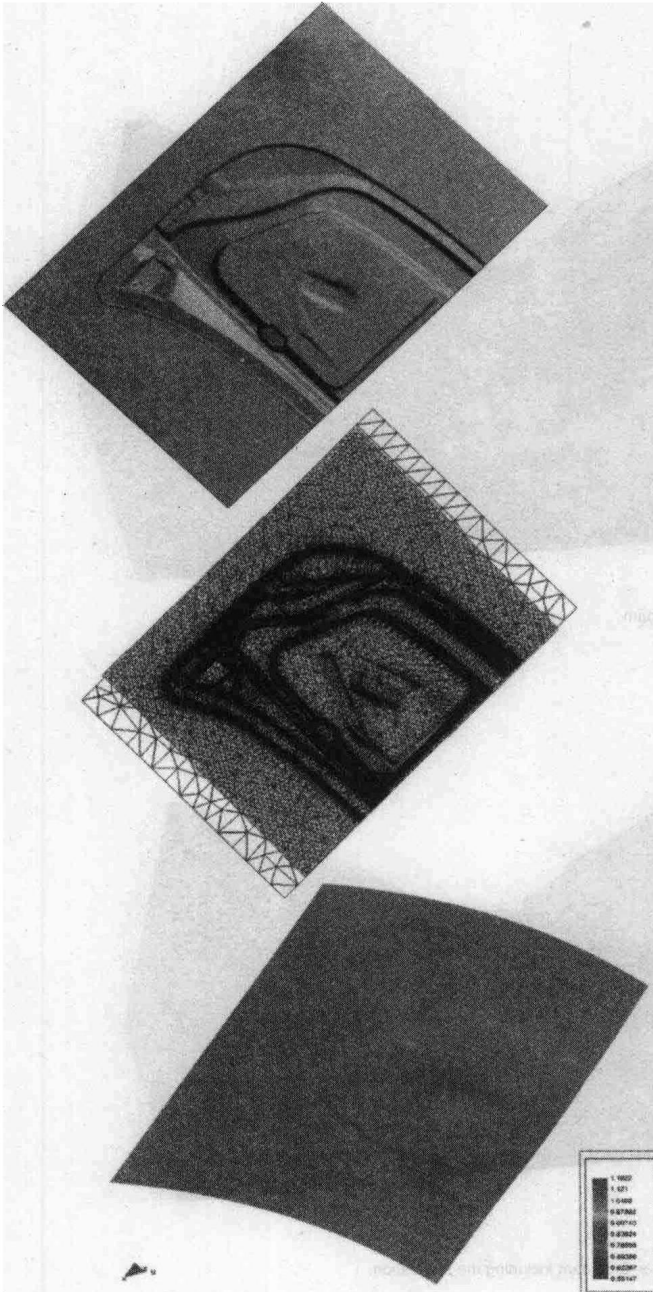
(a) Geometrical model of an arch dam



(b) Finite element discretization of an arch dam including the foundation

Plate 1 Three dimensional non-linear analysis of a dam

Courtesy of Prof. Miguel Cervera, CIMNE, Barcelona. Source: B. Suarez, M. Cervera and J. Miguel Canet, 'Safety assessment of the Suarna arch dam using non-linear damage model', *Proc. Int. Sym. New Trends and Guidelines on Dam Safety*, Barcelona, Spain, 1998. L. Berga (ed.) Balakema, Rotterdam.



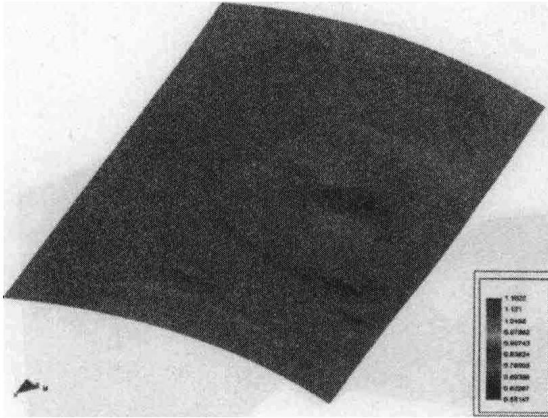
(a) Stamping die for an automotive structural component

(b) Die (black) and sheet (green) discretization into 23522 rotation free BST shell triangles

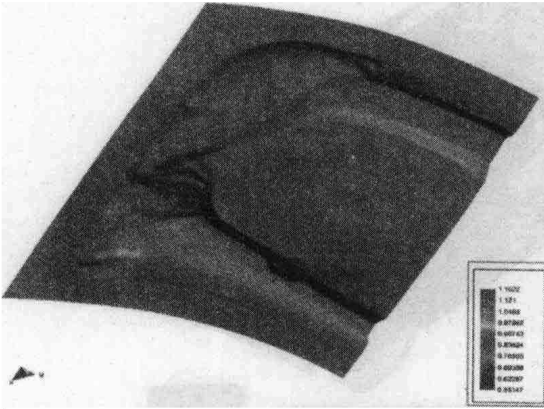
(c) Thickness ratio contours of the sheet at 17mm punch travel

Plate 2 Non-linear metal forming analysis of a car door

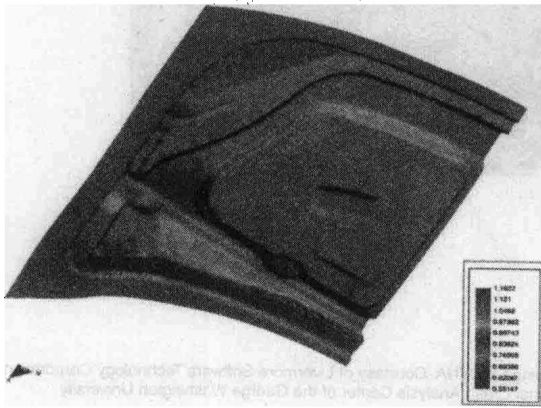
Courtesy of Prof. E. Oñate, CIMNE and DECAD S.A Barcelona. Source: E. Oñate, F. Zarate, J. Rojek, G. Duffet, L. Neamtui, 'Adventures in rotation free elements for sheet stamping analysis'. 4th Int. Conf. Workshop on Numerical Simulation of 3D Sheet Forming Processes (NUMISHEET' 99, Besancon, France, Sept 13-17, 1999)



(d) Thickness ratio contours at 35mm punch travel



(e) Thickness ratio contours at 54mm punch travel



(f) Thickness ratio contours at 72mm punch travel

Plate 2 continued Non-linear metal forming analysis of a car door

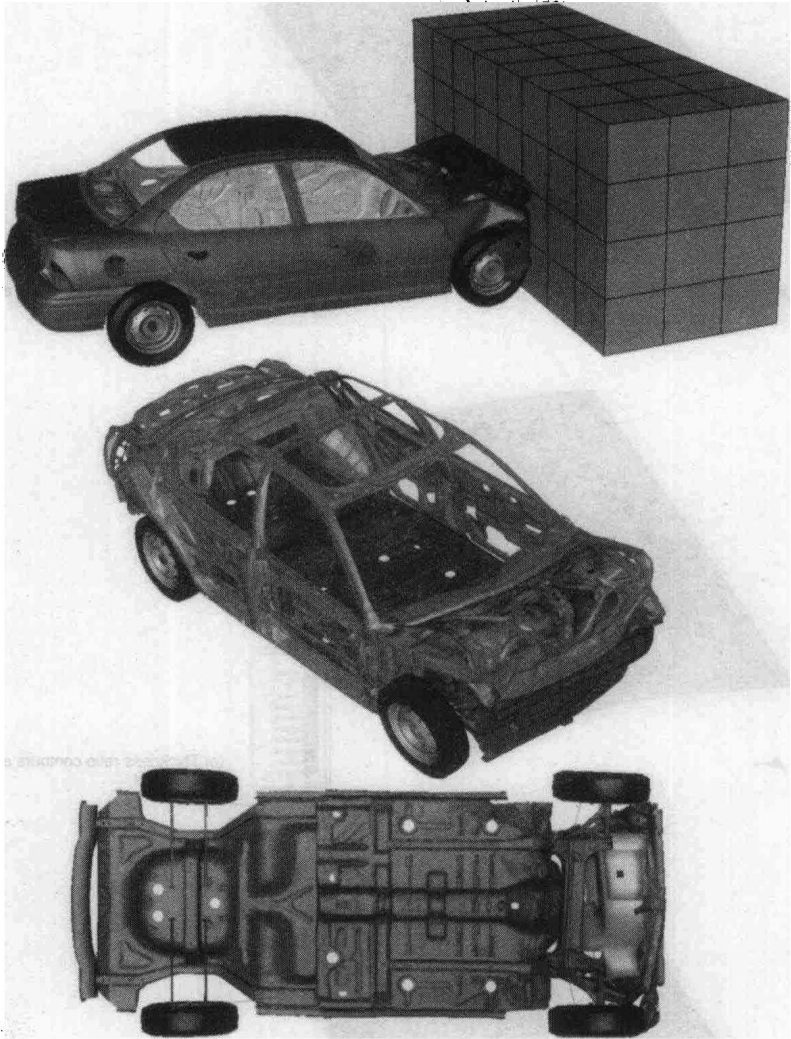


Plate 3 Car crash analysis

Frontal crash of a Neon Car performed using LS-DYNA. Courtesy of Livermore Software Technology Corporation. Model developed by FHWA/NHTSA National Crash Analysis Center of the George Washington University

Contents

<i>Preface</i>	xiii
1. General problems in solid mechanics and non-linearity	1
1.1 Introduction	1
1.2 Small deformation solid mechanics problems	4
1.3 Variational forms for non-linear elasticity	12
1.4 Weak forms of governing equations	14
1.5 Concluding remarks	15
References	15
2. Galerkin method of approximation – irreducible and mixed forms	17
2.1 Introduction	17
2.2 Finite element approximation – Galerkin method	17
2.3 Numerical integration – quadrature	22
2.4 Non-linear transient and steady-state problems	24
2.5 Boundary conditions: non-linear problems	28
2.6 Mixed or irreducible forms	33
2.7 Non-linear quasi-harmonic field problems	37
2.8 Typical examples of transient non-linear calculations	38
2.9 Concluding remarks	43
References	44
3. Solution of non-linear algebraic equations	46
3.1 Introduction	46
3.2 Iterative techniques	47
3.3 General remarks – incremental and rate methods	58
References	60
4. Inelastic and non-linear materials	62
4.1 Introduction	62
4.2 Viscoelasticity – history dependence of deformation	63
4.3 Classical time-independent plasticity theory	72
4.4 Computation of stress increments	80

4.5	Isotropic plasticity models	85
4.6	Generalized plasticity	92
4.7	Some examples of plastic computation	95
4.8	Basic formulation of creep problems	100
4.9	Viscoplasticity – a generalization	102
4.10	Some special problems of brittle materials	107
4.11	Non-uniqueness and localization in elasto-plastic deformations	112
4.12	Non-linear quasi-harmonic field problems	116
4.13	Concluding remarks	118
	References	120
5.	Geometrically non-linear problems – finite deformation	127
5.1	Introduction	127
5.2	Governing equations	128
5.3	Variational description for finite deformation	135
5.4	Two-dimensional forms	143
5.5	A three-field, mixed finite deformation formulation	145
5.6	A mixed-enhanced finite deformation formulation	150
5.7	Forces dependent on deformation – pressure loads	154
5.8	Concluding remarks	155
	References	156
6.	Material constitution for finite deformation	158
6.1	Introduction	158
6.2	Isotropic elasticity	158
6.3	Isotropic viscoelasticity	172
6.4	Plasticity models	173
6.5	Incremental formulations	174
6.6	Rate constitutive models	176
6.7	Numerical examples	178
6.8	Concluding remarks	185
	References	189
7.	Treatment of constraints – contact and tied interfaces	191
7.1	Introduction	191
7.2	Node–node contact: Hertzian contact	193
7.3	Tied interfaces	197
7.4	Node–surface contact	200
7.5	Surface–surface contact	218
7.6	Numerical examples	219
7.7	Concluding remarks	224
	References	224
8.	Pseudo-rigid and rigid–flexible bodies	228
8.1	Introduction	228
8.2	Pseudo-rigid motions	228
8.3	Rigid motions	230

8.4	Connecting a rigid body to a flexible body	234
8.5	Multibody coupling by joints	237
8.6	Numerical examples	240
	References	242
9.	Discrete element methods	245
9.1	Introduction	245
9.2	Early DEM formulations	247
9.3	Contact detection	250
9.4	Contact constraints and boundary conditions	256
9.5	Block deformability	260
9.6	Time integration for discrete element methods	267
9.7	Associated discontinuous modelling methodologies	270
9.8	Unifying aspects of discrete element methods	271
9.9	Concluding remarks	272
	References	273
10.	Structural mechanics problems in one dimension – rods	278
10.1	Introduction	278
10.2	Governing equations	279
10.3	Weak (Galerkin) forms for rods	285
10.4	Finite element solution: Euler–Bernoulli rods	290
10.5	Finite element solution: Timoshenko rods	305
10.6	Forms without rotation parameters	317
10.7	Moment resisting frames	319
10.8	Concluding remarks	320
	References	320
11.	Plate bending approximation: thin (Kirchhoff) plates and C_1 continuity requirements	323
11.1	Introduction	323
11.2	The plate problem: thick and thin formulations	325
11.3	Rectangular element with corner nodes (12 degrees of freedom)	336
11.4	Quadrilateral and parallelogram elements	340
11.5	Triangular element with corner nodes (9 degrees of freedom)	340
11.6	Triangular element of the simplest form (6 degrees of freedom)	345
11.7	The patch test – an analytical requirement	346
11.8	Numerical examples	348
11.9	General remarks	357
11.10	Singular shape functions for the simple triangular element	357
11.11	An 18 degree-of-freedom triangular element with conforming shape functions	360
11.12	Compatible quadrilateral elements	361
11.13	Quasi-conforming elements	362
11.14	Hermitian rectangle shape function	363
11.15	The 21 and 18 degree-of-freedom triangle	364
11.16	Mixed formulations – general remarks	366