Springer 大学数学图书——影印版

Reading, Writing, and Proving

A Closer Look at Mathematics

数学的读写和证明

Ulrich Daepp Pamela Gorkin 著

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内容提要

本书详细讨论数学的读写和证明，将学习数学的过程在某种意义下程序化，以使学习者培养出极好的学习习惯，以及学会如何应付面对的问题。本书对初学高等数学的读者来说特别有意义。

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序　言

在学校教书多年，当学生（特别是本科生）问有什么好的参考书时，我们所能推荐的似乎除了教材还是教材，而且不同教材之间的差别并不明显、特色也不鲜明。所以多年前我们就开始酝酿，希望为本科学生引进一些好的参考书，为此清华大学数学科学系的许多教授与清华大学出版社共同付出了很多心血。

这里首批推出的十余本图书，是从 Springer 出版社的多个系列丛书中精心挑选出来的。在丛书的筹划过程中，我们挑选图书最重要的标准并不是完美，而是有特色并包容各个学派（有些书甚至有争议，比如从数学上看也许不够严格），其出发点是希望我们的学生能够吸纳百家之长；同时，在价格方面，我们也做了很多工作，以使得本系列丛书的价格能让更多学校和学生接受，使得更多学生能够从中受益。

本系列图书按其定位，大体有如下四种类型（一本书可以属于多类，但这里限于篇幅不能一一介绍）。

一、适用面比较广、有特色并可以用作教材或参考书的图书。例如：

● Lovász et al.: Discrete Mathematics. 2003

该书是离散数学的入门类型教材。与现有的教材（包括国外的教材）相比，它涵盖了离散数学新颖而又前沿的研究课题，同时还涉及信息科学方面既基本又有趣的应用；在着力打好数学基础的同时，也强调了数学与信息科学的关联。姚期智先生倡导和主持的清华大学计算机科学试验班，已经选择该书作为离散数学课程的教材。

二、在目前国内的数学教育中，课程主要以学科的纵向发展为主线，而对数学不同学科之间的联系讨论很少。学生缺乏把不同学科视为一个数学整体的训练，这方面的读物尤其欠缺。这是本丛书一个重要的着力点。最典型的是：

● Fine/Rosenberger: The Fundamental Theorem of Algebra. 1997

该书对数学中最重要的定理——代数基本定理给出了六种证明，方法涉及到分析、代数与拓扑；附录中还给出了 Gauss 的证明和 Cauchy 的证明。全书以一个数学问题为主线展开，纵横数学的核心领域；结构严谨、文笔流畅、浅显易懂、引人入胜，是一本少见的能够让读者入迷的好读物，用它来引导学生欣赏和领会“数学的美”绝对不会落于空谈。该书适于自学、讨论，也是极好的短学期课程教材。

● Baker: Matrix Groups. 2001

就内容而言，本书并不超出我国大学线性代数、抽象代数和一般拓扑学课程的内容，但是本书所讲的是李群和李代数的基础理论——这是现代数学和物理学非常重要的工具。各种矩阵群和矩阵代数是李群和李代数最典型和

最重要的例子，同时也能帮助学生建立数学不同学科之间的联系。从矩阵出发，既能把握李群和李代数的实质，又能学会计算和运用，所以这是一本不可多得的好书。

三、科学与技术的发展不断为数学提出新的研究课题，因此在数学学科的发展过程中，来自其他学科的推动力是不能忽视的。本系列中第三种类型的读物就是强调数学与其他学科的联系。例如：

- Woodhouse: Special Relativity. 2003

  该书将物理与数学有机结合，体现了物理学家伽利略的名言："大自然是一部用数学语言写成的巨著。" 不仅如此，本书作者还通过对线性代数、微积分、场论等数学的运用进一步强调并贯穿这样的观点：数学的真谛和发展存在并产生于物理或自然规律及其发现中。精读此书有助于理解物理学和数学的整体关系。

- Britton: Essential Mathematical Biology. 2003

  生命科学在本世纪一定会有很大发展，其对数学的需求和推动是可以预见的。因此生物数学在应用数学中占有日益重要的地位，数学系培养的学生至少一部分人应当对这个领域有所了解。随着生命科学的迅速发展，生物数学也发展很快。本书由浅入深，从经典的问题入手，最后走向学科前沿和近年的热点问题。该书至少可以消除学生对生物学的神秘感。

四、最后一类是适合本科学生的课外读物。这类图书对激发和引导学生学习数学的兴趣会非常有帮助，而且目前国内也急需这样的图书。例如：

- Daepp/Gorkin: Reading, Writing and Proving. 2003

  该书对初学高等数学的读者来说特别有意义。它的基本出发点是引导读者以研究的心态去学习，让读者养成独立思考的习惯，并进而成为研究型的学习者。该书将一个学习数学的过程在某种意义下程序化，努力让学习者养成一个好的学习习惯，以及学会如何应对问题。该书特色鲜明，类似的图书确实很少。

- Brzezniak/Zastawniak: Basic Stochastic Processes. 1998

  随机过程理论在数学、科学和工程中有越来越广泛的应用，本书适合国内的需要。其主要特点是：书中配有的习题是巩固和延伸学习内容的基本手段，而且有十分完整的解答，非常适合自学和作为教学参考书。这是一本难得的好书，它 1999 年出版，到 2000 年已经是第 3 次印刷，到 2003 年则第 6 次重印。

- Anglin/Lambek: The Heritage of Thales. 1995

  该书的基本内容是数学的历史和数学的哲学。数学历史是该书的线索，数学是内容的主体，引申到的是数学哲学。它不是一本史论型的著作，而是采用专题式编写方式，每个专题相对独立，所以比较易读、易懂，是本科生学习数学过程中非常好的课外读物。

本系列丛书中的大部分图书还将翻译为中文出版，以适应更多读者的需要。丛书筹划过程中，冯克勤、郑志勇、卢旭光、郑建华、王殿军、杨利军、叶俊、扈志明等很多清华大学的教授都投入了大量精力。他们之中很多人也将是后面中文版的译者。此外，我们今后还将不断努力丰富引进丛书的种类，同时也会将选书的范围在可能情况下进一步扩大到其他高水平的出版机构。

教育是科学和技术发展的基石，数学教育更是基石的基础。因为是基础所以它重要；也因为是基础所以它显示度不高，容易不被重视。只有将人才培养放到更高的地位上，中国成为创新型国家的目标才会成为可能。

本系列丛书的正式推出，圆了一个我们多年的梦，但这无疑仅仅是开始。

白峰杉

2006 年 6 月于清华园

For Hannes and Madeleine

Preface

You are probably about to teach or take a "first course in proof techniques," or maybe you just want to learn more about mathematics. No matter what the reason, a student who wishes to learn the material in this book likes mathematics, and we hope to keep it that way. At this point, students have an intuitive sense of why things are true, but not the exposure to the detailed and critical thinking necessary to survive in the mathematical world. We have written this book to bridge this gap.

In our experience, students beginning this course have little training in rigorous mathematical reasoning; they need guidance. At the end, they are where they should be; on their own. Our aim is to teach the students to read, write, and do mathematics independently, and to do it with clarity, precision, and care. If we can maintain the enthusiasm they have for the subject, or even create some along the way, our book has done what it was intended to do.

Reading. This book was written for a course we teach to first and second year college students. The style is informal. A few problems require calculus, but these are identified as such. Students will also need to participate while reading proofs, prodded by questions (such as, "Why?"). Many detailed examples are provided in each chapter.

Since we encourage the students to draw pictures, we include many illustrations as well. Exercises, designed to teach certain concepts, are also included. These can be used as a basis for class discussion, or preparation for the class. Students are expected to solve the exercises before moving on to the problems. Complete solutions to almost all of the exercises are provided at the end of each chapter. Problems of varying degrees of difficulty appear at the end of each chapter. Some problems are simply proofs of theorems that students are asked to read and summarize; others supply details to statements in the text. Though many of the remaining problems are standard, we hope that students will solve some of the unique problems presented in each chapter.

Writing. The bad news is that it is not easy to write a proof well. The good news is that with proper instruction, students quickly learn the basics of writing. We try to write in a way that we hope is worthy of imitation, but we also provide students with "tips" on writing, ranging from the (what should be) obvious to the insider's preference ("Don't start a sentence with a symbol.").

Proving. How can someone learn to prove mathematical results? There are many theories on this. We believe that learning mathematics is the same as learning to play an instrument or learning to succeed at a particular sport. Someone must provide the background: the tips, information on the basic skills, and the insider's "know how." Then the student has to practice. Musicians and athletes practice hours a day, and it's not surprising that most mathematicians do, too. We will provide students with the background; the exercises and problems are there for practice. The instructor observes, guides, teaches and, if need be, corrects. As with anything else, the more a student practices, the better she or he will become at solving problems.

Using this book. What should be in a book like this one? Even a quick glance at other texts on this subject will tell you that everyone agrees on certain topics: logic, quantifiers, basic set theoretic concepts, mathematical induction, and the definition and properties of functions. The depth of coverage is open to debate, of course. We try to cover logic and quantifiers fairly quickly, because we believe that students can only fully appreciate the fundamentals of mathematics when they are applied to interesting problems.

What is also apparent is that after these essential concepts, everyone disagrees on what should be included. Even we prefer to vary our approach depending on our students. We have tried to provide enough material for a flexible approach.

- *The Minimal Approach.* If you need only the basics, cover Chapters 1–17. (If you assume the well ordering principle, or decide to accept the principle of mathematical induction without proof, you can also omit Chapter 12.)
- *The Usual Approach.* This approach includes Chapters 1–17 and Chapters 20–22. (This is easily doable in a standard semester, if the class meets three hours per week.)
- *The Algebra Approach.* For an algebraic slant to the course, cover Chapters 1–17 and Chapters 25 and 26.
- *The Analysis Approach.* For a slant towards analysis, cover Chapters 1–22. (This is what we usually cover in our course.) Include as much material from Chapters 23 and 24 as time allows. Students usually enjoy an introduction to metric spaces.
- *Projects.* We have included projects intended to let students demonstrate what they can do when they are on their own. We indicate prerequisites for each project, and have tried to vary them enough that they can be assigned throughout the semester. The results in these projects come from different areas that we find particularly interesting. Students can be guided to a project at their level. Since there are open-ended parts in each project, students can take these projects as far as they want to. We usually encourage the students to work on these in groups.
- *Notation.* A word about some of our symbols is in order here. In an attempt to make this book user-friendly, we indicate the end of a proof with the well-known symbol ■. The end of an example or exercise is designated by ○. If a problem is used later in the text, we designate it by **Problem**$^{\natural}$. We also have a fair number of "nonproofs." These are proofs that are questionable, and students are asked to find the error. We conclude such proofs with the symbol ⍰. Every other symbol will be defined when we introduce you to

it. Definitions are incorporated in the text for ease of reading and the terms defined are given in bold-face type.

Presenting. We also hope that students will make the transition to thinking of themselves as members of a mathematical community. We encourage the students we have in this class to attend talks, give talks, go to conferences, read mathematical books, watch mathematical movies, read journal articles, and talk with their colleagues about the things in this course that interest them. Our (incomplete, but lengthy) list of references should serve a student well as a starting point. Each of the projects works well as the basis of a talk for students, and we have included some background material in each section. We begin the chapter on projects with some tips on speaking about mathematics.

We hope that through reading, writing, proving, and presenting mathematics, we can produce students who will make good colleagues in every sense of the word.

* * *

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We plan to maintain a website with additional material, corrections, and other documentation at

http://www.facstaff.bucknell.edu/udaepp/readwriteprove/

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