CSIAM '98

中国工业与应用数学学会第五次大会论文集

主　编
曾庆存 李大潜 薛炽寿 程乾生

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前　言

随着我国经济建设的巨大发展和"科教兴国"方针的进一步实施，我国工业与应用数学以及数学技术的研究、普及与应用出现了蓬勃的发展势头，在应用数学的基础理论研究和诸多相关领域的应用中取得了可喜的成果。

中国工业与应用数学学会全国大会每两年召开一次。本书是学会第五次大会论文汇编，新收录的 150 余篇论文是学会学术工作委员会组织专家经过认真评审从会议征文中遴选出来的。这些论文覆盖了微分方程、运筹和优化、数理统计、计算数学、经济数学和数学建模等应用数学的诸多方面，在某种程度上反映了我国高等院校、科研院所和工业部门中应用数学工作者的丰硕成果，显示了他们投身国民经济主战场进行研究与开发的风貌。

本论文集的出版得到国家自然科学基金委和中国有色金属工业总公司的大力支持。清华大学出版社和清华大学应用数学系也给予我们鼎力的协助。在此，我们一并表示由衷的感谢。

编者

1998 年 5 月
于北京清华大学

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浅论“数学技术”

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[摘要] 本文论述了“高技术本质上是数学技术”的出处及其背景。由于信息技术及数学软件的迅速发展,数学科学的许多方面已经从工程技术的背后走到了前台;什么是“数学技术”，及其可能的发展。希望本文能对数学技术的进一步的讨论起到抛砖引玉的作用。

一、引 言

曾任美国总统尼克松的科学顾问、现为 EED 股份有限公司总裁的 Edward E. David Jr. 于 1984 年 1 月 25 日在美国数学会 (AMS) 和美国数学协会 (MAA) 联合年会上说:“…… 对数学研究的低水平的资助只能出自对数学带来的好处的完全不适当的估价. 显然,很少的人认识到如今被如此称颂的‘高技术’本质上是数学技术。”(″…… the low level of support for mathematics research can only flow from a totally inadequate appreciation of the benefits it confers. Apparently, too few people recognize that the ‘ high technology’ that is so celebrated today is essentially mathematical technology.″（见文献[1] p.142)。在文献[2] 中又指出,“‘高技术’的出现把我们的社会推进到了数学工程技术的时代。在这个时代里，数学与工程技术以新的方式相互作用着。五十年前，数学虽然也直接为工程技术提供一些工具，但基本方式是间接的：先促进其他科学的发展，再由这些科学提供工程原理和设计的基础。现在不一样了，数学和工程技术之间，在更广阔的范围内和更深刻的程度上，直接地相互作用着，极大地推动了数学和工程科学的发展，也极大地推动了工程科学的发展。”也就是说数学开始从后台走向前台。之后,“‘高技术’本质上是数学技术”的说法在学术界，特别是在数学界广为流传，例如，在欧洲工业数学联合会的宗旨中就引述了 David 的这句话。(见文献[3]) 钱学森教授在 1989 年 8 月 18 日中国数学会召开的数学教育与科研座谈会上的讲话中也指出:“ ……，这是数学技术，即怎样给出一个方法，能使科学的理论通过电子计算机解答具体的科学技术问题。”（见文献[4],p.131） 当然,“‘高技术’本质上是数学技术”的说法也引起了许多不同的看法和争议。

二、什么是数学技术

实际上，技术、工程的概念都是发展的。在文献[5]中对技术的解释为“用于达到商业和工业目的的科学方法和材料”；在文献[6]中的说明为“技术是人类活动的一个专门领域……，技术一词出自希腊文 techne‘工艺、技能’与 logos‘词、讲话’的组合，意思是对造型艺术和应用技术进行论述。当它 17 世纪在英国首次出现时，仅指各种应用技艺。到 20 世纪初技术的含义逐渐扩大，它涉及到工具、机器及其使用方法和过程。到 20 世纪后半期，技术被定义为‘人类改变或控制客观环境的手段或活动。”在文献[5]中对工程的解释为“把科学和数学的原理应用于设计、制造，以及有效又经济的结构、机器、过程和系统的操作的实际目的。”在文献[6]中的说明为，“应用科学知识使自然资源最佳地为人类服务的一种专门技术。工程的设计者称为工程师。…… 科学家的职责是如何认识，而工程师则是如何实现……工程师不同于科学家，他不能自由地选择自己感兴趣的问题，而必须解决面临的问题…… 工程师的主要职能可概括为：1. 研究和发展。研究工程师应用数学和科学概念、实验技术和归纳推理来探求新的工作原理和方法；而发展工程师则把研究成果应用于实际…… 2. 施工和生产…… 3. 操作。 4. 管理和其它职能。”由此可见，把数学中的有些部分看作技术并非是不可理解的。例如，代数与密码技术，Radon 与 CT （计算机层析）技术，大规模线性规划求解技术在经济、管理中的应用，与保险业有关的精算学有关软件，期货、期权交易中的期权定价软件，信息提取与处理软件小波技术在信息科学中的应用，穿甲弹的计算仿真技术，并行计算技术在气象和工程中的应用，等等，都是数学技术。在近年来美国工程院（创建于 1964 年）院士的选举中引人注目的讨论却是“什么是工程？”过去，研制计算机软件的计算机科学家，研制数学软件的计算数学家是不能被选为工程院院士的。但是，在 1997 年 选出的 85 位工程院院士中就有三位数学家，其中一位是研制开发在工业中广泛应用的与整数规划有关软件 CPLEX 的 Robert Bixby （他是 CPLEX Optimization Inc. 的总裁，也是 Rice 大学的教授），该公司已经卖出一万多份软件，广泛应用于石油公司、航空公司、邮局、华尔街、森林业、制造业等工业、企业。另一位是被称为著名的数学软件 MATLAB 之父的 The Math Works, Inc. 董事会主席和首席科学家 Cleve Moler. 德国汽车制造商企图用该公司的数学软件 MATLAB 和 SIMULINK 来设计下一代新车控制系统的芯片中的控制算法。该公司的数学软件还广泛用于汽车和航天工业。（见文献[7]） 在 1998 年 选出的 84 位工程院院士中又有三位数学家，其中一位就是 Texas 大学“地表下地层建模中心”的 Mary Wheeler 教授，她是由于与石油油藏模拟和地下水建模有关的大规模并行计算方面的成就而当选的。我们无须再举更多的例子了。这已经充分说明“数学技术”的出现决非偶然。

综上所述，我们大体上可以认为所谓的“数学技术”必须满足以下四个条件：

1. 对所要解决的问题有一个描述得非常精细的数学模型；
2. 解决该数学模型中的数学问题有很好的数学方法和算法，还有高效的数学软件；
3. 有相当的普适性，即能用于一大类问题或类似的问题；
4. 与实际问题有关的领域的工程、制造有密切的合作，甚至融为一体。

三、结论

其实，问题并不在于对以数学方法为主的技术都要冠以“数学技术”的帽子，问题的实质在于：数学是否开始从后台走向前台，如果是的话，是怎样走向前台呢？数学科学在 21 世纪是否具有关键的重要性？或者，如胡世华先生在文 [9] 中引用 Whitehead 的话：“…… 在今后两千年内，在人类思想领域里具有压倒性的新情况，将是数学地理解问题占统治地位。”（见文献[9],p.16）或如钱学森教授在文献[4]中所说，“…… 数学的发展关系到整个科学技术的发展，而科学技术是第一生产力；所以数学的发展是一件国家大事。”如果大家同意的话，讨论就要容易得多、深入得多。我们相信这样的讨论一定会引起学术界、工程界、金融界、工商界和政府领导人的兴趣，从而将大大推动数学科学的发展，和工程、经济、管理更进一步地结合。

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Some Aspects of Mathematical Finance

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[Abstract] Mathematical finance becomes more and more attractive in recent years. This talk will begin with a brief history of mathematical finance. Several interesting problems, such as option pricing, consol rate problem, etc. will be discussed in some extent. These problems give rise to the study of backward and forward-backward stochastic differential equations. On the other hand, the application of techniques from stochastic control theory and differential games makes the mathematical finance more prosperous.

1. Introduction

The two winners of the 1997 Economic Sciences Nobel Prize are Robert C. Merton and Myron S. Scholes for their significant contributions in mathematical finance. This makes the already popular field of *mathematical finance* more attractive.

Mathematics plays an essential role in the quantitative study of modern finance. In the foreword to the book "*Continuous-Time Finance*" ([21]) of Robert C. Merton, Paul A. Samuelson[2] wrote:

The pole that propelled Merton to Byronic eminence was the **mathematical** *tool of continuous probability à la Norbert Wiener and Kiyoshi Itô. Suddenly what had been complex approximation became beautifully simple truth.*

On the other hand, in the same book, R. C. Merton wrote:

The **mathematical** *models of modern finance contain some of the most beautiful applications of probability and optimization theory.*

..., of course, all that is beautiful in science need not also be practical; and surely, not all that is practical in science is beautiful. Here, we have **both**.

Let us now briefly recall the history of modern mathematical finance. It is now well-accepted that the magnificent dissertation "*On the theory of speculation*" of Louis Bachelier written in 1900 ([2]) pronounced the birth of the mathematical finance. Interesting enough,

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[2]1970 Economic Sciences Nobel Prize winner.

that was also the first time that the later so-called Brownian motion was mathematically formulated and carefully studied.

Bachelier's work had been unknown in the finance literature for more than 50 years. In early 1950s, Paul A. Samuelson rediscovered this work (through the statistician L. J. Savage). The modern finance begins in the later 1950s and 1960s. The most influential works in that period was that of H. Markowitz[3] on the *mean variance theory of portfolio selection.* Later, W. F. Sharpe ([28]) and J. Lintner ([13]) formulated the *Capital Asset Pricing Model* (CAPM), P. A. Samuelson ([27]) and E. Fama ([9]) proposed the *efficient market hypothesis.* In 1973, Fischer Black and Myron S. Scholes published the famous *Black-Scholes formula* solving the pricing of European option. Since then, many researches of mathematical finance have been carrying out. See [22] for some detailed descriptions of the history.

Backward stochastic differential equations (BSDE, for short) was firstly formulated by Bismut ([3]) in studying stochastic optimal controls. Since 1990, due to Pardoux and Peng, the study of BSDE has been re-ignited ([24,25]). the theory of forward-backward stochastic differential equations (FBSDE, for short) have been systematically studied shortly after (see [1,15,14,8,12,29,30], and for relevant results, see [16,17,18,19,26,31]), and these theories turn out to be very useful in mathematical finance (see [8,7,10,6,32,23]). In the rest of this paper, we are going to give a brief description of the theory of BSDE and FBSDE, and their applications in some mathematical finance problems.

2. Simple Description of the Market

We first look at the model of a *bond.* Let $X_0(t)$ be the price of the bond at time t with interest rate r compounded continuously. Then on a period $[t, t+h]$, we have

$$\frac{X_0(t+h) - X_0(t)}{X_0(t)h} \approx r \tag{2.1}$$

which is the relative return per unit time. We may also write the above as

$$X_0(t+h) - X_0(t) \approx rX_0(t)h \tag{2.2}$$

Let $h \to 0$, one has

$$\mathrm{d}X_0(t) = rX_0(t)\mathrm{d}t, \quad X(0) = x \tag{2.3}$$

whose solution is $X_0(t) = xe^{rt}$, $t \in [0, T]$. Clearly, $X_0(\cdot)$ is monotonically increasing. For this reason, we say that the bond is *riskless.*

Next, we look at a *stock*, which is quite different from a bond. Let $X(t)$ be the price of the stock, with the *average return rate* b and the so-called *volatility* σ. Again, consider on $[t, t+h]$, we have

$$X(t+h) - X(t) \approx X(t)[bh + \sigma\eta(t,h)] \tag{2.4}$$

[3] 1990 Economic Sciences Nobel Prize winner

where $\eta(t,h)$ is a (normalized) noise. It could be positive or negative! Thus, the stock is *risky*. In the above, $\eta(t,h)$ is caused by the (independent) behavior of a very large number of investors. Hence, people accept that $\eta(t,h) \sim N(0,h)$, the normal distribution with mean 0 and covariance h.

Let $W(t)$ be the accumulative noise up to time t. This turns out to be a **Brownian motion**. Then (2.4) can be written as

$$X(t+h) - X(t) \approx X(t)\{bh + \sigma[W(t+h) - W(t)]\}$$

Letting $h \to 0$, we have (compare (2.4))

$$\begin{cases} \mathrm{d}X(t) = bX(t)\mathrm{d}t + \sigma X(t)\mathrm{d}W(t), \; t \in [0,T] \\ X(0) = x \end{cases} \tag{2.5}$$

whose solution is given by $X(t) = xe^{bt+\sigma W(t)}$, $t \in [0,T]$.

Now, we look at the general case ([23]). There are $n+1$ assets continuously traded in the market. The 0-th asset is a bond, and the last n are stocks. The price process $X_i(\cdot)$ of the i-th asset satisfies the following SDE:

$$\begin{cases} \mathrm{d}X_0(t) = r(t)X_0(t)\mathrm{d}t \\ \mathrm{d}X_i(t) = b_i(t)X_i(t)\mathrm{d}t + X_i(t)\sum_{j=1}^{d}\sigma_{ij}(t)\mathrm{d}W_j(t) \\ X_i(0) = x_i, \qquad 0 \le i \le n \end{cases} \tag{2.6}$$

Here, $W(\cdot) \equiv (W_1(\cdot), W_2(\cdot), \cdots, W_d(\cdot))$ is a d-dimensional standard Brownian motion defined on some complete filtered probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\ge0}, \mathbf{P})$, such that $\{\mathcal{F}_t\}_{t\ge0}$ is the natural σ-field generated by $W(\cdot)$ (augmented by all the $\mathbf{P}$-null sets in $\mathcal{F}$). In what follows, we denote

$$b(\cdot) = (b_1(\cdot), b_2(\cdot), \cdots, b_n(\cdot))^T, \quad \sigma(\cdot) = (\sigma_{ij}(\cdot))_{n\times d}$$

We call $r(\cdot)$, $b(\cdot)$, and $\sigma(\cdot)$ the interest rate, the appreciation rate, and the volatility, respectively. Recall that the market is said to be *complete* if $n \ge d$ and $\sigma(t)$ is of full rank. Otherwise, the market is said to be *incomplete*.

3. Option Pricing

Consider a market with one bond and one stock whose prices are determined by the following system of SDEs:

$$\begin{cases} \mathrm{d}X_0(t) = r(t)X_0(t)\mathrm{d}t \\ \mathrm{d}X(t) = X(t)b(t)\mathrm{d}t + X(t)\sigma(t)\mathrm{d}W(t) \end{cases} \tag{3.1}$$

where $X_0(t)$, $X(t)$, $r(t)$, $b(t)$, and $\sigma(t)$ are the price of the bond, the price of the stock, the interest rate, the appreciation rate, and the volatility, respectively.

Let us consider the so-called *European call option*. This is a contract which gives the right to buy, say, 1 share of the stock at a given price, q, called the *exercise price*, at a specified time $t = T$, called the *maturity date*. There will be two case:

(a) If at $t = T$, $X(T) > q$, the buyer of the option will come to buy the stock;
(b) If $X(T) < q$, the option will be discarded.

Apparently, such an option has its value (it is not free!). The question is the following: What is the (fair) price of this option? This problem is called the *option pricing* ([4]). We now make some analysis on this problem.

Suppose the price at $t = 0$ is y. Note that the loss of the seller (or writer) of the option at time $t = T$ is: $\big(X(T) - q\big)^+$. To compensate (called *hedge*) this loss, he has to invest the amount y (at time $t = 0$) in the market and to get enough return. Let $Y(t)$ be the wealth of the seller t. Thus,

$$Y(0) = y$$

He wants to invest $Y(t)$ in the market so that

$$Y(T) \geq \big(P(T) - q\big)^+, \quad \text{a.s.} \tag{3.2}$$

Suppose he splits $Y(t)$ in the following way:

$$\begin{cases} \pi(t) & \text{— the amount in stock} \\ Y(t) - \pi(t) & \text{— the amount in bond} \end{cases} \tag{3.3}$$

Clearly, when $\pi(\cdot)$ is given, the allocation of the wealth in bond and stock is determined. We call $\pi(\cdot)$ the *portfolio process*. A simple computation shows that the wealth process $Y(\cdot)$ satisfies the following:

$$\begin{cases} \mathrm{d}Y(t) &= \big\{r(t)Y(t) + [b(t) - r(t)]\sigma(t)^{-1}Z(t)\big\}\mathrm{d}t + Z(t)\mathrm{d}W(t) \\ Y(0) &= y \end{cases} \tag{3.4}$$

Here, we have assumed $\sigma(t) \neq 0$ and have set $Z(t) = \sigma(t)\pi(t)$. Note that the larger the y, the larger the $Y(T)$. Thus, in order the price y of the option being fair, we should have y so that the following holds:

$$Y(T) = (P(T) - q)^+ \tag{3.5}$$

Then we end up with the following system of stochastic differential equations:

$$\begin{cases} \mathrm{d}X(t) &= X(t)b(t)\mathrm{d}t + X(t)\sigma(t)\mathrm{d}W(t) \\ \mathrm{d}Y(t) &= \big\{r(t)Y(t) + [b(t) - r(t)]\sigma(t)^{-1}Z(t)\big\}\mathrm{d}t + Z(t)\mathrm{d}W(t) \\ X(0) &= x, \qquad Y(T) = (X(T) - q)^+ \end{cases} \tag{3.6}$$

We need to find a triple of adapted processes $(X(\cdot), Y(\cdot), Z(\cdot))$. The fair price of the option is given by $y = Y(0)$.

We note that the equation for $X(\cdot)$ is *forward* since the initial value is specified, whereas the equation for $Y(\cdot)$ is backward. For this reason, we call (3.6) a system of *forward-backward stochastic differential equations* (FBSDE, for short). It is seen that (3.6) is a decoupled FBSDE.

Let us look at two interesting cases: