

模糊集

理论与应用

模

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| 刘应明 | 吴从炘 | 王熙照 |
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序 言

自从美国加州贝克莱大学的 L. A. Zadeh 教授于 1965 年发表“Fuzzy Sets”一文之后,模糊数学就成为一门新的数学学科诞生了。在短短的三十多年中,模糊数学的理论和应用都得到了飞速的发展,愈来愈引起人们的高度重视,模糊数学的发展进一步丰富了经典数学的理论,为人们处理模糊信息提供了众多巧妙方法,现已广泛应用于计算机科学、人工智能、信息处理、控制工程、土木工程、机械工程、环境工程、地震工程、知识工程、经济与管理科学、思维科学、社会科学、心理与教育、医学、气象预报及文学艺术等等领域,特别是模糊技术的产业化,模糊产品的问世,取得了显著的经济效益,充分显示出模糊数学在知识经济时代具有强大的生命力。

我国模糊数学理论及其应用的研究,起步较早,发展也很快,并且通过多年的积累,已形成一支实力可观的队伍,取得了一批积极的成果,在世界上产生了很大影响。本书所收集的论文正是对近年来广大研究工作者研究成果的一次较为全面的检阅与总结,该书出版的目也意欲通过广泛的交流、沟通,以期更好的推动我国模糊数学理论及应用研究的深入开展,促进科研成果的转化,实现科技与经济的有机结合,为我国知识经济的发展,实施“科教兴国”的战略,做出应有的贡献。

在中国系统工程学会模糊数学与模糊系统委员会各位同仁们及河北大学数学系的共同努力下,经过近两年的筹备,第九届年会即将举行,这本书也得以问世,谨以此作为对大会召开的祝贺和迎接知识经济时代的一份献礼。

本届大会共收到论文 260 余篇,几乎涉及模糊数学理论与应用的所有领域,经过河北大学数学系组织的审稿小组的一定审查,本书共收入 150 余篇论文。由于时间紧,论文多,本书疏漏之处在所难免,敬请各位同仁予以谅解并批评指正。

衷心感谢同仁们精心为大会撰文以及对大会的诸多支持,衷心感谢中国系统工程学会的大力支持,衷心感谢河北大学数学系及河北大学出版社对出版本书所给予的真诚帮助。

中国系统工程学会
模糊数学与模糊系统委员会
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第一部分

模糊拓扑

Interaction Between Lattice Theoretical Property of Range and L - fuzzy Topological Spaces^{*}

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Abstract Based on L - fuzzy set theory, many propositions in L - fuzzy topology are inevitably related to the lattice theoretical properties of the range L . Besides those L - fuzzy topological propositions which take some lattice theoretical properties of the range as necessary conditions, there are some other more interesting ones (sometimes include generalized analytical propositions) which are equivalent to some lattice theoretical properties. These connections in the form of “sufficient and necessary conditions” more exactly reflect the close internal relationship between structure of lattices and L - fuzzy topology or generalized analysis. In this paper, some of L - fuzzy topological and generalized analytical propositions in this type will be established and be proved.

Keywords Lattice, L - fuzzy Topology, Analysis, Lattices Theoretical Property, L - fuzzy Topological Proposition

1. Preliminaries

In the sequel, X always stands for a non - empty ordinary set and L for a complete lattice. A completely distributive lattice L with an ordering - reversing involution[']: $L \rightarrow L$ is called a *fuzzy lattice*. The smallest element and the largest element of a lattice L , if exist, are denoted by 0_L and 1_L , or 0 and 1 for short, respectively. Call every mapping from X to L an *L - fuzzy subset on X* . Denote the family of all the L - fuzzy subsets on X by L^X . For every $a \in L$, let \underline{a} denote the constant mapping from X to L with value a .

Note that the partial order \leq on L naturally induces a pointwise order \leq on L^X as follows: For every pair $U, V \in L^X$,

$$U \leq V \Leftrightarrow \forall x \in X, U(x) \leq V(x).$$

This order is also a partial order, and it makes L^X to be a fuzzy lattice.

1.1 Definition $\forall a, b \in L$ such that $a \leq b$, denote

$$\uparrow a = \{b \in L : b \geq a\}, \downarrow a = \{b \in L : b \leq a\}, [a, b] = \{c \in L : a \leq c \leq b\}.$$

$a \in L$ is called *join - irreducible*, if for every two $a, b \in L$,

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$$\alpha \leq a \vee b \Rightarrow \alpha \leq a \text{ or } \alpha \leq b;$$

every non - zero join - irreducible element is called a *molecule*. For every $A \subset L$, denote the set of all the molecules contained in A by $M(A)$.

Note that for a non - empty ordinary set X , a fuzzy lattice L and an L - fuzzy subset $A \in L^X$, according to the symbol defined above, $\downarrow A$ is the set of all the elements in L^X smaller than A . So the set of all the molecules of L^X smaller than A is just

$$M(\downarrow A) = \{x_\lambda : x \in X, \lambda \in M(L), \lambda \leq A(x)\}$$

1.2 Definition Let L be a fuzzy lattice. $\delta \subset L^X$ is called an L - fuzzy topology on X , if δ is closed under arbitrary joins and finite meets; especially, $0, 1 \in \delta$, Call (L^X, δ) an L - fuzzy topological space, or call it an L - fts for short. Every $U \in \delta$ is called an *open subset* in (L^X, δ) , and every $P \in L^X$ such that $P' \in \delta$ is called a *closed subset* in (L^X, δ) . Denote the family of all the closed subsets in (L^X, δ) by δ' .

For every L - fts (L^X, δ) , denote $[\delta] = \{U \subset X : \chi_U \in \delta\}$, where $\chi_U : X \rightarrow \{0, 1\} \subset L$ is the characteristic function of $U \subset X$ on X ; call the ordinary topological space $(X, [\delta])$ the *background space* of (L^X, δ) .

1.3 Definition Let (L^X, δ) be an L - fts, $A \in L^X$. Define respectively the *interior* A° and the *closure* A^- of A as

$$A^\circ = \bigvee \{U \in \delta : U \leq A\}, A^- = \bigwedge \{P \in \delta' : P \geq A\}.$$

1.4 Definition Define a relation \leq on L as follows: For every two $a, b \in L$, $b \leq a$ if and only if for every $C \subset L$ such that $\bigvee C \geq a$, there exists $c \in C$ such that $b \leq c$. Denote $\beta(a) = \{b \in L : b \leq a\}$. Every subset $D \subset \beta(a)$ satisfying $\bigvee D = a$ is called a *minimal set* of a in L .

1.5 Theorem^[6, 11, 12, 13] Let L be a complete lattice. Then the following conditions are equivalent:

- (i) L is completely distributive.
- (ii) Every element of L has a minimal set.
- (iii) Every element of L has a minimal set consisting of molecules in L .

1.6 Corollary Every element in a completely distributive lattice can be represented as a join of molecules.

2. Complete Distributivity of Range and Analytical Property, Topological Property of Spaces

2.1 Definition Let L be a complete lattice. The topologies on L generated respectively by subbases $\{L \setminus \uparrow a : a \in L\}$, $\{L \setminus \downarrow a : a \in L\}$ and $\{L \setminus [a, b] : a, b \in L, a \leq b\}$ are called respectively the *upper topology*, the *lower topology* and the *interval topology* of L , and denote them respectively by $\Omega^*(L)$, $\Omega_*(L)$ and $\Omega(L)$. Also denote respectively these topologies on L by Ω^* , Ω_* and Ω for short.

2.2 Definition Let (X, τ) be an ordinary topological space, L a complete lattice. A mapping $f : X \rightarrow L$ is called respectively *upper semicontinuous*, *lower semicontinuous* and

continuous, if f is respectively continuous for the topologies Ω^* , Ω_* and Ω .

2.3 Proposition Let (X, τ) be a topological space, L a complete lattice. Then a mapping $f: X \rightarrow L$ is continuous if and only if f is both upper semicontinuous and lower semicontinuous.

2.4 Definition Denote the category of topological spaces and continuous mappings by **Top**.

Shorten respectively the phrases “upper semicontinuous,” “lower semicontinuous” and “continuous” by “u. s. c.,” “l. s. c.” and “c.”. For a complete lattice L , let (US), (LS) and (CT) denote the following three conditions respectively:

$(X, \tau) \in \text{Ob}(\mathbf{Top})$, $A \in L^X$ is u. s. c. \Rightarrow For every $x \in X$ and every neighborhood base B of x ,

$$A(x) = \bigwedge_{U \in B} \bigvee_{y \in U} A(y);$$

$(X, \tau) \in \text{Ob}(\mathbf{Top})$, $A \in L^X$ is l. s. c. \Rightarrow For every $x \in X$ and every neighborhood base B of x ,

$$A(x) = \bigvee_{U \in B} \bigwedge_{y \in U} A(y);$$

$(X, \tau) \in \text{Ob}(\mathbf{Top})$, $A \in L^X$ is c. \Rightarrow For every $x \in X$ and every neighborhood base B of x ,

$$A(x) = \bigwedge_{U \in B} \bigvee_{y \in U} A(y) = \bigvee_{U \in B} \bigwedge_{y \in U} A(y);$$

clearly, $(US) + (LS) \Rightarrow (CT)$, but the converse is in general false.

2.5 Theorem Every completely distributive lattice L satisfies both conditions (US) and (LS).

2.6 Theorem let $(X, \tau) \in \text{Ob}(\mathbf{Top})$, L a complete lattice. Then for every two mappings $A, B: X \rightarrow L$ the following conclusions hold:

- (i) If for every $x \in X$ there exists a family $B(x)$ of neighborhoods (need not be a neighborhood base) of x such that $A(x) = \bigwedge_{U \in B(x)} \bigvee_{y \in U} A(y)$, then A is upper semicontinuous.
- (ii) If for every $x \in X$ there exists a family $B(x)$ of neighborhoods (need not be a neighborhood base) of x such that $A(x) = \bigvee_{U \in B(x)} \bigwedge_{y \in U} A(y)$, then A is lower semicontinuous.
- (iii) If for every $x \in X$ there exists a neighborhood base $B(x)$ of x such that $B(x) = \bigwedge_{U \in B_x} \bigvee_{y \in U} A(y)$, then B is upper semicontinuous.
- (iv) If for every $x \in X$ there exists a neighborhood base $B(x)$ of x such that $B(x) = \bigvee_{U \in B_x} \bigwedge_{y \in U} A(y)$, then B is lower semicontinuous.

Then we have the following connection between the complete distributivity of ranges and some analytical properties of topological spaces:

2.7 Theorem Let L be a distributive complete lattice. Then the following conditions are equivalent:

- (i) L is completely distributive.
- (ii) L satisfies both conditions (US) and (LS).

Now we turn to a generalization of ordinary staircase functions in topological spaces:

2.8 Definition Let (X, τ) be an ordinary topological space, L a complete lattice.

For every $a \in L$, $U \in \tau$, denote

$$\begin{aligned} F_*(a, U) &= aU, F^*(a, U) = \underline{a} \vee \chi_{X \setminus U}, \\ stb_{*L}(\tau) &= \{F_*(a, U) : a \in L, U \in \tau\} \subset L^X, \\ stb^*_{*L}(\tau) &= \{F^*(a, U) : a \in L, U \in \tau\} \subset L^X, \\ stt_{*L}(\tau) &= \{A : A \subset stb_{*L}(\tau)\} \subset L^X, \\ stt^*_{*L}(\tau) &= \{A : A \subset stb^*_{*L}(\tau)\} \subset L^X, \end{aligned}$$

Call $stb_{*L}(\tau)$ the staircase base associated with τ , $stb^*_{*L}(\tau)$ the staircase co-base associated with τ , $stt_{*L}(\tau)$ the staircase topology associated with τ , $stt^*_{*L}(\tau)$ the staircase co-topology associated with τ .

Denote the family of all the lower semicontinuous mappings from (X, τ) to L by $lc_L(\tau)$, the family of all the upper semicontinuous mappings from (X, τ) to L by $uc_L(\tau)$.

2.9 Proposition Let (X, τ) be an ordinary topological space, L a complete lattice. Then

- (i) $stt_{*L}(\tau) \subset lc_L(\tau)$.
- (ii) $stt^*_{*L}(\tau) \subset uc_L(\tau)$.

2.10 Proposition Let (X, τ) be an ordinary topological space, L a complete lattice, $x \in X$, B a neighborhood base of x in X . Then

- (i) $A \in stt_{*L}(\tau) \Rightarrow A(x) = \bigvee_{U \in B} \bigwedge_{y \in U} A(y)$.
- (ii) $A \in stt^*_{*L}(\tau) \Rightarrow A(x) = \bigwedge_{U \in B} \bigwedge_{y \in U} A(y)$.

Theorem 2.7 characterizes completely distributive law from an angle of analysis, then we can consider the following conclusion as a topological way to do the same thing:

2.11 Theorem Let L be a distributive complete lattice. Then the following conditions are equivalent:

- (i) L is completely distributive.
- (ii) For every ordinary topological space (X, τ) , $lc_L(\tau) \subset stt_{*L}(\tau)$, $uc_L(\tau) \subset stt^*_{*L}(\tau)$.

2.12 Remark In real analysis, as well-known, semicontinuous functions can be approximated by staircase functions. Theorem 2.11 extends this result into the case of lattice. Moreover, this theorem tells us: As a value domain, a distributive complete lattice is completely distributive if and only if every semicontinuous mappings into it can be approximated by staircase mappings. So Theorem 2.11 shows the special importance of completely distributive lattice in topology on lattice.

3. Lattice Theoretical Property and L - fuzzy Topological Propositions

As a preparation, we introduce the concepts of lattice-valued weakly induced spaces and lattice-valued induced spaces as follows:

3.1 Definition Let (L^X, δ) be an L -fts. δ or (L_X, δ) is called L -valued weakly in-

duced or *weakly induced* for short, if every $U \in \delta$ is a lower semicontinuous mapping from the background space $(X, [\delta])$ to L ; is called *L-valued stratified* or *stratified* for short, if $\underline{a} \in \delta$ for every $a \in L$; is called *L-valued induced* or *induced* for short, if (L^X, δ) is both weakly induced and stratified.

3.2 Definition Let (L^X, δ) be an L -fts, $A, B \in L^X$.

A and B are called *separated*, if

$$A^- \wedge B = A \wedge B^- = \underline{0}$$

A is called *connected*, if there not exist separated $C, D \in L^X \setminus \{\underline{0}\}$ such that $A = C \vee D$,

$\text{Call}(L^X, \delta)$ is *connected*, if the largest L -fuzzy subset $\underline{1}$ is connected.

3.3 Definition A lattice L is called *anti-diamond-type*, if there not exists a sublattice of L which is isomorphic to the diamond-type lattice; i. e there not exist $a, b \in L \setminus \{0, 1\}$ such that $a \wedge b = 0, a \vee b = 1$.

Then we have the following results:

3.4 Theorem Let L be a fuzzy lattice. Then the following conditions are equivalent:

- (i) L is anti-diamond-type.
- (ii) For every weakly induced L -fts (L^X, δ) , (L^X, δ) is connected if and only if $(X, [\delta])$ is connected.
- (iii) For every induced L -fts (L^X, δ) , (L^X, δ) is connected if and only if $(X, [\delta])$ is connected.

3.5 Definition Let (L^X, δ) be an L -fts, $A \in L^X$, Define the *density* $dn(\delta)$ of (L^X, δ) by

$$dn(\delta) = \min \{ |A| : A \in L^X, A^- = \underline{1} \}$$

The relation between the densities of an L -fuzzy product space and its coordinate spaces is tightly combined with the property of their range just as the following theorem shows:

3.6 Theorem Let $\{(L^X, \delta_t) : t \in T\}$ be a family of L -fts, (L^X, δ) their L -fuzzy product space, $1 \in M(L)$, $\kappa \geq \omega$. Then

- (i) If $dn(\delta_t) \leq \kappa$ for every $t \in T$, $|T| \leq 2^\kappa$, then $dn(\delta) \leq \kappa$
- (ii) If $dn(\delta_t) \leq \kappa$ for every $t \in T$, $|T| > 2^\kappa$, then $dn(\delta) \leq |T|$.

Since the following investigation involves classes, we need to generalize the concept of mapping. Sure, it is just a parallel generalization:

3.7 Definition Let A, B be classes. Denote the class of all the ordered pairs (a, b) , where $a \in A, b \in B$, by $A \times B$. Similarly define $A_1 \times \cdots \times A_n$ for finite number of classes A_1, \cdots, A_n . If $A_i = A$ for $i = 1, \cdots, n$, denote $A_1 \times \cdots \times A_n$ by A^n . A subclass f of $A \times B$ is called a *general mapping* from A to B , denoted by $f: A \rightarrow B$, if for every $a \in A$, there exists exactly one $b \in B$ such that $(a, b) \in f$. For every general mapping f from A to B and every $a \in A$, denote $f(a) = b$ if and only if $(a, b) \in f$.

3.8 Definition Denote the category of all cardinal numbers and all the order preserving mappings among them by **Card**.

Denote the category of all completely distributive lattices and all the complete lattice homomorphisms among them by **CDL**.

3.9 Theorem Let L be a fuzzy lattice. Then the following conditions are equivalent:

- (i) $1 \in M(L)$.
- (ii) There exist general mappings

$$l: Ob(\mathbf{CDL}) \rightarrow Ob(\mathbf{Card}), f: Ob(\mathbf{Card})^3 \rightarrow Ob(\mathbf{Card})$$

such that for every $\kappa \geq \omega$, every family $\{(L^x, \delta_t): t \in T\}$ of L -fts' with property $dn(\delta_t) \leq \kappa$ for every $t \in T$, and their L -fuzzy product space (L^X, δ) , $dn(\delta) \leq f(\kappa, |T|, l(L))$.

3.10 Remark The preceding theorem is interesting. It tells us, without some certain lattice theoretical property of the range, it is even impossible to find any mathematical result – no matter what method will be used – on some relations, although they all hold in the crisp case.

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Some Considerations on Convergence of Filters in A Lattice – Ordered Group*

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Abstract In this paper we give a general definition of filters, and discuss the f – convergence and f – convergence structure.

Keywords lattice ordered group, R – filter, f – convergence

1. Introduction and Notations

It is well known that the Fuzzy Control and Network of Neurons play important roles in Artificial Intelligence and Intelligent Control. The Control procedure is often represented by language rules that different from the classical control. The classical stability theory is useless here. The stability of intelligent control is still an open problem. The key to discuss this problem is how to characterize and analysis the control procedure by an appropriate mathematical tool. For this reason, we introduce the analytical theory based on l^* – module. This paper continues the work in the papers[1, 2]. The convergence structure on a lattice, especially on a lattice – order group, is a very interesting goal to study. Many people devoted to do this work and got some important results. There are some authors worthy to be mentioned. As is nearly always true in the field of lattice ordered groups, the research about l – convergence follows a path first trod by P. F. Conrad^[8] and W. C. Holland^[11]. The contributions of F. Papangelou^[4, 5] and G. O. Kenny^[6] have also been important. Some authors^[7, 9] developed the topological convergence on a lattice group while other authors^[3, 10] employed the techniques of order – convergence and Cauchy strucure for lattice ordered groups. As we know that filter plays a crucial role in Set Theory, Lattice Theory and Logic. In this paper we give a general definition of filters, and discuss the f – convergence and its Cauchy convergence structure. At first we give some notations. Throughout this paper, G is a lattice group and L is a lattice. For a subset A of G (or L). Let Φ and Ψ be collections of subsets of G (or L). For subsets A and B of Φ , we define

$$A \vee B = \{x \vee y, x \in A \text{ and } y \in B\},$$

$$A \wedge B = \{x \wedge y, x \in A \text{ and } y \in B\},$$

$$AB = \{xy, x \in A \text{ and } y \in B\} \text{ and } A^{-1} = \{x^{-1}, x \in A\}.$$

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