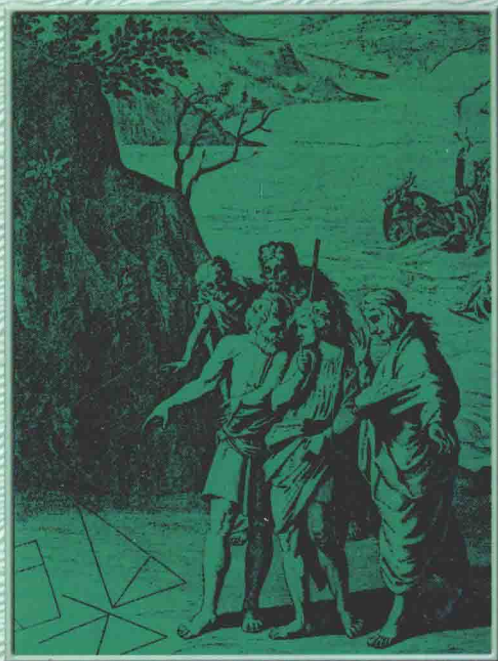


欧美初等数学经典系列(第一辑)

幻方和魔方

W.S. Andrew 著 刘培杰数学工作室(注)



● 富兰克林幻方

● 幻方的数学研究

● 六阶幻方

● 诸多幻方类型

● 多种构造方法

● 幻圆

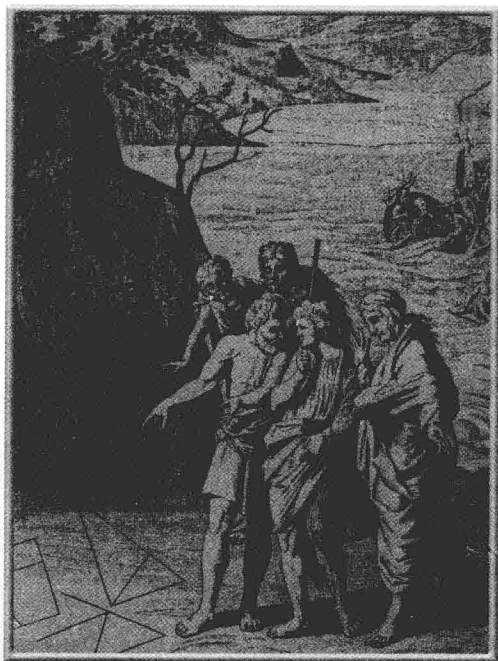


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内容简介

本书列举诸多幻方和魔方的例子,研究幻方和魔方所具备的特性及构筑方法,生动地展示幻方和魔方的神奇之处。主要包括幻方的数学研究,六阶幻方,幻方类型,构造方法,幻圆等十五章内容。适合在校学生的学习研究,以及幻方和魔方爱好者作为兴趣读物。

图书在版编目(CIP)数据

幻方和魔方/(美)安德鲁斯著;刘培杰数学工作室注. —哈尔滨:哈尔滨工业大学出版社,2012.5

ISBN 978 - 7 - 5603 - 3576 - 6

I. ①幻… II. ①安… ②刘… III. ①组合数学 - 青年读物②组合数学 - 少年读物 IV. ①O157 - 49

中国版本图书馆 CIP 数据核字(2012)第 070987 号

策划编辑 刘培杰 张永芹

责任编辑 陈 洁

封面设计 孙茵艾

出版发行 哈尔滨工业大学出版社

社 址 哈尔滨市南岗区复华四道街 10 号 邮编 150006

传 真 0451 - 86414749

网 址 <http://hitpress.hit.edu.cn>

印 刷 哈尔滨市石桥印务有限公司

开 本 787mm×960mm 1/16 印张 34 字数 375 千字

版 次 2012 年 5 月第 1 版 2012 年 5 月第 1 次印刷

书 号 ISBN 978 - 7 - 5603 - 3576 - 6

定 价 68.00 元

(如因印装质量问题影响阅读,我社负责调换)



PUBLISHERS' PREFACE

The essays which comprise this volume appeared first in *The Monist* at different times during the years 1905 to 1916, and under different circumstances. Some of the diagrams were photographed from the authors' drawings, others were set in type, and different authors have presented the results of their labors in different styles. In compiling all these in book form the original presentation has been largely preserved, and in this way uniformity has been sacrificed to some extent. Clarity of presentation was deemed the main thing, and so it happens that elegance of typographical appearance has been considered of secondary importance. Since mathematical readers will care mainly for the thoughts presented, we hope they will overlook the typographical shortcomings. The first edition contained only the first eight chapters, and these have now been carefully revised. The book has been doubled in volume through the interest aroused by the first edition in mathematical minds who have contributed their labors to the solution of problems along the same line.

In conclusion we wish to call attention to the title vignette which is an ancient Tibetan magic square borne on the back of the cosmic tortoise.



INTRODUCTION

The peculiar interest of magic squares and all *lusus numerorum* in general lies in the fact that they possess the charm of mystery. They appear to betray some hidden intelligence which by a preconceived plan produces the impression of intentional design, a phenomenon which finds its close analogue in nature.

Although magic squares have no immediate practical use, they have always exercised a great influence upon thinking people. It seems to me that they contain a lesson of great value in being a palpable instance of the symmetry of mathematics, throwing thereby a clear light upon the order that pervades the universe wherever we turn, in the infinitesimally small interrelations of atoms as well as in the immeasurable domain of the starry heavens, and order which, although of a different kind and still more intricate, is also traceable in the development of organized life, and even in the complex domain of human action.

Pythagoras says that number is the origin of all things, and certainly the law of number is the key that unlocks the secrets of the universe. But the law of number possesses an immanent order, which is at first sight mystifying, but on a more intimate acquaintance we easily understand it to be intrinsically necessary; and this law of

number explains the wondrous consistency of the laws of nature. Magic squares are conspicuous instances of the intrinsic harmony of number, and so they will serve as an interpreter of the cosmic order that dominates all existence. Though they are a mere intellectual play they not only illustrate the nature of mathematics, but also, incidentally, the nature of existence dominated by mathematical regularity.

In arithmetic we create a universe of figures by the process of counting; in geometry we create another universe by drawing lines in the abstract field of imagination, laying down definite directions; in algebra we produce magnitudes of a still more abstract nature, expressed by letters. In all these cases the first step producing the general conditions in which we move, lays down the rule to which all further steps are subject, and so every one of these universes is dominated by a consistency, producing a wonderful symmetry.

There is no science that teaches the harmonies of nature more clearly than mathematics, and the magic squares are like a mirror which reflects the symmetry of the divine norm immanent in all things, in the immeasurable immensity of the cosmos and in the construction of the atom not less than in the mysterious depths of the human mind.

PAUL CARUS



哈尔滨工业大学出版社刘培杰数学工作室

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CHAPTER I

MAGIC SQUARES

The study of magic squares (幻方) probably dates back to prehistoric (史前的) times. Examples have been found in Chinese literature written about A. D. 1125^① which were evidently copied from still older documents. It is recorded that as early as the ninth century magic squares were used by Arabian astrologers in their calculations of horoscopes etc. Hence the probable origin of the term "magic" which has survived to the present day.

THE ESSENTIAL CHARACTERISTICS
OF MAGIC SQUARES

A magic square consists of a series of numbers so arranged in a square, that the sum of each row and column and of both the corner diagonals (对角线) shall be the same amount which may be termed (命名) the *summation* (*S*). Any square arrangement of numbers that fulfils these conditions may properly be called a magic square. Various features may be added to such a square which may enhance its value as a mathematical curio, but these must be considered non-essentials.

① See page 19 of *Chinese Philosophy* by Paul Carus.

There are thus many different kinds of magic squares, but this chapter will be devoted principally to the description of *associated* or *regular* magic squares, in which the sum of any two numbers that are located in cells diametrically(直径地) equidistant(等距的) from the center of the square equals the sum of the first and last terms of the series, or $n^2 + 1$.

Magic squares with an odd number of cells are usually constructed by methods which differ from those governing the construction of squares having an even number of cells, so these two classes will be considered under separate headings.

ASSOCIATED OR REGULAR MAGIC SQUARES OF ODD NUMBERS

The square of 3×3 shown in Fig. 1 covers the smallest aggregation(集合) of numbers that is capable of magic square arrangement, and it is also the only possible arrangement of nine different numbers, relatively to each other, which fulfils the required conditions. It will be seen that the sum of each of the three vertical(垂直的), the three horizontal(水平的), and the two corner diagonal columns in this square is 15, making in all eight columns having that total; also that the sum of any two opposite numbers is 10, which is twice the center number, or $n^2 + 1$.

The next largest odd magic square is that of 5×5 , and there are a great many different arrangements(布置) of twenty-five numbers, which will show magic results,