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计算机程序设计艺术

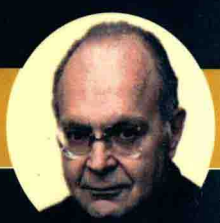
第3卷 排序和查找

The Art of Computer Programming

Volume 3: Sorting and Searching, Second Edition

（英文版·第2版）

（美）Donald E. Knuth 斯坦福大学 著



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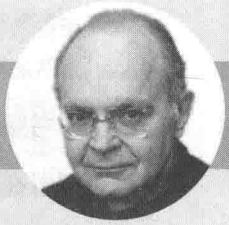

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本书版权登记号：图字：01-2007-2440

图书在版编目（CIP）数据

计算机程序设计艺术　第3卷：排序和查找（英文版·第2版）/（美）克努特（Knuth, D. E.）著. -北京：机械工业出版社，2008.1

书名原文：The Art of Computer Programming, Volume 3: Sorting and Searching, Second Edition

ISBN 978-7-111-22717-5

Ⅰ. 计…　Ⅱ. 克…　Ⅲ. 程序设计-英文　Ⅳ. TP311.1

中国版本图书馆CIP数据核字（2007）第171849号

机械工业出版社（北京市西城区百万庄大街22号　邮政编码　100037）

责任编辑：迟振春

北京瑞德印刷有限公司印刷·新华书店北京发行所发行

2008年1月第1版第1次印刷

170mm×242mm · 50.5印张（含0.5印张插页）

定价：109.00元

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出版者的话

文艺复兴以降，源远流长的科学精神和逐步形成的学术规范，使西方国家在自然科学的各个领域取得了垄断性的优势；也正是这样的传统，使美国在信息技术发展的六十多年间名家辈出、独领风骚。在商业化的进程中，美国的产业界与教育界越来越紧密地结合，计算机学科中的许多泰山北斗同时身处科研和教学的最前线，由此而产生的经典科学著作，不仅擘划了研究的范畴，还揭橥了学术的源变，既遵循学术规范，又自有学者个性，其价值并不会因年月的流逝而减退。

近年，在全球信息化大潮的推动下，我国的计算机产业发展迅猛，对专业人才的需求日益迫切。这对计算机教育界和出版界都既是机遇，也是挑战；而专业教材的建设在教育战略上显得举足轻重。在我国信息技术发展时间较短、从业人员较少的现状下，美国等发达国家在其计算机科学发展的几十年间积淀的经典教材仍有许多值得借鉴之处。因此，引进一批国外优秀计算机教材将对我国计算机教育事业的发展起积极的推动作用，也是与世界接轨、建设真正的世界一流大学的必由之路。

机械工业出版社华章图文信息有限公司较早意识到“出版要为教育服务”。自1998年开始，华章公司就将工作重点放在了遴选、移译国外优秀教材上。经过几年的不懈努力，我们与Prentice Hall，Addison-Wesley，McGraw-Hill，Morgan Kaufmann等世界著名出版公司建立了良好的合作关系，从它们现有的数百种教材中甄选出Tanenbaum，Stroustrup，Kernighan，Jim Gray等大师名家的一批经典作品，以“计算机科学丛书”为总称出版，供读者学习、研究及庋藏。大理石纹理的封面，也正体现了这套丛书的品位和格调。

“计算机科学丛书”的出版工作得到了国内外学者的鼎力襄助，国内的专家不仅提供了中肯的选题指导，还不辞劳苦地担任了翻译和审校的工作；而原书的作者也相当关注其作品在中国的传播，有的还专程为其书的中译本作序。迄今，“计算机科学丛书”已经出版了近260个品种，这些书籍在读者中树立了良好的口碑，并被许多高校采用为正式教材和参考书籍，为进一步推广与发展打下了坚实的基础。

随着学科建设的初步完善和教材改革的逐渐深化，教育界对国外计算机教材的需求和应用都步入一个新的阶段。为此，华章公司将加大引进教材的力度，除“计算机科学丛书”之外，对影印版的教材，则单独开辟出“经典原版书库”。为了保证这两套丛书的权威性，同时也为了更好地为学校和老师们服务，华章

公司聘请了中国科学院、北京大学、清华大学、国防科技大学、复旦大学、上海交通大学、南京大学、浙江大学、中国科技大学、哈尔滨工业大学、西安交通大学、中国人民大学、北京航空航天大学、北京邮电大学、中山大学、解放军理工大学、郑州大学、湖北工学院、中国国家信息安全测评认证中心等国内重点大学和科研机构在计算机的各个领域的著名学者组成“专家指导委员会”，为我们提供选题意见和出版监督。

这两套丛书是响应教育部提出的使用外版教材的号召，为国内高校的计算机及相关专业的教学度身订造的。其中许多教材均已为M. I. T.，Stanford，U.C. Berkeley，C. M. U. 等世界名牌大学所采用。不仅涵盖了程序设计、数据结构、操作系统、计算机体系结构、数据库、编译原理、软件工程、图形学、通信与网络、离散数学等国内大学计算机专业普遍开设的核心课程，而且各具特色——有的出自语言设计者之手、有的历经三十年而不衰、有的已被全世界的几百所高校采用。在这些圆熟通博的名师大作的指引之下，读者必将在计算机科学的宫殿中由登堂而入室。

权威的作者、经典的教材、一流的译者、严格的审校、精细的编辑，这些因素使我们的图书有了质量的保证，但我们的目标是尽善尽美，而反馈的意见正是我们达到这一终极目标的重要帮助。教材的出版只是我们的后续服务的起点。华章公司欢迎老师和读者对我们的工作提出建议或给予指正，我们的联系方法如下：

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（按姓氏笔画顺序）

PREFACE

Cookery is become an art,
a noble science;
cooks are gentlemen.

— TITUS LIVIUS, *Ab Urbe Condita* XXXIX.vi
(Robert Burton, *Anatomy of Melancholy* 1.2.2.2)

THIS BOOK forms a natural sequel to the material on information structures in Chapter 2 of Volume 1, because it adds the concept of linearly ordered data to the other basic structural ideas.

The title "Sorting and Searching" may sound as if this book is only for those systems programmers who are concerned with the preparation of general-purpose sorting routines or applications to information retrieval. But in fact the area of sorting and searching provides an ideal framework for discussing a wide variety of important general issues:

- How are good algorithms discovered?
- How can given algorithms and programs be improved?
- How can the efficiency of algorithms be analyzed mathematically?
- How can a person choose rationally between different algorithms for the same task?
- In what senses can algorithms be proved "best possible"?
- How does the theory of computing interact with practical considerations?
- How can external memories like tapes, drums, or disks be used efficiently with large databases?

Indeed, I believe that virtually *every* important aspect of programming arises somewhere in the context of sorting or searching!

This volume comprises Chapters 5 and 6 of the complete series. Chapter 5 is concerned with sorting into order; this is a large subject that has been divided chiefly into two parts, internal sorting and external sorting. There also are supplementary sections, which develop auxiliary theories about permutations (Section 5.1) and about optimum techniques for sorting (Section 5.3). Chapter 6 deals with the problem of searching for specified items in tables or files; this is subdivided into methods that search sequentially, or by comparison of keys, or by digital properties, or by hashing, and then the more difficult problem of secondary key retrieval is considered. There is a surprising amount of interplay

between both chapters, with strong analogies tying the topics together. Two important varieties of information structures are also discussed, in addition to those considered in Chapter 2, namely priority queues (Section 5.2.3) and linear lists represented as balanced trees (Section 6.2.3).

Like Volumes 1 and 2, this book includes a lot of material that does not appear in other publications. Many people have kindly written to me about their ideas, or spoken to me about them, and I hope that I have not distorted the material too badly when I have presented it in my own words.

I have not had time to search the patent literature systematically; indeed, I decry the current tendency to seek patents on algorithms (see Section 5.4.5). If somebody sends me a copy of a relevant patent not presently cited in this book, I will dutifully refer to it in future editions. However, I want to encourage people to continue the centuries-old mathematical tradition of putting newly discovered algorithms into the public domain. There are better ways to earn a living than to prevent other people from making use of one's contributions to computer science.

Before I retired from teaching, I used this book as a text for a student's second course in data structures, at the junior-to-graduate level, omitting most of the mathematical material. I also used the mathematical portions of this book as the basis for graduate-level courses in the analysis of algorithms, emphasizing especially Sections 5.1, 5.2.2, 6.3, and 6.4. A graduate-level course on concrete computational complexity could also be based on Sections 5.3, and 5.4.4, together with Sections 4.3.3, 4.6.3, and 4.6.4 of Volume 2.

For the most part this book is self-contained, except for occasional discussions relating to the MIX computer explained in Volume 1. Appendix B contains a summary of the mathematical notations used, some of which are a little different from those found in traditional mathematics books.

Preface to the Second Edition

This new edition matches the third editions of Volumes 1 and 2, in which I have been able to celebrate the completion of TEX and METAFONT by applying those systems to the publications they were designed for.

The conversion to electronic format has given me the opportunity to go over every word of the text and every punctuation mark. I've tried to retain the youthful exuberance of my original sentences while perhaps adding some more mature judgment. Dozens of new exercises have been added; dozens of old exercises have been given new and improved answers. Changes appear everywhere, but most significantly in Sections 5.1.4 (about permutations and tableaux), 5.3 (about optimum sorting), 5.4.9 (about disk sorting), 6.2.2 (about entropy), 6.4 (about universal hashing), and 6.5 (about multidimensional trees and tries).


The Art of Computer Programming is, however, still a work in progress. Research on sorting and searching continues to grow at a phenomenal rate. Therefore some parts of this book are headed by an "under construction" icon, to apologize for the fact that the material is not up-to-date. For example, if I were teaching an undergraduate class on data structures today, I would surely discuss randomized structures such as treaps at some length; but at present, I am only able to cite the principal papers on the subject, and to announce plans for a future Section 6.2.5 (see page 478). My files are bursting with important material that I plan to include in the final, glorious, third edition of Volume 3, perhaps 17 years from now. But I must finish Volumes 4 and 5 first, and I do not want to delay their publication any more than absolutely necessary.

I am enormously grateful to the many hundreds of people who have helped me to gather and refine this material during the past 35 years. Most of the hard work of preparing the new edition was accomplished by Phyllis Winkler (who put the text of the first edition into TEX form), by Silvio Levy (who edited it extensively and helped to prepare several dozen illustrations), and by Jeffrey Oldham (who converted more than 250 of the original illustrations to METAPOST format). The production staff at Addison–Wesley has also been extremely helpful, as usual.

I have corrected every error that alert readers detected in the first edition — as well as some mistakes that, alas, nobody noticed — and I have tried to avoid introducing new errors in the new material. However, I suppose some defects still remain, and I want to fix them as soon as possible. Therefore I will cheerfully pay $2.56 to the first finder of each technical, typographical, or historical error. The webpage cited on page iv contains a current listing of all corrections that have been reported to me.

Stanford, California D. E. K.
February 1998

There are certain common Privileges of a Writer,
the Benefit whereof, I hope, there will be no Reason to doubt;
Particularly, that where I am not understood, it shall be concluded,
that something very useful and profound is coucht underneath.

— JONATHAN SWIFT, *Tale of a Tub*, Preface (1704)

NOTES ON THE EXERCISES

THE EXERCISES in this set of books have been designed for self-study as well as classroom study. It is difficult, if not impossible, for anyone to learn a subject purely by reading about it, without applying the information to specific problems and thereby being encouraged to think about what has been read. Furthermore, we all learn best the things that we have discovered for ourselves. Therefore the exercises form a major part of this work; a definite attempt has been made to keep them as informative as possible and to select problems that are enjoyable as well as instructive.

In many books, easy exercises are found mixed randomly among extremely difficult ones. This is sometimes unfortunate because readers like to know in advance how long a problem ought to take — otherwise they may just skip over all the problems. A classic example of such a situation is the book *Dynamic Programming* by Richard Bellman; this is an important, pioneering work in which a group of problems is collected together at the end of some chapters under the heading "Exercises and Research Problems," with extremely trivial questions appearing in the midst of deep, unsolved problems. It is rumored that someone once asked Dr. Bellman how to tell the exercises apart from the research problems, and he replied, "If you can solve it, it is an exercise; otherwise it's a research problem."

Good arguments can be made for including both research problems and very easy exercises in a book of this kind; therefore, to save the reader from the possible dilemma of determining which are which, *rating numbers* have been provided to indicate the level of difficulty. These numbers have the following general significance:

Rating *Interpretation*

00 An extremely easy exercise that can be answered immediately if the material of the text has been understood; such an exercise can almost always be worked "in your head."

10 A simple problem that makes you think over the material just read, but is by no means difficult. You should be able to do this in one minute at most; pencil and paper may be useful in obtaining the solution.

20 An average problem that tests basic understanding of the text material, but you may need about fifteen or twenty minutes to answer it completely.

30 A problem of moderate difficulty and/or complexity; this one may involve more than two hours' work to solve satisfactorily, or even more if the TV is on.

40 Quite a difficult or lengthy problem that would be suitable for a term project in classroom situations. A student should be able to solve the problem in a reasonable amount of time, but the solution is not trivial.

50 A research problem that has not yet been solved satisfactorily, as far as the author knew at the time of writing, although many people have tried. If you have found an answer to such a problem, you ought to write it up for publication; furthermore, the author of this book would appreciate hearing about the solution as soon as possible (provided that it is correct).

By interpolation in this "logarithmic" scale, the significance of other rating numbers becomes clear. For example, a rating of *17* would indicate an exercise that is a bit simpler than average. Problems with a rating of *50* that are subsequently solved by some reader may appear with a *45* rating in later editions of the book, and in the errata posted on the Internet (see page iv).

The remainder of the rating number divided by 5 indicates the amount of detailed work required. Thus, an exercise rated *24* may take longer to solve than an exercise that is rated *25*, but the latter will require more creativity.

The author has tried earnestly to assign accurate rating numbers, but it is difficult for the person who makes up a problem to know just how formidable it will be for someone else to find a solution; and everyone has more aptitude for certain types of problems than for others. It is hoped that the rating numbers represent a good guess at the level of difficulty, but they should be taken as general guidelines, not as absolute indicators.

This book has been written for readers with varying degrees of mathematical training and sophistication; as a result, some of the exercises are intended only for the use of more mathematically inclined readers. The rating is preceded by an *M* if the exercise involves mathematical concepts or motivation to a greater extent than necessary for someone who is primarily interested only in programming the algorithms themselves. An exercise is marked with the letters "*HM*" if its solution necessarily involves a knowledge of calculus or other higher mathematics not developed in this book. An "*HM*" designation does *not* necessarily imply difficulty.

Some exercises are preceded by an arrowhead, "▸"; this designates problems that are especially instructive and especially recommended. Of course, no reader/student is expected to work *all* of the exercises, so those that seem to be the most valuable have been singled out. (This is not meant to detract from the other exercises!) Each reader should at least make an attempt to solve all of the problems whose rating is *10* or less; and the arrows may help to indicate which of the problems with a higher rating should be given priority.

Solutions to most of the exercises appear in the answer section. Please use them wisely; do not turn to the answer until you have made a genuine effort to

solve the problem by yourself, or unless you absolutely do not have time to work this particular problem. *After* getting your own solution or giving the problem a decent try, you may find the answer instructive and helpful. The solution given will often be quite short, and it will sketch the details under the assumption that you have earnestly tried to solve it by your own means first. Sometimes the solution gives less information than was asked; often it gives more. It is quite possible that you may have a better answer than the one published here, or you may have found an error in the published solution; in such a case, the author will be pleased to know the details. Later editions of this book will give the improved solutions together with the solver's name where appropriate.

When working an exercise you may generally use the answers to previous exercises, unless specifically forbidden from doing so. The rating numbers have been assigned with this in mind; thus it is possible for exercise $n + 1$ to have a lower rating than exercise n, even though it includes the result of exercise n as a special case.

Summary of codes:			
		00	Immediate
		10	Simple (one minute)
		20	Medium (quarter hour)
▸	Recommended	*30*	Moderately hard
M	Mathematically oriented	*40*	Term project
HM	Requiring "higher math"	*50*	Research problem

EXERCISES

▸ **1.** [*00*] What does the rating "*M20*" mean?

2. [*10*] Of what value can the exercises in a textbook be to the reader?

3. [*HM45*] Prove that when n is an integer, $n > 2$, the equation $x^n + y^n = z^n$ has no solution in positive integers x, y, z.

Two hours' daily exercise ... will be enough to keep a hack fit for his work.

— M. H. MAHON, *The Handy Horse Book* (1865)

Character code:

00	01	02	03	04	05	06	07	08	09	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
␣	A	B	C	D	E	F	G	H	I	Δ	J	K	L	M	N	O	P	Q	R	Σ	Π	S	T	U

00	*1*	**01**	*2*	**02**	*2*	**03**	*10*

No operation
`NOP(0)`
		rA ← rA + V
		`ADD(0:5)`
		`FADD(6)`
				rA ← rA − V
				`SUB(0:5)`
				`FSUB(6)`
						rAX ← rA × V
						`MUL(0:5)`
						`FMUL(6)`	

08	*2*	**09**	*2*	**10**	*2*	**11**	*2*
rA ← V `LDA(0:5)`		rI1 ← V `LD1(0:5)`		rI2 ← V `LD2(0:5)`		rI3 ← V `LD3(0:5)`	
16	*2*	**17**	*2*	**18**	*2*	**19**	*2*
rA ← −V `LDAN(0:5)`		rI1 ← −V `LD1N(0:5)`		rI2 ← −V `LD2N(0:5)`		rI3 ← −V `LD3N(0:5)`	
24	*2*	**25**	*2*	**26**	*2*	**27**	*2*
M(F) ← rA `STA(0:5)`		M(F) ← rI1 `ST1(0:5)`		M(F) ← rI2 `ST2(0:5)`		M(F) ← rI3 `ST3(0:5)`	
32	*2*	**33**	*2*	**34**	*1*	**35**	$1+T$
M(F) ← rJ `STJ(0:2)`		M(F) ← 0 `STZ(0:5)`		Unit F busy? `JBUS(0)`		Control, unit F `IOC(0)`	
40	*1*	**41**	*1*	**42**	*1*	**43**	*1*
rA : 0, jump `JA[+]`		rI1 : 0, jump `J1[+]`		rI2 : 0, jump `J2[+]`		rI3 : 0, jump `J3[+]`	
48	*1*	**49**	*1*	**50**	*1*	**51**	*1*

rA ← [rA]? ± M
`INCA(0) DECA(1)`
`ENTA(2) ENNA(3)`
		rI1 ← [rI1]? ± M
		`INC1(0) DEC1(1)`
		`ENT1(2) ENN1(3)`
				rI2 ← [rI2]? ± M
				`INC2(0) DEC2(1)`
				`ENT2(2) ENN2(3)`
						rI3 ← [rI3]? ± M
						`INC3(0) DEC3(1)`
						`ENT3(2) ENN3(3)`	

56	*2*	**57**	*2*	**58**	*2*	**59**	*2*

CI ← rA(F) : V
`CMPA(0:5)`
`FCMP(6)`
		CI ← rI1(F) : V
		`CMP1(0:5)`
				CI ← rI2(F) : V
				`CMP2(0:5)`
						CI ← rI3(F) : V
						`CMP3(0:5)`	

General form:

C	*t*
Description `OP(F)`	

C = operation code, (5 : 5) field of instruction
F = op variant, (4 : 4) field of instruction
M = address of instruction after indexing
V = M(F) = contents of F field of location M
`OP` = symbolic name for operation
`(F)` = normal F setting
t = execution time; T = interlock time

25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55
V	W	X	Y	Z	0	1	2	3	4	5	6	7	8	9	.	,	(	)	+	-	*	/	=	$	<	>	@	;	:	'

04	*12*	**05**	*10*	**06**	*2*	**07**	*1 + 2F*

rA ← rAX/V
rX ← remainder
`DIV(0:5)`
`FDIV(6)`
		Special
		`NUM(0)`
		`CHAR(1)`
		`HLT(2)`
				Shift M bytes
				`SLA(0)` `SRA(1)`
				`SLAX(2)` `SRAX(3)`
				`SLC(4)` `SRC(5)`
						Move F words from M to rI1
						`MOVE(1)`	

12	*2*	**13**	*2*	**14**	*2*	**15**	*2*
rI4 ← V `LD4(0:5)`		rI5 ← V `LD5(0:5)`		rI6 ← V `LD6(0:5)`		rX ← V `LDX(0:5)`	
20	*2*	**21**	*2*	**22**	*2*	**23**	*2*
rI4 ← −V `LD4N(0:5)`		rI5 ← −V `LD5N(0:5)`		rI6 ← −V `LD6N(0:5)`		rX ← −V `LDXN(0:5)`	
28	*2*	**29**	*2*	**30**	*2*	**31**	*2*
M(F) ← rI4 `ST4(0:5)`		M(F) ← rI5 `ST5(0:5)`		M(F) ← rI6 `ST6(0:5)`		M(F) ← rX `STX(0:5)`	
36	*1 + T*	**37**	*1 + T*	**38**	*1*	**39**	*1*

Input, unit F
`IN(0)`
		Output, unit F
		`OUT(0)`
				Unit F ready?
				`JRED(0)`
						Jumps
						`JMP(0)` `JSJ(1)`
						`JOV(2)` `JNOV(3)`
						also `[*]` below	

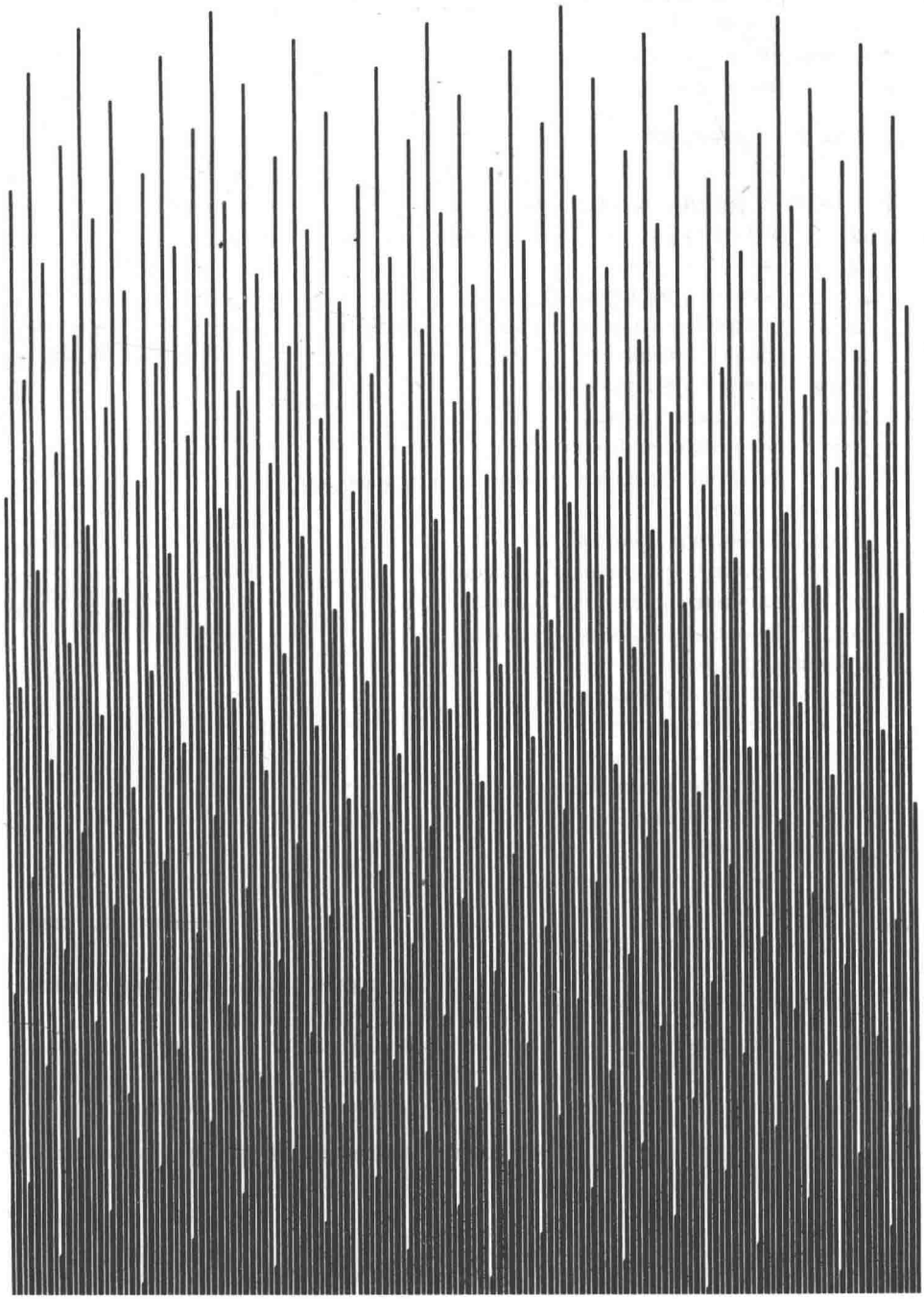
44	*1*	**45**	*1*	**46**	*1*	**47**	*1*
rI4 : 0, jump `J4[+]`		rI5 : 0, jump `J5[+]`		rI6 : 0, jump `J6[+]`		rX : 0, jump `JX[+]`	
52	*1*	**53**	*1*	**54**	*1*	**55**	*1*

rI4 ← [rI4]? ± M
`INC4(0)` `DEC4(1)`
`ENT4(2)` `ENN4(3)`
		rI5 ← [rI5]? ± M
		`INC5(0)` `DEC5(1)`
		`ENT5(2)` `ENN5(3)`
				rI6 ← [rI6]? ± M
				`INC6(0)` `DEC6(1)`
				`ENT6(2)` `ENN6(3)`
						rX ← [rX]? ± M
						`INCX(0)` `DECX(1)`
						`ENTX(2)` `ENNX(3)`	

60	*2*	**61**	*2*	**62**	*2*	**63**	*2*
CI ← rI4(F) : V `CMP4(0:5)`		CI ← rI5(F) : V `CMP5(0:5)`		CI ← rI6(F) : V `CMP6(0:5)`		CI ← rX(F) : V `CMPX(0:5)`	

rA = register A
rX = register X
rAX = registers A and X as one
rIi = index register i, $1 \le i \le 6$
rJ = register J
CI = comparison indicator

`[*]:`		`[+]:`
`JL(4)`	$<$	`N(0)`
`JE(5)`	$=$	`Z(1)`
`JG(6)`	$>$	`P(2)`
`JGE(7)`	$\ge$	`NN(3)`
`JNE(8)`	$\neq$	`NZ(4)`
`JLE(9)`	$\le$	`NP(5)`



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