丘成桐大学生数学竞赛

Questions and Solutions of S.-T. Yau College Student Mathematics Contests (2010—2013)

丘成桐大学生数学竞赛试题及解答

(2010—2013)

丘成桐 主编

高等教育出版社

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出版前言

为促进我国数学教育的发展, 提高数学本科生的专业水平, 我们从 2010 年开始举办大学生数学竞赛. 这项竞赛的主要目的是使我们本科生的数学知识面更广, 基础打得更牢, 进一步缩小其与国外一流院校优秀本科生之间的差距. 与此同时, 通过竞赛的科目大纲及参考书建议, 使有志者可以通过自学学好学校里无法开设的数学基础课, 以弥补其知识缺陷. 这对普通院校的学生来说尤其重要, 它可以帮助缩小普通院校与传统强校之间的差距, 从而使大家共同提高, 也使竞争平台更宽广、更公平.

竞赛每年一度于夏天举行. 所有大陆、台湾和香港地区的在读本科生都可以报名参加. 竞赛分五个科目: 代数与数论; 分析与微分方程; 应用数学与计算数学; 几何与拓扑; 概率与统计. 每个科目有个人赛和团体赛两套试题. 具体安排和规则可见竞赛网页 `http://www.cms.zju.edu.cn//conference/YCMC/rules-c.html`

为方便读者, 我们在本书中也列出了各科目的大纲和参考书目, 以及各竞赛委员会的成员名单.

竞赛的试题主要取材于美国各高校数学系的研究生资格考试, 也结合一些竞赛委员会成员们收集或自编的问题. 与中学数学奥林匹克竞赛或美国大学生的 Putnam 竞赛不同, 我们这个竞赛主要是测试学生对本科数学基础知识的全面掌握程度, 而不是在一些比较初等的内容上考拐弯抹角的小技巧. 目前我国的数学研究生入学考试过分注重于大一的课程, 如数学分析、线性代数, 而一些有志于出国深造的学生则因忙于准备 GRE, TOFEL 考试而忽略掉一些高年级数学课程的学习. 这些都非常不利于对基础知识的学习和训练. 我们创办这个竞赛的目的之一也是希望能对纠正这种片面现象起到一定的帮助作用, 进一步增强本科数学教育水平.

在本书中, 我们收集整理了 2010 年至 2013 年的试题和解答. 我们希望读者从这些试题里能进一步了解本竞赛的范围和风格, 也希望更多的大陆、台湾和香港地

区的大学数学本科生能踊跃参加今后的竞赛, 并通过学习更好地掌握本科数学的基础知识.

数学作为现代科学技术的基础, 是一个国家综合实力的重要指标之一. 我们生活在民族复兴的大时代, 肩负着振兴我国数学的使命. 希望我们年轻一代中的有志者能打好基础、奋发图强, 使自己变成能为数学大树添枝加叶的来者, 也为我国早日成为数学强国贡献力量. 你们是中国数学的未来!

丘成桐

2015 年 5 月于杭州

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鸣谢

我们首先要感谢丘成桐大学生数学竞赛的组织和学术委员会的成员们, 正是他们的不懈努力, 使第一届到第五届丘成桐大学生数学竞赛得以顺利举行和圆满成功. 从竞赛的筹备、组织到命题、评审、提供解答, 都离不开他们的辛勤劳动和无私奉献. 竞赛组委会由加州大学洛杉矶分校的刘克峰教授担任主席, 由清华大学的肖杰教授、中国科学技术大学的胡森教授、浙江大学的许洪伟教授、中国科学院的王跃飞教授共同担任副主席. 组织委员会的成员名单请见本书的委员会名单部分.

学术委员会由丘成桐教授和中国科学院的杨乐院士担任主席. 五个分部中, 代数与数论组由普林斯顿大学的张寿武教授、中国科学院的席南华院士、斯坦福大学的李骏教授担任主席; 分析与微分方程组由台湾大学的林长寿教授和中国科学院的王跃飞教授担任主席; 应用数学与计算数学组由布朗大学的舒其望教授和杜克大学的刘建国教授担任主席; 概率与统计组由中国科学院的严加安院士和哈佛大学的刘军教授担任主席; 几何与拓扑组由加州大学洛杉矶分校的刘克峰教授担任主席. 诸位教授在繁忙的科研教学工作中抽出了很多时间和精力, 投入到竞赛的组织工作中去, 在此一并致谢. 各学术委员会成员的名单也列在了本书中.

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竞赛的奖章和奖杯, 第一届是在郭开朗副省长的直接领导下, 由湖南省人民政府出资赞助的, 第二届至第五届则是由贺正需教授个人出资赞助的. 奖章和奖杯是袁蕾女士设计的. 在此我们一并致以诚挚的谢意.

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我们也要感谢参加竞赛的来自两岸三地的众多学生, 正是他们的踊跃参与和良好表现, 使竞赛得到成功. 我们还要感谢所有参与竞赛的大学和研究所, 它们为考试提供了场地和后勤支持, 使竞赛和阅卷得以顺利进行.

在本书的编写过程中, 我们得到了多位教授的大力帮助. 我们这里要特别感谢中国科学院数学所的方明教授, 他帮助给出了 2011 年代数试题的解答. 我们也感谢肖青、沈可美、楼筱静、孙庆有、吴柳峰诸位老师的多方协助, 使本书的编写工作得以顺利完成. 最后, 我们要特别感谢 International Press of Boston 的秦立新先生和邓宇善先生以及高等教育出版社的李鹏先生, 他们为本书的编辑出版付出了大量的时间和精力, 使本书得以面世.

本书编委

2014 年 11 月

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About S.-T. Yau College Student Mathematics Contests

Welcome Letter

In the training of a good mathematician, a crucial step is to have students to know basic subjects and skills in mathematics before they do research. To achieve this major, universities in the world set up qualify exams for beginning graduate students. Good students majoring in mathematics in Chinese universities should be good enough to pass qualifying exams in any major universities in the world, e.g. Harvard, Stanford, Berkeley, Princeton, Columbia, etc. However in US we are facing the problem that some good Chinese students could not pass while good students from other countries could, especially those from Eastern European countries. The College Student Mathematics Contest shall be broad enough to cover the subjects in those qualifying exams and it shall be helpful to train many good students in mathematics for China and for the world.

It should be a fair competition so that students from "second rated" universities in China are given a second chance to compete. This is very important. There are many reasons that a high school student did not get into top universities such as bad luck, poor health, poverty, or immature. But it is often the case that situations changed when they go to college. They become more mature and suddenly they decide to be good. With the present situation, students of such colleges have hard time to go to the best universities to do graduate study. We should give them a chance. After all, when I was educated in Hong Kong, it was difficult for me to go to the first rated universities. It was through my hard work and good luck that I became recognized.

To make sure those top university students to know that there are continuous challenges for their mathematics life so that they have to work hard at a fundamental level. It is very popular to see those students from college in China to talk about philosophy of mathematics while they have no idea on how to sit down to do a mathematics computation, this can be changed most effectively by such a competition.

We shall invite world-class mathematicians to give students oral tests (for the top 15 candidates) and their names will endorse the quality of those students and

it will be helpful for them to go to the best graduate schools in the world. I am sure that many mathematicians would like to meet those brilliant students.

In short I believe that the competition will bring the ability of our students to a much higher level. We welcome college students to participate the College Student Mathematics Contest.

Shing-Tung Yau
Chair of the Scientific and Questions Committee
Chair of Department of Mathematics at Harvard University

Opening Letter

Based on Prof. Shing-Tung Yau's suggestions it is proposed to organize college student mathematics contests. By competing with their peers it would help to enhance undergraduates in their basic college training in mathematics. The examination shall cover a wide range of subjects. With deep and interesting questions this would motivate students to learn. There shall be an outstanding panel to oversee it.

Young talents in China are abundant. The aim of the competition is to help many students in China to acquire basic trainings in modern mathematics. It is a basic step to build up a solid foundation in the developments of mathematics in China. We believe this will bring college mathematics education in China to a higher level and it would help to transform a country of vast population into a country of vast human resources. The test results shall be important references to admitting graduate students both in and outside of China.

Kefeng Liu
Chair of the Competition Organizing Committee

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