

TUP-Springer Project

陈喜群 李力 史其信 著

Xiqun (Michael) Chen · Li Li · Qixin Shi

动态交通流的随机演化 建模与应用

Stochastic Evolutions of Dynamic Traffic Flow

Modeling and Applications



清华大学出版社



Springer



陈喜群 李力 史其信 著

Xiqun (Michael) Chen · Li Li · Qixin Shi

动态交通流的随机演化 建模与应用

Stochastic
Evolutions of
Dynamic Traffic
Flow

Modeling and Applications



清华大学出版社
北京

 Springer

内 容 简 介

随机交通流模型是智能交通系统、交通工程设计、交通管理与控制等领域的应用基础,对丰富现代交通流理论体系具有重要意义。道路交通流具有复杂性、动态性和随机性特征,新一代智能交通系统对交通流理论提出更高要求。本书应用多元异构数据,建立基于车辆轨迹信息的随机交通流模型,揭示交通流复杂动态性和随机演化性的内在机理。本书主要研究内容和成果表现在数据挖掘、微观关联、宏观关联、匝道瓶颈建模等方面。

本书适合交通流和交通大数据领域的相关研究人员和学生参考。

本书封面贴有清华大学出版社防伪标签,无标签者不得销售。

版权所有,侵权必究。侵权举报电话:010-62782989 13701121933

图书在版编目(CIP)数据

动态交通流的随机演化:建模与应用/陈喜群,李力,史其信著.--北京:清华大学出版社,2015

ISBN 978-7-302-37479-4

I. ①动… II. ①陈… ②李… ③史… III. ①交通流—数据模型 IV. ①U491.1

中国版本图书馆 CIP 数据核字(2014)第 170753 号

责任编辑:王一玲

封面设计:傅瑞学

责任印制:沈 露

出版发行:清华大学出版社

网 址: <http://www.tup.com.cn>, <http://www.wqbook.com>

地 址:北京清华大学学研大厦 A 座 邮 编:100084

社 总 机:010-62770175 邮 购:010-62786544

投稿与读者服务:010-62776969, c-service@tup.tsinghua.edu.cn

质量反馈:010-62772015, zhiliang@tup.tsinghua.edu.cn

课件下载: <http://www.tup.com.cn>, 010-62775954

印 装 者:三河市中晟雅豪印务有限公司

经 销:全国新华书店

开 本:155mm×235mm 印 张:13.5 字 数:234 千字

版 次:2015 年 6 月第 1 版 印 次:2015 年 6 月第 1 次印刷

印 数:1~1000

定 价:89.00 元

产品编号:057860-01

To Dandan, Cynthia, Yuexin and Shuhuai
The families of Xiqun (Michael) Chen

Preface

Road traffic flow is intrinsic with stochastic and dynamic characteristics so that traditional deterministic theory no longer satisfies requirements of the evolution analysis. Stochastic traffic flow modeling aims to study relationships of transportation components. The kernel is an investigation of both stochastic characteristics and traffic congestion evolution mechanism using headway, spacing, and velocity distributions. The primary contents include empirical observations, connections with microscopic and macroscopic traffic flow models, and traffic breakdown analysis of highway bottlenecks.

The book first analyzes characteristics of empirical traffic flow measurements to reveal the underlying mechanism of complexity and stochastic evolutions. By using *Eulerian* measurements (e.g., inductive loop data) and *Lagrangian* measurements (e.g., vehicular trajectory data), we study headway-spacing-velocity distributions quantitatively and qualitatively. Meanwhile, disturbances of congested platoons (jam queues) and time-frequency properties of oscillations, which establish the empirical foundation for stochastic traffic flow modeling.

Then we establish a Markov car-following model by incorporating the connection between headway-spacing-velocity distributions and microscopic car-following models using the transition probability matrix to describe random choices of headways/spacings by drivers. Results show that the stochastic model more veritably reflects the dynamic evolution characteristics of traffic flow. As discussions of the connection between headway-spacing-velocity distributions and the macroscopic fundamental diagram model, we analyze the probability densities and probabilistic boundaries of congested flow in flow-density plot by proposing a stochastic extension of Newell's simplified model to study wide scattering features of flow-density points.

For applications to highway on-ramp bottlenecks, a traffic flow breakdown probability model is proposed based on headway/spacing distributions. We reveal the mechanism of transitions from disturbances to traffic congestion, and the phase diagram analysis based on a spatial-temporal queueing model that is beneficial to obtain optimal control strategies to improve the reliability of road traffic flow.

We would like to acknowledge the following people for their contributions in bringing this book to completion. We are grateful to Prof. Meng Li and Prof. Zhiheng Li for their sustaining guidance and encouragement. They share perspectives on dynamic transportation planning and traffic control approaches for the analysis of stochastic traffic phenomena. It has been a privilege for us to work with them. Thanks to Profs. Huapu Lu, Jing Shi, Xinmiao Yang, and Ruimin Li, who provided a number of valuable comments and suggestions that substantially improved this book.

We would like to show our gratitude to Prof. Wei-Bin Zhang, Drs. Liping Zhang, Kun Zhou, and Jing-Quan Li for their kind help in the *FHWA—Advanced Traffic Signal Control Algorithms* project. They studied the dynamic all-read extension and minimized the yellow arrival rate using vehicular trajectories, connected vehicles and eco-driving challenges on urban arterials while at California PATH, University of California at Berkeley. In addition, the book was inspired by the work of Prof. Pravin Varaiya on the exploration of magnetic sensors for travel time analysis, and Prof. Alex Skabardonis's work on PeMS for traditional loop data applications. The book has been advanced by several discussions with Prof. H. Michael Zhang, who pointed toward the deeper understanding of stochastic fundamental diagram and traffic breakdown probability model.

We also thank our former students and friends both in research and in life at Tsinghua University; they are Jin Wang, Xuyan Qin, Zhen Qian, Yunfei Sha, Liguang Sun, Runhua Qian, Jinxin Cao, Chun Xu, Shen Dong, Xinxin Yu, Xiongfei Zhang, Yuan Fang, Xiangqian Chen, Jia Wang, Chenyi Chen, Weijun Xie, Zhaomiao Guo, Yan Yan, Lijun Sun, and all other students in our program. You have provided us support on a number of research projects, brainstorming ideas in seminars or collaboration on articles. We also sincerely appreciate the friendship and generous help from my colleagues in Institute of Transportation Studies, University of California at Berkeley, Weihua Gu, Bo Zou, Yiguang Xuan, Haotian Liu, Xiaofei Hu, Lu Hao, Haoyu Chen, and Yang Yang.

Finally, Dr. Xiqun (Michael) Chen sends special thanks to his parents Yuexin Chen and Shuhuai Guo, his wife Dandan Li, his daughter Cynthia Chen, who will be born in the autumn of 2014, and elder sister Xili Chen. It would not have been possible for him to finish this book without their love and unwavering support.

May 2014

Xiqun (Michael) Chen
Li Li
Qixin Shi

Acronyms

Mathematical Symbols

a, b	Scale and translation parameters of Wavelet Transform
a_{\max}, v_{\max}	Maximal acceleration and maximal speed
$a_n(t)$	Acceleration of the n th vehicle at time t
A	Perturbation matrix
$\{\mathcal{A}_i, i \in \mathbb{N}^+\}$	Mutually exclusive events
b_i	Deceleration of the following vehicle
c_0	Substitution of speed
C_h	Coefficient of variation
$\mathcal{C}, \mathcal{C}', \hat{\mathcal{C}}$	Copula, its density function and empirical copula
d_i	Spacing of full stop
$E(\cdot)$	Average wavelet energy
$E[\cdot], \text{Var}[\cdot], \sqrt{S^2[\cdot]}$	Expectation, variance and standard deviation operators
$f(h \alpha, \beta, h_0)$	PDF of Gamma distribution
$f(h \lambda)$	PDF of negative exponential distribution
$f(h \lambda, h_0)$	PDF of shifted exponential distribution
$f(h \mu_h, \sigma_h, h_0)$	PDF of lognormal distribution
F, F_1, F_2	CDF
$F(h \alpha, \beta, h_0)$	CDF of Gamma distribution
$F(h \lambda)$	CDF of negative exponential distribution
$F(h \lambda, h_0)$	CDF of shifted exponential distribution
$F(h \mu_h, \sigma_h, h_0)$	CDF of lognormal distribution
$\hat{F}, \hat{F}_1, \hat{F}_2$	Empirical CDF
$F_c(q_s)$	CDF of upstream flow q_s , $1 - F_c(q_s)$ is lifetime function
$\hat{F}_c(q_s)$	PLM estimation

$F_{H v}, F_{S v}$	Conditional distribution of headway/spacing with respect to v
$F_{H,\alpha v}^{-1}$	α quantile of headway distribution
\bar{h}	Average headway
h_0, \hat{h}_0	Shift coefficient and its estimate
$h_{0,p}, h_{0,q}$	Shift coefficients of car-following state and free flow state
h_{free}	Critical headway of free flow
h_i	Observation samples of headway, $i = 1, 2, \dots, n$
$h_k(t)$	Headway
h_{\min}	Minimal headway
h_{random}	Random headway from uniform distribution $\mathcal{U}(H_j^-, H_j^+)$
\tilde{h}, \tilde{v}	Mean headway and speed of \tilde{Y}
$\tilde{h}_{n,n-1}, \tilde{v}_n$	Mathematical expectations of headway and speed
$H(\cdot)$	Heaviside function
H_i^-, H_i^+	Upper and lower limits of the i th headway state
H_n	Summation of headway
$H_n^{(1)}, H_n^{(2)}$	Disturbance propagation time to the n th vehicle in ac/deceleration waves
$i = \sqrt{-1}$	Imaginary unit
\mathcal{J}	Population
\mathcal{J}_{ij}	Sample size of that $h_k(t)$ belongs to state i and $h_k(t + \Delta t)$ belongs to state j
\mathcal{J}_k	Number of triple, analogically, $\mathcal{J}_k^*, \mathcal{J}_k^-, \mathcal{J}_k^+, \mathcal{J}_k^\Delta$
k_1, k_2	Weight coefficients of density and speed gradients
L	Circular road length
L_{cong}	Jam queue length
L_{free}	Free flow traffic length
$L_{qs}, L_{\rho,v}$	Likelihood function
\mathcal{L}	Lagrangian function
\mathbb{N}, \mathbb{M}	Finite dimensional state space, satisfying $\mathbb{N} = \{1, 2, \dots, n\}, \mathbb{M} = \{1, 2, \dots, m\}$
\mathbb{N}^+	The set of positive integers
$\mathcal{N}, \text{Log}-\mathcal{N}$	Normal and lognormal distributions
$o(\cdot)$	Infinitesimal of higher order
$p(h)$	PDF of car-following state headway
$p^*(\lambda)$	Laplace transform of $p(h)$
p_{slow}	Slow-to-start probability
\hat{p}_k, \bar{p}	Probability estimate
P_B	Traffic flow breakdown probability
$\tilde{P}_0(v)$	Distribution of speed expectation

$\tilde{P}(v x, t)$	Speed distribution
P_1, P_2	Pressure terms, satisfying $\partial_\rho P_1 \leq 0$ and $\partial_v P_2 \leq 0$
P_{GUE}	Gaussian unitary ensemble, GUE
P_{Poisson}	Poisson distribution
\mathbf{P}	Transition probability matrix, satisfying $\mathbf{P} = (P_{ij})$
\mathbf{P}_l, π_l	Transition probability matrix and stationary distribution of headway in the l th velocity range
$\mathcal{P}(n, t)$	Probability of a jam queue with n vehicles at time t
$\mathcal{P}_{(z, t)}$	Probability of being state z at time t
$\mathcal{P}_S(t)$	Probability of being state S at time t
$q(h)$	PDF of headway in free flow state
$q_e(\rho), v_e(\rho)$	Equilibrium functions of flow and speed
$q_{\text{lower}}, q_{\text{upper}}$	Maximal and minimal traffic flow rates, analogically, $\rho_{\text{lower}}, \rho_{\text{upper}}$
$q_{\text{main}}, q_{\text{ramp}}$	Mainline flow and ramp flow
q_{max}	Maximal flow rate
\bar{q}_S	Breakdown traffic flow rate
r	Iteration times
R	Least squares residual
\mathcal{R}	Response function
$s^A(v), s^D(v)$	Spacings of ac/deceleration curves at speed v
$s_{\text{before}}, s_{\text{after}}$	Spacings when joining and departing from jam queues
$s_{\text{break}}, s_{\text{free}}$	Spacing thresholds of braking and free flow states
s_{cong}	Spacing in a jam queue
$s_{\text{start}}, s_{\text{stop}}$	Spacing thresholds of starting and stopping states
$s_i(t)$	Spacing
$s_{\text{min}}, s_{\text{max}}$	Minimal and maximal critical spacings
$s_{n, n-1}, h_{n, n-1}$	Headway and spacing of the n th vehicle
$s_{\text{stop}}^A, s_{\text{stop}}^D$	Fully stopping spacings of ac/deceleration curves
$s_{\text{upper}}^A, s_{\text{upper}}^D$	Maximal and minimal spacings corresponding to the maximal speed \tilde{v}^+ in metastable traffic flow, analogically, $s_{\text{lower}}^A, s_{\text{lower}}^D$
\mathbb{S}_k	Sample set, analogically, $\mathbb{S}_k^*, \mathbb{S}_k^-, \mathbb{S}_k^+, \mathbb{S}_k^\Delta$
\mathcal{S}	Arbitrary discrete state of traffic flow
S^A, S^D	Probabilistic boundaries of microscopic fundamental diagram
t	Time
$(t_{a,i}^{(k)}, x_{a,i}^{(k)})$	Acceleration point of the i th vehicle, analogically, $(t_{d,i}^{(k)}, x_{d,i}^{(k)})$
T	Entering time interval of ramping vehicles
\bar{T}	Average travel time

T, T^-, T^+	Tau factor
T^*, \bar{T}^*	Tau factor Related statistics
$\hat{T}_k^-, \hat{T}_k^+, \hat{T}_k^\pm$	Estimate of Tau factor
v, u	Velocity
$v(x, t)$	Mean velocity at location x and time t
$v_{\text{cong}}, v_{\text{free}}$	Congestion wave speed and free flow speed
$v_n(t)$	Speed of the n th vehicle at time t
$v_{n,\text{safe}}$	Lower bound of safety speed
$v_{\text{opt}}(x_n(t))$	Optimal speed function
$v + c_\pm$	Characteristic speed
$\bar{v}(x, t)$	Expected speed
$\tilde{v}, \tilde{\rho}$	Perturbation amplitudes of speed and density
w	Congestion wave speed, analogically, $w_i, -w_i, -W_n$
$w(t - k)$	Window function
w^A, w^D	Ac/deceleration congestion wave speeds
\mathcal{W}	Weibull
x	Location
$x_n(t)$	Location of the n th vehicle at time t
$\tilde{X} = (\tilde{\rho}, \tilde{v})^T$	Column vector of perturbation amplitude
$(x, v) \mapsto (y, u)$	Transition rate from state (x, v) to state (y, u) of traffic flow
z, z'	Arbitrary continuous traffic flow states
(Z_1, Z_2)	Bivariate random variables
$(Z_1^{(i)}, Z_2^{(i)})$	Observations of bivariate random variables, $i \in \mathbb{N}$
$\{z_t\}$	Traffic flow discrete time series data, $t = 0, 1, \dots, T - 1$
Z_n	Summation of random variables
$Z_{n,\text{typical}}$	Extreme points of PDF of Z_n
α, β	Parameters of Weibull distribution
$\hat{\alpha}, \hat{\beta}$	Parameter estimations of Weibull distribution
$\gamma_i^S, \gamma_i^B, \gamma_i^E$	Net spacings before, within and after the deceleration of the i th vehicle
Γ	Gamma function
$\delta v, \delta \rho$	Speed and density variations
Δ	Field data measurement interval
Δt	Update time step
$\Delta v_n, \Delta x_n$	Speed and location differences between the n th and the $n - 1$ the vehicles
$\epsilon(x, t)$	Noise function at location x and time t
$\eta(\rho, v)$	Inertial coefficient of driving behaviors
$\eta_{\text{lower}}, \eta_{\text{upper}}$	$\alpha/2$ and $(1 - \alpha/2)$ percentiles of standard normal distribution

θ_1, θ_2	Substitution parameters
$\theta_{\text{OCT}}, \theta_{\text{HCT}}$	Dimensionless critical coefficients
Θ	Indicator function, analogically, $\mathbf{1}(\cdot) \mapsto \{0, 1\}$
$\Theta(x, t)$	States and parameter vector at location x and time t
$\vartheta_i^-, \vartheta_i^+$	Upper and lower bounds of the i th vehicle headway
$\kappa, \tilde{\kappa}$	First vehicle delay after perturbations
$\lambda, \tilde{\lambda}$	Parameters and their estimations
λ_p, λ_q	Parameters of car-following and free flow states
μ, σ	Lognormal distribution parameters, analogically, headway (μ_h, σ_h) , spacing (μ_s, σ_s) , reaction time (μ_τ, σ_τ) , bivariate lognormal distribution $(\mu_{z_1 z_2}, \sigma_{z_1 z_2})$
$\hat{\mu}, \hat{\sigma}$	Lognormal distribution parameter estimation
ξ, ζ	Random variable
$\xi(\rho, v)$	Anticipation coefficient of driving behavior
$\rho(x, t)$	Average density at location x and time t
ρ_0	Steady state density, initial density
ρ_z	Correlation coefficient
$\bar{\rho}(x, t, v)$	Phase space density
$\zeta, \bar{\zeta}$	Vehicle gap and average vehicle gap
τ	Latency time, relaxation time and reaction time
$\tau_{\text{in}}, \tau_{\text{out}}$	Interarrival time and service time, analogically, $\tau_{\text{in},i}, \tau_{\text{in}}^{(k)}, \tilde{\tau}_{\text{out}}, \hat{\tau}_{\text{out}}$
$\tau_{\text{in}}(m)$	Interarrival time summation of m vehicle
τ_i^A, τ_i^D	Reaction time of ac/deceleration
$(\tau^A, w^A s_{\text{stop}}^A)$	Characteristic parameters of acceleration curve, analogically, $(\tau^D, w^D s_{\text{stop}}^D)$
$\tilde{\gamma}, \gamma_k$	Inhomogeneous platoon and homogeneous sub-platoon
$\phi_1, \phi_2, \varphi_1, \varphi_2$	Substitution parameters
$\Phi(\cdot)$	Standard normal distribution
φ	Proportionality coefficient
$\varphi_{FF}, \varphi_{PLC}$	Dimensionless critical coefficient
$\psi(t), \hat{\psi}(f)$	Mother wavelet function and its Fourier transform, the conjugate function is $\hat{\psi}^*(f)$
ω	Digital frequency
$\omega(k), k \in \mathbb{N}^+$	Angular frequency
$\omega_{zz'}$	Transition rate from state z to z'
$\omega_+(n), \omega_-(n)$	Transition rate of jam queue length from $(n \mapsto n+1)$ and $(n \mapsto n-1)$
ω_{\pm}	Complex solution of ω
$\omega_{SS'}$	Transition rate from state S to S'

Ω	Analog frequency
$\ell(\cdot)$	Likelihood function
\Re, \Im	Real part and imaginary part
\emptyset	Empty set

Abbreviations

ACTM	Asymmetric Cell Transmission Model
A-curve	Acceleration Curve
CA	Cellular Automaton
CCTM	Compositional Cell Transmission Model
CDF	Cumulative Distribution Function
CTM	Cell Transmission Model
CWT	Continuous Wavelet Transform
D-curve	Deceleration Curve
DFT	Discrete Fourier Transform
DTA	Dynamic Traffic Assignment
DWT	Discrete Wavelet Transform
EKF	Extended Kalman Filter
ELCTM	Enhanced Lagged CTM
EM	Error Mean
FD	Fundamental Diagram
FF	Free Flow
FHWA	Federal Highway Administration
FT	Fourier Transform
G/D/1	General Determinant 1
G/G/1	General General 1
GKT	Gas-Kinetic-based Traffic Model
GUE	Gaussian Unitary Ensemble
HCT	Homogeneous Congested Traffic
i.i.d.	Independent and Identically Distributed
IDM	Intelligent Driver Model
ITS	Intelligent Transportation System
K-S	Kolmogorov-Smirnov Test
LCTM	Lagged Cell Transmission Model
LPO	Log Periodic Oscillations
LSCTM	Location Specific CTM
LSR	Least Squares Regression
MCTM	Modified Cell Transmission Model
MLC	Moving Local Cluster
NGSIM	Next Generation Simulation
OCT	Oscillated Congested Traffic
OVM	Optimal Velocity Model

PA	Perturbation Analysis
PDF	Probability Density Function
PeMS	Performance Measurement System
PLC	Pinned Local Cluster
PLM	Product Limit Method
RMSE	Root-Mean-Square Error
RMT	Random Matrix Theory
SCTM	Stochastic Cell Transmission Model
SSM	State Switching Model
STFT	Short-Term Fourier Transform
TF-BP	Traffic Flow Breakdown Probability
TSG/SGW	Triggered Stop-and-Go Waves
WSS	Second-order Wide-sense Stationary Process
WT	Wavelet Transform

Contents

1	Introduction	1
1.1	Motivation	1
1.2	Objectives	3
1.3	Contributions	3
1.4	Organization	5
2	Literature Review	9
2.1	Introduction	9
2.2	Historical Development of Traffic Flow Theory	9
2.2.1	Macroscopic Modeling	9
2.2.2	Mesosopic Modeling	12
2.2.3	Microscopic Modeling	14
2.2.4	Stochastic Modeling	15
2.3	Probabilistic Headway/Spacing Distributions	19
2.3.1	Simple Univariable Distributions	19
2.3.2	Compositional Distributions	22
2.3.3	Mixed Distributions	23
2.3.4	Random Matrix Model	24
2.4	Summary	25
3	Empirical Observations of Stochastic and Dynamic Evolutions of Traffic Flow	27
3.1	Introduction	27
3.2	Characteristics of Headway/Spacing/Velocity	27
3.3	Congested Platoon Oscillations	33
3.4	Time-Frequency Properties	36
3.5	Summary	46
4	A Markov Model Based on Headway/Spacing Distributions	49
4.1	Introduction	49
4.2	A Markov Model for Headway/Spacing Distributions	50

4.2.1	Background	50
4.2.2	Markov-Process Simulation Models	52
4.2.3	Simulation Results	58
4.2.4	Discussions	61
4.3	Asymmetric Stochastic Tau Theory in Car-Following	63
4.3.1	Asymmetric Stochastic Extension of the Tau Theory	64
4.3.2	Testing Results	68
4.3.3	Discussions	78
5	Stochastic Fundamental Diagram Based on Headway/Spacing Distributions	81
5.1	Introduction	81
5.2	Newell's Simplified Model and Its Stochastic Extension	83
5.3	The Homogeneous Platoon Model	89
5.3.1	Basic Idea	89
5.3.2	Summation of Lognormal Random Variables	91
5.3.3	Average Headway Distribution	92
5.3.4	Model Validation	97
5.3.5	Sensitivity Analysis	105
5.4	The Heterogeneous Platoon Model	109
5.4.1	Average Headway Distribution	109
5.4.2	Validation	111
5.4.3	Boundaries of Congested Flows	113
5.5	Summary	115
6	Traffic Flow Breakdown Model Based on Headway/Spacing Distributions	117
6.1	Introduction	117
6.2	Nonparametric Lifetime Statistics Approach	118
6.3	Queueing Models for Breakdown Probability	121
6.3.1	Backgrounds	121
6.3.2	Some Previous Models	123
6.3.3	G/G/1 Queueing Model	125
6.3.4	Discussions	140
6.3.5	Model Validation	141
6.3.6	Summary	143
6.4	Phase Diagram Analysis	143
6.4.1	Backgrounds	143
6.4.2	The Spatial-Temporal Queueing Model	145
6.4.3	The Analytical Solution for Phase Diagram	151
6.4.4	Numerical Example	157
6.5	Discussions	161

7 Conclusions and Future Work 163

**Appendix A: Linear Stability Analysis of the Higher-Order
Macroscopic Model. 167**

**Appendix B: Linear Stability Analysis of the Multi-Anticipative
Car-Following Models 171**

References. 177

Index 189

Chapter 1

Introduction

1.1 Motivation

Traffic congestion results in a number of negative effects on: (1) *Mobility*. Travel delays and wasting time of passengers or goods reduce the efficiency of transportation systems and increase opportunity costs; (2) *Safety*. Higher probability of serious injuries and death crashes as a result of human fallibility in congested flows; (3) *Sustainability*. Increased travel time and oscillatory acceleration/braking maneuvers in traffic congestion induce significant environmental impacts, such as fuel consumption, greenhouse gas emissions, air pollution, noises, etc.

Road traffic flow is influenced by various random factors, including both external factors (e.g., weather) and internal factors (e.g., transportation facilities, vehicle characteristics, driver behaviors, etc.). These stochastic influences make deterministic traffic flow models difficult to accurately estimate and predict dynamic evolutions. To overcome this problem, numerous stochastic approaches were developed for continuous traffic flow on the basis of microscopic/macrosopic traffic flow models. Particularly, different kinds of drivers (e.g., aggressive versus passive, young versus old, skilled versus greenhand, rigorous versus fatigued) run different kinds of vehicles (e.g., cars versus trucks, buses) on the same road, and thus, traffic flow is heterogeneous.

Since headway/spacing/velocity perform fundamental roles in stochastic traffic flow modeling, it is significant to study their stochastic characteristics in traffic flow evolutions. According to *Highway Capacity Manual 2000* (on page 48 of Transportation Research Board of the National Academies (2000)),

Definition 1.1 Headway (time headway, h) is the time, in seconds, between two successive vehicles as they pass a point on the roadway, measured from the same common feature of both vehicles.

Studies on headway distributions received continuous interests since the birth of traffic flow research, because of their wide applications ranging from measuring road capacity to scheduling traffic signals. Headway distribution model is one of