Classics in Mathematics

Tosio Kato

Perturbation Theory for Linear Operators

线性算子的微扰理论

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Classics in Mathematics

Tosio Kato Perturbation Theory for Linear Operators

Tosio Kato was born in 1917 in a village to the north of Tokyo. He studied theoretical physics at the Imperial University of Tokyo. After several years of inactivity during World War II due to poor health, he joined the Faculty of Science at the University of Tokyo in 1951. From 1962 he was Professor of Mathematics at the University of California, Berkeley, where he is now Professor Emeritus.

Kato was a pioneer in modern mathematical physics. He worked in the areas of operator theory, quantum mechanics, hydrodynamics, and partial differential equations, both linear and nonlinear.

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To the memory
of my parents

Preface to the Second Edition

In view of recent development in perturbation theory, supplementary notes and a supplementary bibliography are added at the end of the new edition. Little change has been made in the text except that the paragraphs V-§ 4.5, VI-§ 4.3, and VIII-§ 1.4 have been completely rewritten, and a number of minor errors, mostly typographical, have been corrected. The author would like to thank many readers who brought the errors to his attention.

Due to these changes, some theorems, lemmas, and formulas of the first edition are missing from the new edition while new ones are added. The new ones have numbers different from those attached to the old ones which they may have replaced.

Despite considerable expansion, the bibliography is not intended to be complete.

Berkeley, April 1976 TOSIO KATO

Preface to the First Edition

This book is intended to give a systematic presentation of perturbation theory for linear operators. It is hoped that the book will be useful to students as well as to mature scientists, both in mathematics and in the physical sciences.

Perturbation theory for linear operators is a collection of diversified results in the spectral theory of linear operators, unified more or less loosely by their common concern with the behavior of spectral properties when the operators undergo a small change. Since its creation by RAYLEIGH and SCHRÖDINGER, the theory has occupied an important place in applied mathematics; during the last decades, it has grown into a mathematical discipline with its own interest. The book aims at a mathematical treatment of the subject, with due consideration of applications.

The mathematical foundations of the theory belong to functional analysis. But since the book is partly intended for physical scientists, who might lack training in functional analysis, not even the elements of that subject are presupposed. The reader is assumed to have only a basic knowledge of linear algebra and real and complex analysis. The necessary tools in functional analysis, which are restricted to the most elementary part of the subject, are developed in the text as the need for them arises (Chapters I, III and parts of Chapters V, VI).

An introduction, containing a brief historical account of the theory, precedes the main exposition. There are ten chapters, each prefaced by a summary. Chapters are divided into sections, and sections into paragraphs. I-§ 2.3, for example, means paragraph three of section two of chapter one; it is simply written § 2.3 when referred to within the same chapter and par. 3 when referred to within the same section. Theorems, Corollaries, Lemmas, Remarks, Problems, and Examples are numbered in one list within each section: Theorem 2.1, Corollary 2.2, Lemma 2.3, etc. Lemma I-2.3 means Lemma 2.3 of chapter one, and it is referred to simply as Lemma 2.3 within the same chapter. Formulas are numbered consecutively within each section; I-(2.3) means the third formula of section two of chapter one, and it is referred to as (2.3) within the same chapter. Some of the problems are disguised theorems, and are quoted in later parts of the book.

Numbers in [ ] refer to the first part of the bibliography containing articles, and those in ⦃ ⦄ to the second part containing books and monographs.

There are a subject index, an author index and a notation index at the end of the book.

The book was begun when I was at the University of Tokyo and completed at the University of California. The preparation of the book has been facilitated by various financial aids which enabled me to pursue research at home and other institutions. For these aids I am grateful to the following agencies: the Ministry of Education, Japan; Commissariat Général du Plan, France; National Science Foundation, Atomic Energy Commission, Army Office of Ordnance Research, Office of Naval Research, and Air Force Office of Scientific Research, U.S.A.

I am indebted to a great many friends for their suggestions during the long period of writing the book. In particular I express my hearty thanks to Professors C. CLARK, K. O. FRIEDRICHS, H. FUJITA, S. GOLDBERG, E. HILLE, T. IKEBE, S. KAKUTANI, S. T. KURODA, G. NEUBAUER, R. S. PHILLIPS, J. and O. TODD, F. WOLF, and K. YOSIDA. I am especially obliged to Professor R. C. RIDDELL, who took the pains of going through the whole manuscript and correcting innumerable errors, mathematical as well as linguistic. I am indebted to Dr. J. HOWLAND, Dr. F. MCGRATH, Dr. A. MCINTOSH, and Mr. S.-C. LIN for helping me in proofreading various parts of the book. I wish to thank Professor F. K. SCHMIDT who suggested that I write the book and whose constant encouragement brought about the completion of the book. Last but not least my gratitudes go to my wife, MIZUE, for the tedious work of typewriting the manuscript.

Berkeley TOSIO KATO

August, 1966

Introduction

Throughout this book, "perturbation theory" means "perturbation theory for linear operators". There are other disciplines in mathematics called perturbation theory, such as the ones in analytical dynamics (celestial mechanics) and in nonlinear oscillation theory. All of them are based on the idea of studying a system deviating slightly from a simple ideal system for which the complete solution of the problem under consideration is known; but the problems they treat and the tools they use are quite different. The theory for linear operators as developed below is essentially independent of other perturbation theories.

Perturbation theory was created by RAYLEIGH and SCHRÖDINGER (cf. SZ.-NAGY [1]). RAYLEIGH gave a formula for computing the natural frequencies and modes of a vibrating system deviating slightly from a simpler system which admits a complete determination of the frequencies and modes (see RAYLEIGH [1], §§ 90, 91). Mathematically speaking, the method is equivalent to an approximate solution of the eigenvalue problem for a linear operator slightly different from a simpler operator for which the problem is completely solved. SCHRÖDINGER developed a similar method, with more generality and systematization, for the eigenvalue problems that appear in quantum mechanics (see SCHRÖDINGER [1], [1]).

These pioneering works were, however, quite formal and mathematically incomplete. It was tacitly assumed that the eigenvalues and eigenvectors (or eigenfunctions) admit series expansions in the small parameter that measures the deviation of the "perturbed" operator from the "unperturbed" one; no attempts were made to prove that the series converge.

It was in a series of papers by RELLICH that the question of convergence was finally settled (see RELLICH [1]–[5]; there were some attempts at the convergence proof prior to RELLICH, but they were not conclusive; see e. g. WILSON [1]). The basic results of RELLICH, which will be described in greater detail in Chapters II and VII, may be stated in the following way. Let $T(\varkappa)$ be a bounded selfadjoint operator in a Hilbert space H, depending on a real parameter $\varkappa$ as a convergent power series

$$T(\varkappa) = T + \varkappa T^{(1)} + \varkappa^2 T^{(2)} + \cdots . \tag{1}$$

Suppose that the unperturbed operator $T = T(0)$ has an isolated eigenvalue λ (isolated from the rest of the spectrum) with a finite multiplicity m. Then $T(\varkappa)$ has exactly m eigenvalues $\mu_j(\varkappa)$, $j = 1, \ldots, m$

(multiple eigenvalues counted repeatedly) in the neighborhood of λ for sufficiently small $|\varkappa|$, and these eigenvalues can be expanded into convergent series

$$(2) \qquad \mu_j(\varkappa) = \lambda + \varkappa\, \mu_j^{(1)} + \varkappa^2\, \mu_j^{(2)} + \cdots, \quad j = 1, \ldots, m .$$

The associated eigenvectors $\varphi_j(\varkappa)$ of $T(\varkappa)$ can also be chosen as convergent series

$$(3) \qquad \varphi_j(\varkappa) = \varphi_j + \varkappa\, \varphi_j^{(1)} + \varkappa^2\, \varphi_j^{(2)} + \cdots, \quad j = 1, \ldots, m ,$$

satisfying the orthonormality conditions

$$(4) \qquad (\varphi_j(\varkappa), \varphi_k(\varkappa)) = \delta_{jk} ,$$

where the φ_j form an orthonormal family of eigenvectors of T for the eigenvalue λ.

These results are exactly what were anticipated by RAYLEIGH, SCHRÖDINGER and other authors, but to prove them is by no means simple. Even in the case in which H is finite-dimensional, so that the eigenvalue problem can be dealt with algebraically, the proof is not at all trivial. In this case it is obvious that the $\mu_j(\varkappa)$ are branches of algebroidal functions of $\varkappa$, but the possibility that they have a branch point at $\varkappa = 0$ can be eliminated only by using the selfadjointness of $T(\varkappa)$. In fact, the eigenvalues of a selfadjoint operator are real, but a function which is a power series in some fractional power $\varkappa^{1/p}$ of $\varkappa$ cannot be real for both positive and negative values of $\varkappa$, unless the series reduces to a power series in $\varkappa$. To prove the existence of eigenvectors satisfying (3) and (4) is much less simple and requires a deeper analysis.

Actually RELLICH considered a more general case in which $T(\varkappa)$ is an unbounded operator; then the series (1) requires new interpretations, which form a substantial part of the theory. Many other problems related to the one above were investigated by RELLICH, such as estimates for the convergence radii, error estimates, simultaneous consideration of all the eigenvalues and eigenvectors and the ensuing question of uniformity, and non-analytic perturbations.

Rellich's fundamental work stimulated further studies on similar and related problems in the theory of linear operators. One new development was the creation by FRIEDRICHS of the perturbation theory of continuous spectra (see FRIEDRICHS [2]), which proved extremely important in scattering theory and in quantum field theory. Here an entirely new method had to be developed, for the continuous spectrum is quite different in character from the discrete spectrum. The main problem dealt with in Friedrichs's theory is the similarity of $T(\varkappa)$ to T, that is, the existence of a non-singular operator $W(\varkappa)$ such that $T(\varkappa) = W(\varkappa)\, T\, W(\varkappa)^{-1}$.

The original results of RELLICH on the perturbation of isolated eigenvalues were also generalized. It was found that the analytic theory gains in generality as well as in simplicity by allowing the parameter $\varkappa$ to be complex, a natural idea when analyticity is involved. However, one must then abandon the assumption that $T(\varkappa)$ is selfadjoint for all $\varkappa$, for an operator $T(\varkappa)$ depending on $\varkappa$ analytically cannot in general be selfadjoint for all $\varkappa$ of a complex domain, though it may be selfadjoint for all real $\varkappa$, say. This leads to the formulation of results for non-selfadjoint operators and for operators in Banach spaces, in which the use of complex function theory prevails (SZ.-NAGY [2], WOLF [1], T. KATO [6]). It turns out that the basic results of RELLICH for selfadjoint operators follow from the general theory in a simple way.

On the other hand, it was recognized (TITCHMARSH [1], [2], T. KATO [1]) that there are cases in which the formal power series like (2) or (3) diverge or even have only a finite number of significant terms, and yet approximate the quantities $\mu_j(\varkappa)$ or $\varphi_j(\varkappa)$ in the sense of asymptotic expansion. Many examples, previously intractable, were found to lie within the sway of the resulting asymptotic theory, which is closely related to the singular perturbation theory in differential equations.

Other non-analytic developments led to the perturbation theory of spectra in general and to stability theorems for various spectral properties of operators, one of the culminating results being the index theorem (see GOHBERG and KREIN [1]).

Meanwhile, perturbation theory for one-parameter semigroups of operators was developed by HILLE and PHILLIPS (see PHILLIPS [1], HILLE and PHILLIPS [1]). It is a generalization of, as well as a mathematical foundation for, the so-called time-dependent perturbation theory familiar in quantum mechanics. It is also related to time-dependent scattering theory, which is in turn closely connected with the perturbation of continuous spectra. Scattering theory is one of the subjects in perturbation theory most actively studied at present.

It is evident from this brief review that perturbation theory is not a sharply-defined discipline. While it incorporates a good deal of the spectral theory of operators, it is a body of knowledge unified more by its method of approach than by any clear-cut demarcation of its province. The underpinnings of the theory lie in linear functional analysis, and an appreciable part of the volume is devoted to supplying them. The subjects mentioned above, together with some others, occupy the remainder.

Contents

Chapter Two

Chapter Three

Introduction to the theory of operators in Banach spaces