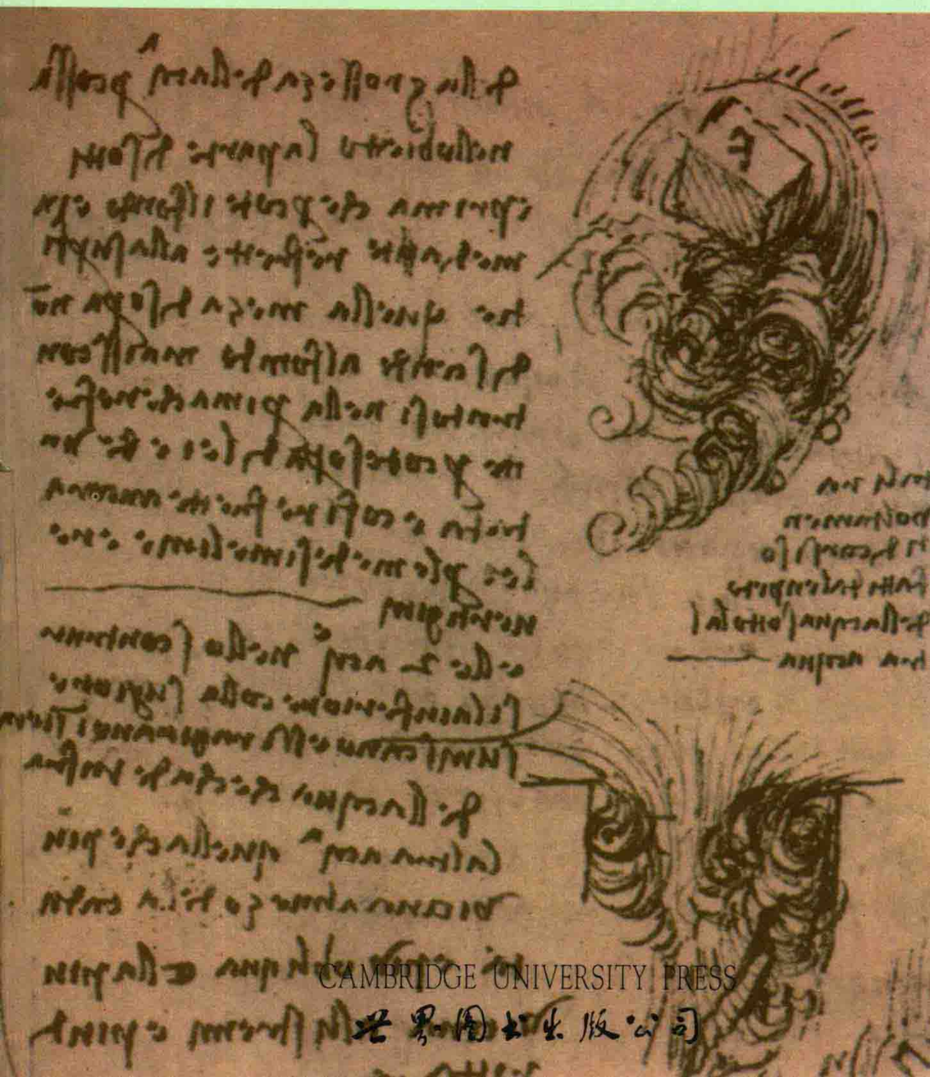
TURBULENCE

湍流

Uriel Frisch



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THE LEGACY OF A.N. KOLMOGOROV

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This textbook presents a modern account of turbulence, one of the greatest challenges in physics. The state-of-the-art is put into historical perspective five centuries after the first studies of Leonardo and half a century after the first attempt by A. N. Kolmogorov to predict the properties of flow at very high Reynolds numbers. Such "fully developed turbulence" is ubiquitous in both cosmical and natural environments, in engineering applications and in everyday life.

First, a qualitative introduction is given to bring out the need for a probabilistic description of what is in essence a deterministic system. Kolmogorov's 1941 theory is presented in a novel fashion with emphasis on symmetries (including scaling transformations) which are broken by the mechanisms producing the turbulence and restored by the chaotic character of the cascade to small scales. Considerable material is devoted to intermittency, the clumpiness of small-scale activity, which has led to the development of fractal and multifractal models. Such models, pioneered by B. Mandelbrot, have applications in numerous fields besides turbulence (diffusion limited aggregation, solid-earth geophysics, attractors of dynamical systems, etc). The final chapter contains an introduction to analytic theories of the sort pioneered by R. Kraichnan, to the modern theory of eddy transport and renormalization and to recent developments in the statistical theory of two-dimensional turbulence. The book concludes with a guide to further reading.

The intended readership for the book ranges from first-year graduate students in mathematics, physics, astrophysics, geosciences and engineering, to professional scientists and engineers. Elementary presentations of dynamical systems ideas, of probabilistic methods (including the theory of large deviations) and of fractal geometry make this a self-contained textbook.

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Preface

Andrei Nikolaevich Kolmogorov's work in 1941 remains a major source of inspiration for turbulence research. Great classics, when revisited in the light of new developments, may reveal hidden pearls, as is the case with Kolmogorov's very brief third 1941 paper 'Dissipation of energy in locally isotropic turbulence' (Kolmogorov 1941c). It contains one of the very few exact and nontrivial results in the field, as well as very modern ideas on scaling, ideas which cannot be refuted by the argument Lev Landau used to criticize the universality assumptions of the first 1941 paper.

Revisiting Kolmogorov's fifty-year-old work on turbulence was one goal of the lectures on which this book is based. The lectures were intended for first-year graduate students in 'Turbulence and Dynamical Systems' at the University of Nice–Sophia–Antipolis. My presentation deliberately emphasizes concepts which are central in dynamical systems studies, such as symmetry-breaking and deterministic chaos. The students had some knowledge of fluid dynamics, but little or no training in modern probability theory. I have therefore included a significant amount of background material. The presentation uses a physicist's viewpoint with more emphasis on systematic arguments than on mathematical rigor. Also, I have a marked preference for working in coordinate space rather than in Fourier space, whenever possible.

Modern work on turbulence focuses to a large extent on trying to understand the reasons for the partial failure of the 1941 theory. This 'intermittency' problem has received here considerable coverage. Kolmogorov himself became a pioneer in this line of investigation in 1961, following the work of his collaborator A.M. Obukhov (Kolmogorov 1961). Although some of their suggestions can be criticized as mathematically or physically inconsistent, their 1961 work has been and remains a major source of inspiration. For pedagogical reasons, I have chosen to discuss historical aspects only after presentation of more recent work on 'fractal' and 'multifractal' models of turbulence.

Some of the material on Kolmogorov presented here has appeared in a special issue of the *Proceedings of the Royal Society* 'Kolmogorov's ideas 50 years on', which also contains a whole range of alternative views on Kolmogorov and on what matters for turbulence research (Frisch 1991). Other useful references on Kolmogorov are the selected works (Tikhomirov 1991), the obituary (Kendall 1990), the review of the turbulence work of one of his close collaborators (Yaglom 1994) and the personal recollections concerned more with the mathematician and the man (Arnold 1994).

In an introductory course on turbulence, of about thirty hours of lecturing, many aspects had to be left out. I have included at the end of this book a guided tour to further reading as a partial remedy. It is also intended to convey briefly my — possibly very biased — views of what matters. No attempt has been made to present a balanced historical perspective of a subject now at least five centuries old (see p. 112); the reader will nevertheless find a number of historical sections and remarks and may discover for example that the concept of eddy viscosity was introduced in the middle of the nineteenth century (see p. 223).

More information on the organization of this book may be found in Section 1.2 (see p. 11).

The intended readership for the book ranges from first-year graduate students in mathematics, physics, astrophysics, geophysics and engineering, to professional scientists and engineers. Primarily, it is intended for those interested in learning about the basics of turbulence or wanting to take a fresh look at the subject. Much of the material on probabilistic background, on fractals and multifractals also has applications beyond fluid mechanics, for instance, to solid-earth geophysics.

I am deeply grateful to J.P. Rivet who in many respects has given life to this book and I am particularly indebted to A.M. Yaglom for numerous discussions and comments. Very useful remarks and suggestions were received from V.I. Arnold, G. Barenblatt, G.K. Batchelor, L. Biferale, M. Blank, M.E. Brachet, G. Eyink, H. Frisch, H.L. Grant, M. Hénon, J. Jiménez, R. Kraichnan, B. Legras, A. Migdal, G.M. Molchan, A. Noullez, K. Ohkitani, S.A. Orszag, A. Praskovsky, A. Pumir, Z.S. She, Ya. Sinai, J. Sommeria, P.L. Sulem, M. Vergassola, E. Villermaux and B. Villone. M.C. Vergne has realized some of the figures. I also wish to thank the students of the 'DEA Turbulence et

Systèmes Dynamiques' of the University of Nice–Sophia–Antipolis who have helped me with their questions, since I started teaching this material as a graduate course in 1990.

Part of the work for this book was done while I was visiting Princeton University (Center for Fluid Dynamics Research). Significant support was received from the 'Direction des Recherches et Moyens Techniques', from various programs of the European Union and from the 'Fondation des Treilles'.

I would like to dedicate this second printing (August 1996) to Giovanni Paladin who died in a mountaineering accident on June 29, 1996.

Finally, it was a pleasure and a privilege to work in close collaboration with Alison, Maureen, Simon and Stephanie at Cambridge University Press.

Nice, France U. Frisch
July 1995

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1
Introduction

1.1 Turbulence and symmetries

In Chapter 41 of his *Lectures on Physics*, devoted to hydrodynamics and turbulence, Richard Feynman (1964) observes this:

Often, people in some unjustified fear of physics say you can't write an equation for life. Well, perhaps we can. As a matter of fact, we very possibly already have the equation to a sufficient approximation when we write the equation of quantum mechanics:

$$H\psi = -\frac{\hbar}{i}\frac{\partial \psi}{\partial t}. \tag{1.1}$$

Of course, if we only had this equation, without detailed observation of biological phenomena, we would be unable to reconstruct them. Feynman believes, and this author shares his viewpoint, that an analogous situation prevails in *turbulent* flow of an incompressible fluid. The equation, generally referred to as the Navier–Stokes equation, has been known since Navier (1823):

$$\partial_t \boldsymbol{v} + \boldsymbol{v} \cdot \nabla \boldsymbol{v} = -\nabla p + \nu \nabla^2 \boldsymbol{v}, \tag{1.2}$$

$$\nabla \cdot \boldsymbol{v} = 0. \tag{1.3}$$

It must be supplemented by initial and boundary conditions (such as the vanishing of $\boldsymbol{v}$ at rigid walls). We shall come back later to the choice of notation.

The Navier–Stokes equation probably contains all of turbulence. Yet it would be foolish to try to guess what its consequences are without looking at experimental facts. The phenomena are almost as varied as in the realm of life.

A good way to make contact with the rich world of turbulence phenomena is through the book of Van Dyke (1982) *An Album of Fluid Motion*. To communicate a first impression of the experimental facet of turbulence, we shall mainly use pictures from this book.

In this Introduction, we have chosen to stress the ideas of *broken symmetries* and of *restored symmetries*. Symmetry consideration are indeed central to the study of both *transition phenomena* and *fully developed turbulence*. For the time being we shall leave aside the quantitative aspects of experimental data with the exception of the control parameter, the Reynolds number, which is defined as

$$R = \frac{LV}{\nu}, \tag{1.4}$$

L and V being respectively a characteristic scale and velocity of the flow, and ν its (kinematic) viscosity.[1] Remember a consequence of the *similarity principle* for incompressible flow: for a given geometrical shape of the boundaries, the Reynolds number is the only control parameter of the flow.

With this in mind, let us observe what happens when increasing the Reynolds number in flow past a cylinder. We have chosen a cylinder in order to ensure some degree of symmetry, while selecting an *external* flow. External flow is more difficult to control and to study but has more life than internal flow which is confined by its boundaries, such as Rayleigh–Bénard convection or Taylor–Couette flow.

As shown in Fig. 1.1, we consider a flow of uniform velocity $\boldsymbol{V} = (V, 0, 0)$ (at infinity), parallel to the x-axis, incident from the left on an infinite cylinder, of circular cross-section with diameter L, the axis being along the z-direction.

Fig. 1.2 is a visualization of the flow at $R = 0.16$. At first, the flow appears to possess the following symmetries:

- *Left–right* (x-reversal),
- *Up–down* (y-reversal),
- *Time-translation* (t-invariance),
- *Space-translation* parallel to the axis of the cylinder (z-invariance).

All these symmetries, except the first, are consistent with the Navier–Stokes equation and the boundary conditions. Let us be a little bit more

[1] In c.g.s. units the kinematic viscosity is about one-seventh for air and one-hundredth for water.

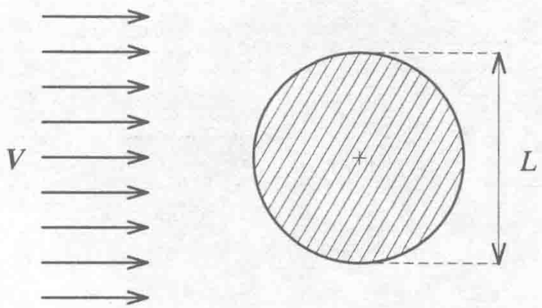

Fig. 1.1. Uniform flow with velocity $\boldsymbol{V}$, incident on a cylinder of diameter L.

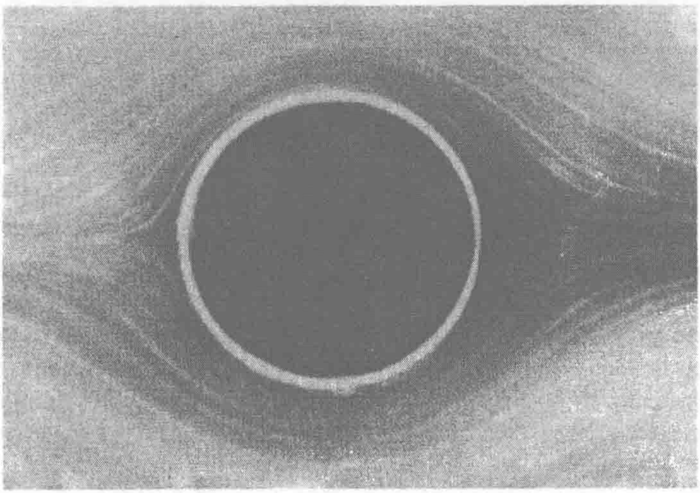

Fig. 1.2. Uniform flow past a cylinder at $R = 0.16$ (Van Dyke 1982). Photograph S. Taneda.

specific. We denote by (u, v, w) the components of the velocity. The left–right symmetry is

$$(x, y, z) \to (-x, y, z), \qquad (u, v, w) \to (u, -v, -w). \tag{1.5}$$

The up–down symmetry is

$$(x, y, z) \to (x, -y, z), \qquad (u, v, w) \to (u, -v, w). \tag{1.6}$$

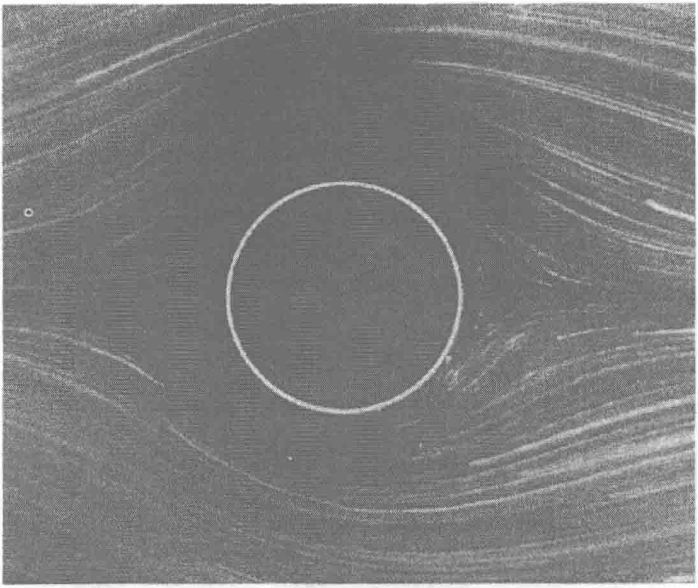

Fig. 1.3. Circular cylinder at $R = 1.54$ (Van Dyke 1982). Photograph S. Taneda.

It is easily checked that the left–right symmetry is not consistent with the Navier–Stokes equation, although it is consistent with the Stokes equation, obtained by dropping the nonlinear term. Actually, closer inspection of Fig. 1.2 shows that the left–right symmetry is not exact: it is slightly *broken*. This is an effect of the residual nonlinearity, which would get even weaker if we were to let the Reynolds number become much smaller.

Fig. 1.3 shows the flow at $R = 1.54$. There is now a marked left–right asymmetry. Around $R = 5$ the flow begins to separate behind the cylinder. Although no symmetry-breaking occurs, there is a change in the topology of the flow associated with the formation of recirculating standing eddies, shown in Fig. 1.4 for various values of R from 9.6 to 26.

Around $R = 40$ the first true loss of symmetry occurs by an Andronov–Hopf bifurcation which makes the flow time-periodic; in other words, the continuous t-invariance is broken in favor of a discrete t-invariance. The flow in the immediate neighborhood of the bifurcation point is shown in Fig. 1.5. At higher values of R, such as shown in Figs. 1.6, 1.7 and

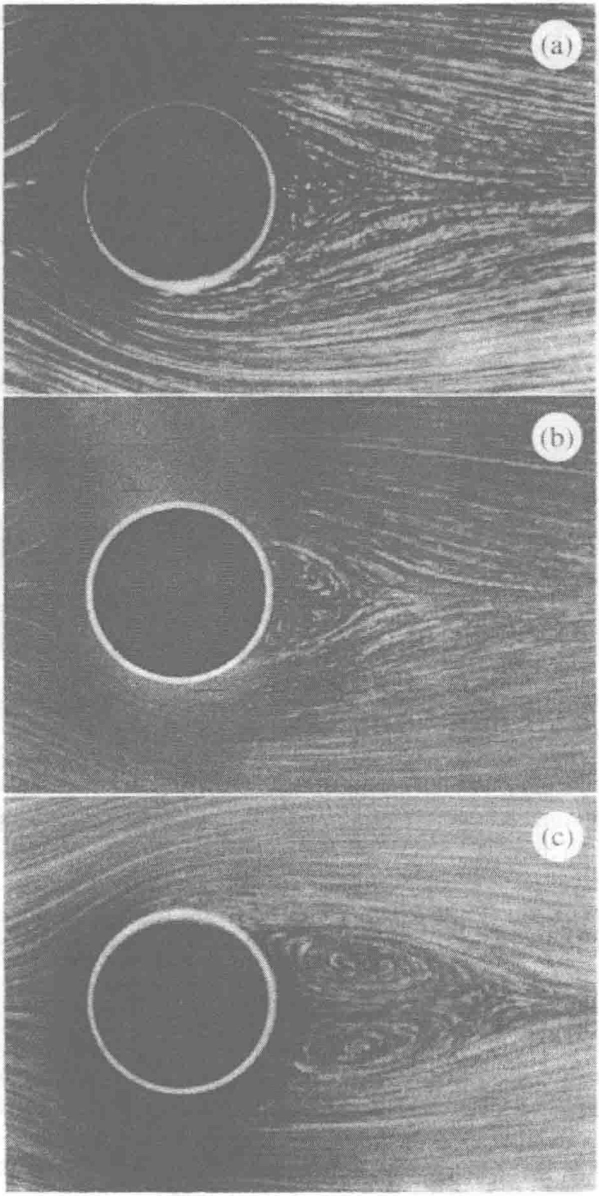

Fig. 1.4. Circular cylinder at $R = 9.6$ (a), $R = 13.1$ (b) and $R = 26$ (c) (Van Dyke 1982). Photograph S. Taneda.