

H. Schlichting  
K. Gersten

# Boundary Layer Theory

8th Revised and Enlarged Edition

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Hermann Schlichting · Klaus Gersten

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# Boundary-Layer Theory

With contributions  
from Egon Krause and Herbert Oertel Jr.

Translated by Katherine Mayes

8th Revised and Enlarged Edition  
With 287 Figures and 22 Tables



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# Boundary-Layer Theory

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Popular-Layer Theory

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by Hermann Schlichting and Klaus Gersten

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## Preface to the Eighth English Edition

According to the tradition of this book, a German edition has always been soon followed by the English translation. I am very grateful to Springer-Verlag for undertaking this version and for securing a translator. My particular thanks go to Katherine Mayes for this excellent translation. In the course of the translation, some errors in the German edition were corrected and a number of additions carried out. In this connection I am very thankful to Prof. Dr. W. Schneider, Vienna, for several suggestions and improvements. I would like to thank Ursula Beitz again for her careful checking of the bibliography. I hope that the English edition will attain the same positive resonance as the ninth German edition.

Bochum, May 1999

*Klaus Gersten*

## Preface to the Ninth German Edition

There is no doubt that *Boundary-Layer Theory* by Hermann Schlichting is one of most important books within the sphere of fluid mechanics to appear in the last decade. Shortly before his death, Hermann Schlichting brought out the eighth edition which he revised together with his friend and former colleague Wilhelm Riegels.

When this edition went out of print and a new edition was desired by the publishers, I was very glad to take on the task. During the fifteen years I spent at the institute of my highly respected teacher Hermann Schlichting, I had already been involved with earlier editions of the book and had revised some chapters. The burden was also eased by the fact that boundary-layer theory in its widest sense has been my preferred direction of research for many years.

It quickly became clear that a complete revision was necessary; indeed this was also known to Hermann Schlichting. In the preface to the eighth edition he wrote: "Noting the systematic of our knowledge of today, it would have been desirable to fully revise this work; however such a process would have pushed back the appearance of this book by years." Compared to the eighth edition, the literature of the last 15 years had to be taken into account and recent developments, in turbulence models for example, had to be incorporated. In order to keep the size of the book tractable, some results – those which no longer seem so important with today's computing potential – had to be curtailed, or in some cases, left out altogether.

Thus the necessity to completely rewrite the text emerged. The fundamental divisions within the book were retained; as before it consists of the four major sections: basic laws of the flows of viscous fluids, laminar boundary layers, the onset of turbulence, turbulent boundary layers. However a new fifth section on numerical methods in boundary-layer theory has been added.

The partition into chapters had to be somewhat modified in order to improve the style of presentation of the material. Because of the necessary restrictions on the material, the aim was to concentrate on boundary-layer theory as the theory of high Reynolds number flows. Accordingly the chapter on "creeping flows", that is flows at very small Reynolds numbers, was omitted.



It seemed natural to steer towards the style and level of presentation with the same target audience as with Hermann Schlichting.

The research area of boundary-layer theory is continuously growing, and it has become so extensive that no single person can possess a complete overview. Consequently I am extremely grateful to two colleagues who supported me actively. Professor E. Krause wrote the new additional chapter on numerical methods in boundary-layer theory, and Professor H. Oertel provided the revision of the section on the onset of turbulence (stability theory).

Further assistance was furnished from different sources. I am indebted to Dr.-Ing. Peter Schäfer and Dr.-Ing. Detlev Vieth for a great many new sample calculations. Dr. Vieth also read the entire text discerningly. I am grateful to him for numerous improving suggestions. Renate Gölzenleuchtner deserves particular thanks for generating the figures which almost all had to be newly drawn up. I would like to thank Ursula Beitz particularly for her careful and exhaustive checking of the bibliography, while Marianne Ferdinand and Eckhard Schmidt were of first class assistance. It was by far impossible to adopt all citations, so that it may be necessary to revert to the eighth edition for specific references to earlier pieces of work.

The printing firm of Jörg Steffenhagen is due particular praise for an extremely fruitful collaboration. My thanks also go to Springer-Verlag for our most agreeable work together.

I hope we have been able to carry on the work of Hermann Schlichting as he would have wished.

Bochum, October 1996

Klaus Gersten

# Introduction

## Short historical review

At the end of the 19th century, fluid mechanics had split into two different directions which hardly had anything more in common. On one side was the science of *theoretical hydrodynamics*, emanating from Euler's equations of motion and which had been developed to great perfection. However this had very little practical importance, since the results of this so-called classical hydrodynamics were in glaring contradiction to everyday experience. This was particularly true in the very important case of pressure loss in tubes and channels, as well as that of the drag experienced by a body moved through a fluid. For this reason, engineers, on the other side, confronted by the practical problems of fluid mechanics, developed their own strongly empirical science, *hydraulics*. This relied upon a large amount of experimental data and differed greatly from theoretical hydrodynamics in both methods and goals.

It is the great achievement of Ludwig Prandtl which, at the beginning of this century, set forth the way in which these two diverging directions of fluid mechanics could be unified. He achieved a high degree of correlation between theory and experiment, which, in the first half of this century, has led to unimagined successes in modern fluid mechanics. It was already known then that the great discrepancy between the results in classical hydrodynamics and reality was, in many cases, due to neglecting the *viscosity effects* in the theory. Now the complete equations of motion of viscous flows (the Navier Stokes equations) had been known for some time. However, due to the great mathematical difficulty of these equations, no approach had been found to the mathematical treatment of viscous flows (except in a few special cases). For technically important fluids such as water and air, the viscosity is very small, and thus the resulting viscous forces are small compared to the remaining forces (gravitational force, pressure force). For this reason it took a long time to see why the viscous forces ignored in the classical theory should have an important effect on the motion of the flow.

— In his lecture on “Über Flüssigkeitsbewegung bei sehr kleiner Reibung” (On Fluid Motion with Very Small Friction) at the Heidelberg mathematical congress in 1904, L. Prandtl (1904) showed how a theoretical treatment could be used on viscous flows in cases of great practical importance. Using theoretical considerations together with some simple experiments, Prandtl

showed that the flow past a body can be divided into two regions: a very thin layer close to the body (*boundary layer*) where the viscosity is important, and the remaining region outside this layer where the viscosity can be neglected. With the help of this concept, not only was a physically convincing explanation of the importance of the viscosity in the drag problem given, but simultaneously, by hugely reducing the mathematical difficulty, a path was set for the theoretical treatment of viscous flows. Prandtl supported his theoretical work by some very simple experiments in a small, self-built water channel, and in doing this reinitiated the lost connection between theory and practice. The theory of the *Prandtl boundary layer* or the *frictional layer* has proved to be exceptionally useful and has given considerable stimulation to research into fluid mechanics since the beginning of this century. Under the influence of a thriving flight technology, the new theory developed quickly and soon became, along with other important advances – airfoil theory and gas dynamics – a keystone of modern fluid mechanics.

One of the most important applications of boundary-layer theory is the calculation of the friction drag of bodies in a flow, e.g. the drag of a flat plate at zero incidence, the friction drag of a ship, an airfoil, the body of an airplane, or a turbine blade. One particular property of the boundary layer is that, under certain conditions, a reverse flow can occur directly at the wall. A *separation* of the boundary layer from the body and the formation of large or small eddies at the back of the body can then occur. This results in a great change in the pressure distribution at the back of the body, leading to the *form or pressure drag* of the body. This can also be calculated using boundary-layer theory. Boundary-layer theory answers the important question of what shape a body must have in order to avoid this detrimental separation. It is not only in flow past a body where separation can occur, but also in flow through a duct. In this way boundary-layer theory can be used to describe the flow through blade cascades in compressors and turbines, as well as through diffusers and nozzles. The processes involved in maximum lift of an airfoil, where separation is also important, can only be understood using boundary-layer theory. The boundary layer is also important for heat transfer between a body and the fluid around it.

Initially boundary-layer theory was developed mainly for the laminar flow of an incompressible fluid, where Stokes law of friction could be used as an ansatz for the viscous forces. This area was later researched in very many pieces of work, so that today it can be considered to be fully understood. Later the theory was extended to the practically important turbulent incompressible boundary-layer flows. Around 1890, O. Reynolds (1894) had already introduced the fundamentally important concept of apparent turbulent stresses, but this did not yet permit the theoretical treatment of turbulent flows. The introduction of the concept of the Prandtl mixing length, cf. L. Prandtl (1925), contributed considerable advances and, together with systematic experiments, allowed turbulent flows to be treated theoretically

with the help of boundary-layer theory. Even today a rational theory of fully developed turbulent flows remains to be found. Thanks to the great increase of velocities in flight technology, boundary layers in compressible flows were subsequently also thoroughly examined. As well as the boundary layer in the velocity field, a thermal boundary layer also forms; this is of great importance for the heat transfer between the flow and the body. Because of internal friction (dissipation) at high Mach numbers, the body surface heats up greatly. This causes many problems, particularly in flight technology and satellite flights ("thermal barrier").

The transition from laminar to turbulent flow, important for all of fluid mechanics, was first examined in pipe flow at the end of the last century by O. Reynolds (1883). Using the flow about a sphere, in 1914 Prandtl was able to show experimentally that the boundary layer also can be both laminar or turbulent and that the process of separation and thus the drag problem are controlled by this laminar-turbulent transition, cf. L. Prandtl (1914). The theoretical investigations into this transition assume Reynolds' idea of the instability of the laminar flow. This was treated by Prandtl in 1921. After some futile attempts, W. Tollmien (1929) and H. Schlichting (1933) were able to theoretically calculate the indifference Reynolds number for the flat plate at zero incidence. However it took more than ten years before the theory could be confirmed by careful experiments by H.L. Dryden (1946-1948) and his coworkers. The effect of other parameters on the transition (pressure gradient, suction, Mach number, heat transfer) were clarified using the stability theory of the boundary layer. This theory has found important application with, among other things, airfoils with very low drag (laminar airfoils).

An important characteristic of modern research into fluid mechanics in general and more specifically into the branch of boundary-layer theory is the close connection between theory and experiment. The most crucial advances have been achieved through a few fundamental experiments together with theoretical considerations. Many years ago, A. Betz (1949) produced a review of the development of boundary-layer theory, with particular emphasis on the mutual fructification of theory and experiment. Research into boundary layers, inspired by Prandtl from 1904, were, in the first 20 years up until Prandtl's Wilbur Wright memorial lecture at the Royal Aeronautical Society in London, (L. Prandtl (1927)) almost exclusively confined to Prandtl's institute in Göttingen. It is only since 1930 that other researchers have been involved in the further expansion of boundary-layer theory, initially in England and the USA. Today boundary-layer theory has spread over the whole world; together with other branches it forms one of the most important pillars of fluid mechanics.

In the mid-fifties, mathematical methods into singular perturbation theory were being systematically developed, cf. S. Kaplun (1954), S. Kaplun; P.A. Lagerstrom (1957), M. Van Dyke (1964b), also W. Schneider (1978).

It became clear that the boundary-layer theory heuristically developed by Prandtl was a classic example of the solution of a singular perturbation problem. Thus boundary-layer theory is a rational asymptotic theory of the solution of the Navier–Stokes equations for high Reynolds numbers, cf. K. Gersten (2000). This opened the possibility of a systematic development to higher order boundary-layer theory, cf. M. Van Dyke (1969), K. Gersten (1972), K. Stewartson (1974), K. Gersten; J.F. Gross (1976), V.V. Sychev et al. (1998), I.J. Sobey (2000). The asymptotic methods which were first developed for laminar flows were then, at the start of the seventies, carried over to turbulent flows, cf. K.S. Yajnik (1970), G.L. Mellor (1972). Reviews of asymptotic theory of turbulent flows are to be found in K. Gersten (1987), K. Gersten (1989c), A. Kluwick (1989a), as well as W. Schneider (1991). K. Gersten; H. Herwig (1992) have presented a systematic application of asymptotic methods (regular and singular perturbation methods) to the theory of viscous flows. The book by P.A. Libby (1998) also gives preferential treatment to asymptotic methods. Most of the characteristics of the asymptotic theory for high Reynolds-number flows can already be found in Prandtl's work, cf. K. Gersten (2000).

In turbulence modelling, the mixing length hypothesis developed by L. Prandtl (1925) led to an algebraic turbulence model. Twenty years later, L. Prandtl (1945) showed how transport equations for turbulent quantities such as the kinetic energy of the random motion, the dissipation and the Reynolds shear stress could be applied to improve turbulence models. Calculation methods for turbulent boundary layers with highly refined turbulence models have been developed by, for example, P. Bradshaw et al. (1967), W.P. Jones; B.E. Launder (1973), K. Hanjalić; B.E. Launder (1976), as well as J.C. Rotta (1973). Overviews on turbulence modelling are presented by W.C. Reynolds (1976) and V.C. Patel et al. (1985). In two extremely noteworthy events at Stanford University in the years 1968 and 1980/81, the existing methods for calculating boundary layers were compared and examined in specially chosen experiments; see the reports by S.J. Kline et al. (1968) and S.J. Kline et al. (1981). A review on Reynolds number effects in wall-bounded turbulent flows by M. Gad-el-Hak; P.R. Bandyopadhyay (1994) is also worth mentioning.

The following tendency is emerging from the rapid developments in the area of supercomputing: the future will consist more of direct numerical solutions of the Navier–Stokes equations without any simplifications, and also of the computation of turbulent flows using direct numerical simulation (DNS), i.e. without using a turbulence model or by modelling only high frequency turbulent fluctuations ("large-eddy simulations"), cf. D.R. Chapman (1979). However numerical methods in computing flows at high Reynolds numbers only become efficient if the particular layered structure of the flow, as given by the asymptotic theory, is taken into account, as occurs if a suitable grid

is used for computation. Boundary-layer theory will therefore retain its fundamental place in the calculation of high Reynolds number flows.

The first summary of boundary-layer theory is to be found in two short articles by W. Tollmien (1931) in the *Handbuch der Experimentalphysik*. Some years later Prandtl's comprehensive contribution appeared in *Aerodynamic Theory*, edited by W.F. Durand, L. Prandtl (1935). In the six decades since then, the extent of this research area has become extraordinarily large. cf. H. Schlichting (1960) and also I. Tani (1977), A.D. Young (1989), K. Gersten (1989a), A. Kluwick (1998) and T. Cebeci; J. Cousteix (1999). According to a review by H.L. Dryden (1955), about 100 articles appeared in the year 1955, and now, 45 years later, this number has grown to about 800 per year. This branch of fluid mechanics, as with other branches, has become so large that all its different areas can barely be kept track of by one single researcher. This, on the other hand, is obvious proof of the great importance of boundary-layer theory.

# Contents

Introduction .....	XIX
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## Part I. Fundamentals of Viscous Flows

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<b>1. Some Features of Viscous Flows .....</b>	<b>3</b>
1.1 Real and Ideal Fluids .....	3
1.2 Viscosity .....	4
1.3 Reynolds Number .....	6
1.4 Laminar and Turbulent Flows .....	12
1.5 Asymptotic Behaviour at Large Reynolds Numbers .....	14
1.6 Comparison of Measurements Using the Inviscid Limiting Solution .....	14
1.7 Summary .....	26
<b>2. Fundamentals of Boundary-Layer Theory .....</b>	<b>29</b>
2.1 Boundary-Layer Concept .....	29
2.2 Laminar Boundary Layer on a Flat Plate at Zero Incidence ..	30
2.3 Turbulent Boundary Layer on a Flat Plate at Zero Incidence ..	33
2.4 Fully Developed Turbulent Flow in a Pipe .....	36
2.5 Boundary Layer on an Airfoil .....	38
2.6 Separation of the Boundary Layer .....	39
2.7 Overview of the Following Material .....	48
<b>3. Field Equations for Flows of Newtonian Fluids .....</b>	<b>51</b>
3.1 Description of Flow Fields .....	51
3.2 Continuity Equation .....	52
3.3 Momentum Equation .....	52
3.4 General Stress State of Deformable Bodies .....	53
3.5 General State of Deformation of Flowing Fluids .....	57
3.6 Relation Between Stresses and Rate of Deformation .....	62
3.7 Stokes Hypothesis .....	65
3.8 Bulk Viscosity and Thermodynamic Pressure .....	66
3.9 Navier-Stokes Equations .....	68

3.10	Energy Equation .....	69
3.11	Equations of Motion for Arbitrary Coordinate Systems (Summary) .....	73
3.12	Equations of Motion for Cartesian Coordinates in Index Notation .....	76
3.13	Equations of Motion in Different Coordinate Systems .....	79
<b>4.</b>	<b>General Properties of the Equations of Motion .....</b>	<b>83</b>
4.1	Similarity Laws .....	83
4.2	Similarity Laws for Flow with Buoyancy Forces (Mixed Forced and Natural Convection) .....	86
4.3	Similarity Laws for Natural Convection .....	90
4.4	Vorticity Transport Equation .....	91
4.5	Limit of Very Small Reynolds Numbers .....	93
4.6	Limit of Very Large Reynolds Numbers .....	94
4.7	Mathematical Example of the Limit $Re \rightarrow \infty$ .....	96
4.8	Non-Uniqueness of Solutions of the Navier-Stokes Equations. ....	99
<b>5.</b>	<b>Exact Solutions of the Navier-Stokes Equations .....</b>	<b>101</b>
5.1	Steady Plane Flows .....	101
5.1.1	Couette-Poiseuille Flows .....	101
5.1.2	Jeffery-Hamel Flows (Fully Developed Nozzle and Diffuser Flows) .....	104
5.1.3	Plane Stagnation-Point Flow .....	110
5.1.4	Flow Past a Parabolic Body .....	115
5.1.5	Flow Past a Circular Cylinder .....	115
5.2	Steady Axisymmetric Flows .....	116
5.2.1	Circular Pipe Flow (Hagen-Poiseuille Flow) .....	116
5.2.2	Flow Between Two Concentric Rotating Cylinders ....	117
5.2.3	Axisymmetric Stagnation-Point Flow .....	118
5.2.4	Flow at a Rotating Disk .....	119
5.2.5	Axisymmetric Free Jet .....	124
5.3	Unsteady Plane Flows .....	126
5.3.1	Flow at a Wall Suddenly Set into Motion (First Stokes Problem) .....	126
5.3.2	Flow at an Oscillating Wall (Second Stokes Problem) .....	129
5.3.3	Start-up of Couette Flow .....	130
5.3.4	Unsteady Asymptotic Suction .....	131
5.3.5	Unsteady Plane Stagnation-Point Flow .....	131
5.3.6	Oscillating Channel Flow .....	137
5.4	Unsteady Axisymmetric Flows .....	139
5.4.1	Vortex Decay .....	139
5.4.2	Unsteady Pipe Flow .....	139
5.5	Summary .....	141



---

**Part II. Laminar Boundary Layers**


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<b>6. Boundary-Layer Equations in Plane Flow; Plate Boundary Layer</b>	145
6.1 Setting up the Boundary-Layer Equations	145
6.2 Wall Friction, Separation and Displacement	150
6.3 Dimensional Representation of the Boundary-Layer Equations	152
6.4 Friction Drag	155
6.5 Plate Boundary Layer	156
<b>7. General Properties and Exact Solutions of the Boundary-Layer Equations for Plane Flows</b>	165
7.1 Compatibility Condition at the Wall	166
7.2 Similar Solutions of the Boundary-Layer Equations	167
7.2.1 Derivation of the Ordinary Differential Equation	167
A Boundary Layers with Outer Flow	169
B Boundary Layers Without Outer Flow	172
7.2.2 Wedge Flows	172
7.2.3 Flow in a Convergent Channel	174
7.2.4 Mixing Layer	175
7.2.5 Moving Plate	177
7.2.6 Free Jet	177
7.2.7 Wall Jet	180
7.3 Coordinate Transformation	182
7.3.1 Görtler Transformation	182
7.3.2 v. Mises Transformation	183
7.3.3 Crocco Transformation	184
7.4 Series Expansion of the Solutions	184
7.4.1 Blasius Series	184
7.4.2 Görtler Series	186
7.5 Asymptotic Behaviour of Solutions Downstream	187
7.5.1 Wake Behind Bodies	187
7.5.2 Boundary Layer at a Moving Wall	190
7.6 Integral Relations of the Boundary Layer	191
7.6.1 Momentum-Integral Equation	191
7.6.2 Energy-Integral Equation	192
7.6.3 Moment-of-Momentum Integral Equations	194
<b>8. Approximate Methods for Solving the Boundary-Layer Equations for Steady Plane Flows</b>	195
8.1 Integral Methods	196