D. F. Walls
Gerard J. Milburn

Quantum Optics

Second Edition

量子光学

第2版

Springer

世界图书出版公司
www.wpcbj.com.cn

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图书在版编目（CIP）数据

量子光学：第2版＝Quantum Optics Second Edition：英文/（新西兰）沃尔斯（Walls，D. F.）著．—影印本．—北京：世界图书出版公司北京公司，2015. 11
ISBN 978-7-5192-0549-2

Ⅰ.①量…　Ⅱ.①沃…　Ⅲ.①量子光学—研究生—教材—英文　Ⅳ.①O431.2

中国版本图书馆CIP数据核字（2015）第292971号

Quantum Optics Second Edition
量子光学 第2版

著　　者：D. F. Walls and Gerard J. Milburn
责任编辑：刘　慧　岳利青
装帧设计：任志远

出版发行：世界图书出版公司北京公司
地　　址：北京市东城区朝内大街137号
邮　　编：100010
电　　话：010－64038355（发行）　64015580（客服）　64033507（总编室）
网　　址：http://www.wpcbj.com.cn
邮　　箱：wpcbjst@vip.163.com
销　　售：新华书店
印　　刷：三河市国英印务有限公司
开　　本：711mm×1245mm　1/24
印　　张：18.5
字　　数：356千
版　　次：2016年7月第1版　　2016年7月第1次印刷
版权登记：01-2015-6930
ISBN 978-7-5192-0549-2　　　定价：69.00元

Quantum Optics

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ISBN: 978-3-540-28573-1 e-ISBN: 978-3-540-28574-8

Library of Congress Control Number: 2007936291

Preface to Second Edition

The field of quantum optics today is very different form the field that Dan Walls and I surveyed in 1994 for the first Edition of this book. Some of the new fields that have emerged over the years were hinted at in the earlier edition: quantum information has at least some roots in the study of Bell's Inequalities, while the fields of ion trapping and quantum condensed gases have their roots in the old chapter on light forces. However such is the growth of activity in each of these areas that I have found it necessary to write four new chapters for this edition. In order to keep the book to a reasonable size this has meant cutting some of the material from the first edition. The old chapter on Intracavity Atomic Systems is largely gone with parts distributed in the new chapter on Cavity QED and elsewhere. Likewise the old chapter on Resonance Fluorescence has been redistributed across Chaps. 10 and 11 in this edition. No doubt more cutting could have been made but I have tried to keep some continuity with the previous edition. In any case an emphasis on experimental realisations has been retained in the new material. Preparing this edition was not as much fun as the first. With Dan Walls untimely death in 1999, I have been denied the consolations of a shared task and soldiered on alone (although I must admit to hearing his voice from time to time as I cut and pasted). I can only hope that I have not lost his vision for the book in my unchallenged role of sole author.

Brisbane, Australia,
October 2007.

G.J. Milburn

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Chapter 1
Introduction

The first indication of the quantum nature of light came in 1900 when Planck discovered he could account for the spectral distribution of thermal light by postulating that the energy of a simple harmonic oscillator was quantized. Further evidence was added by Einstein who showed in 1905 that the photoelectric effect could be explained by the hypothesis that the energy of a light beam was distributed in discrete packets later known as photons.

Einstein also contributed to the understanding of the absorption and emission of light from atoms with his development of a phenomenological theory in 1917. This theory was later shown to be a natural consequence of the quantum theory of electromagnetic radiation.

Despite this early connection with the quantum theory, physical optics developed more or less independently of quantum theory. The vast majority of physical-optics experiments can be adequately explained using classical theory of electromagnetism based on Maxwell's equations. An early attempt to find quantum effects in an optical interference experiment by G.I. Taylor in 1909 gave a negative result. Taylor's experiment was an attempt to repeat Young's famous two slit experiment with one photon incident on the slits. The classical explanation based in the interference of electric field amplitudes and the quantum explanation based on the interference of probability amplitudes both correctly explain the phenomenon in this experiment. Interference experiments of Young's type do not distinguish between the predictions of the classical theory and the quantum theory. It is only in higher order interference experiments, involving the interference of intensities, that differences between the predictions of classical and quantum theory appear. In such an experiment the probability amplitudes to detect a photon from two different fields interfere on a detector. Whereas classical theory treats the interference of intensities, in quantum theory the interference is still at the level of probability amplitudes. This is one of the most important differences between the classical and the quantum theory.

The first experiment in intensity interferometry was the famous experiment of R. Hanbury Brown and R.Q. Twiss. This experiment studied the correlation in the photocurrent fluctuations fro two detectors. Later experiments were based on photon counting, and the correlation between photon number was studied.

The Hanbury–Brown and Twiss experiment observed an enhancement in the two-time correlation function of short time delays for a thermal light source, known as photon bunching. This was a consequence of the large intensity fluctuations in the thermal source. Such photon bunching phenomenon may be adequately explained using a classical theory with a fluctuating electric field amplitude. For a perfectly amplitude stabilized light field, such as an ideal laser operating well above threshold, there is no photon bunching. A photon counting experiment where the number of photons arriving in an interval of time T are counted, shows that there is still randomness in the arrival time of the photons. The photon number distribution for an ideal laser is Poissonian. For thermal light a super-Poissonian photocount distribution results.

While the these results may be derived form a classical and quantum theory, the quantum theory makes additional unique predictions. This was first elucidated by R.J. Glauber in his quantum formulation of optical coherence theory in 1963. Glauber was jointly awarded the 2005 Nobel Prize in physics for this work. One such prediction is photon anti bunching, in which the initial slope of the two-time photon correlation function is positive. This corresponds to an enhancement, on average, of the temporal separation between photo counts at a detector, or photon anti-bunching. The photo-count statistics may also be sub-Poissonian. A classical theory of fluctuating field amplitudes would require negative probability in order to give anti-bunching. In the quantum picture it is easy to visualize photon arrivals more regular than Poissonian.

It was not until 1975 that H.J. Carmichel and D.F. Walls predicted that light generated in resonance fluorescence fro a two-level atom would exhibit photon anti-bunching that a physically accessible system exhibiting non-classical behaviour was identified. Photon anti-bunching in this system was observed the following year by H.J. Kimble, M. Dagenais and L. Mandel. This was the first non classical effect observed in optics and ushered in a new era of quantum optics.

The experiments of Kimble et al. used an atomic beam and hence the photon anti-bunching was convoluted with the atomic number fluctuations in the beam. With the development of ion trap technology it is now possible to trap a single ion for many minute and observe fluorescence. H. Walther and co workers in Munich have studied resonance fluorescence from a single ion in a trap and observed both photon bunching and anti-bunching.

In the 1960s improvements in photon counting techniques proceeded in tandem with the development of new laser light sources. Light from incoherent (thermal) and coherent (laser) sources could now be distinguished by their photon counting statistics. The groups of F.T. Arecchi in Milan, L. Mandel in Rochester and R. Pike in Malvern measured the photo count statistics of the laser. These experiments showed that the photo-count statistics went from super-Poissonian below threshold to Poissonian far above threshold. Concurrently the quantum theory of the laser was being developed by H. Haken in Stuttgart, M.O. Scully and W. Lamb in Yale and M. Lax and W.H. Louisell in New Jersey. In these theories both the atomic variables and the electromagnetic field were quantized. The results of these calculations were that the laser functioned as an essentially classical device. In fact H. Risken showed that it could be modeled as a van der Pol Oscillator.

In the late 80s the role of noise in the laser pumping process was shown to obscure the quantum aspects of the laser. If the noise in the pump can be suppressed the laser may exhibit sub-Poissonian statistics. In other words the intensity fluctuations may be reduced below the shot noise level of normal lasers. Y. Yamamoto first in Tokyo and then Stanford has pioneered experimental developments of semiconductor lasers with suppressed pump noise. More recently, Yamamoto and others have pioneered the development of the single photon source. This is a source of transform-limited pulsed light with one and only one photon per pulse: the ultimate limit of an anti-bunched source. The average field amplitude of such a source is zero while the intensity is definite. Such sources are highly non classical and have applications in quantum communication and computation.

It took another nine years after the first observation of photon anti-bunching for another prediction of the quantum theory of light to be observed – squeezing of quantum fluctuations. The electric field of a nearly monochromatic plane wave may be decomposed into two quadrature component amplitudes of an oscillatory sine term and a cosine term. In a coherent state, the closest quantum counter-part to a classical field, the fluctuations in the two quadrature amplitudes are equal and saturate the lower bound in the Heisenberg uncertainty relation. The quantum fluctuations in a coherent state are equal to the zero point fluctuations of the vacuum and are randomly distributed in phase. In a squeezed state the fluctuations are phase dependent. One quadrature phase amplitude may have reduced fluctuations compared to the vacuum while, in consequence, the other quadrature phase amplitude will have increased fluctuations, with the product of the uncertainties still saturating the lower bound in the Heisenberg uncertainty relation.

The first observation of squeezed light was made by R.E. Slusher in 1985 at AT&T Bell Laboratories in four wave mixing. Shortly after squeezing was demonstrated using optical parametric oscillators, by H.J. Kimble and four wave mixing in optical fibres by M.D. Levenson. Since then, greater and greater degrees of quantum noise suppression have been demonstrated, currently more than 7 dB, driven by new applications in quantum communication protocols such as teleportation and continuous variable quantum key distribution.

In the nonlinear process of parametric down conversion, a high frequency photon is absorbed and two photons are simultaneously produced with lower frequencies. The two photons produced are correlated in frequency, momentum and possibly polarisation. This results in very strong intensity correlations in the down converted beams that results in strongly suppressed intensity difference fluctuations as demonstrated by E. Giacobino in Paris and P. Kumar in Evanston.

Early uses of such correlated twin beams included accurate absorption measurements in which the sample was placed in one arm with the other beam providing a reference. when the twin beams are detected and the photo currents are subtracted, the presence of very weak absorption can be seen because of the small quantum noise in the difference current. More recently the strong intensity correlations have been used to provide an accurate calibration of photon detector efficiency by A. Migdall at NIST and also in so called quantum imaging in which an object paced in one path changes the spatial pattern of intensity correlations between the two twin beams.

The high degree of correlation between the down converted photons enables some of the most stringent demonstrations of the violation of the Bell inequalities in quantum physics. In 1999 P. Kwiat obtained a violation by more than 240 standard deviations using polarisation correlated photons produced by type II parametric down conversion. The quadrature phase amplitudes in the twin beams generated in down conversion carry quantum correlations of the Einstein-Podolsky-Rosen type. This enabled the continuous variable version of quantum teleportation, proposed by L. Vaidmann, to be demonstrated by H.J. Kimble in 1998. More recently P.K. Lam, using the same quadrature phase correlations, demonstrated a continuous variable quantum key distributions.

These last examples lie at the intersection of quantum optics with the new field of quantum information. Quantum entanglement enables new communication and computational tasks to be performed that are either difficult or impossible in a classical world. Quantum optics provides an ideal test bed for experimental investigations in quantum information, and such investigations now form a large part of the experimental agenda in the field.

Quantum optics first entered the business of quantum information processing with the proposal of Cirac and Zoller in 1995 to use ion trap technology. Following pioneering work by Dehmelt and others using ion traps for high resolution spectroscopy, by the early 1990s it was possible to trap and cool a single ion to almost the ground state of its vibrational motion. Cirac and Zoller proposed a scheme, using multiple trapped ions, by which quantum information stored in the internal electronic state of each ion could be processed using an external laser to correlate the internal states of different ions using collective vibrational degrees of freedom. Ion traps currently provide the most promising approach to quantum information processing with more than eight qubits having been entangled in the labs of D. Wineland at NIST in Colorado and R. Blatt in Innsbruck.

Quantum computation requires the ability to strongly entangle independent degrees of freedom that are used to encode information, known as qubits. It was initially thought however that the very weak optical nonlinearities typically found in quantum optics would not be powerful enough to implement such entangling operations. This changed in 2001 when E. Knill, R. Laflamme and G.J. Milburn, followed shortly thereafter by T. Pittman and J. Franson, proposed a way to perform conditional entangling operations using information encoded on single photons, and photon counting measurements. Early experimental demonstrations of simple quantum gates soon followed.

At about the same time another measurement based protocol for quantum computing was devised by R. Raussendorf and H. Breigel. Nielsen showed how this approach could be combined with the single photon methods introduced by Knill et al., to dramatically simplify the implementation of conditional gates. The power of this approach was recently demonstrated by A. Zeilinger's group in Vienna. Scaling up this approach to more and more qubits is a major activity of experimental quantum optics.

These schemes provide a powerful incentive to develop a totally new kind of light source: the single photon pulsed source. This is a pulsed light source that produces one and only one photon per pulse. Such sources are in development in many laboratories around the world. A variety of approaches are being pursued. Sources based on excitons in semiconductor quantum dots are being developed by A. Imamoglu in Zurich, A. Shields in Toshiba Cambridge, and Y. Yamamoto and J. Vukovic in Stanford. NV centres in diamond nanocrystal are under development by S. Prawer in Melbourne. An interesting approach based on down conversion in optical fibers is being studied by A. Migdall in NIST. Sources based on single atoms in optical cavities have been demonstrated by H. Walther in Munich and P. Grangier in Paris. Once routinely available, single photon sources will enable a new generation of experiments in single photon quantum optics.

Beginning in the early 1980s a number of pioneers including G. Ashkin, C. Cohen Tannoudji and S. Chu began to study the forces exerted on atoms by light. This work led to the ability to cool and trap ensembles of atoms, or even single atoms, and culminated in the experimental demonstration by E. Cornell and C. Weimann of a Bose Einstein condensate using a dilute gas of rubidium atoms at NIST in 1995, followed soon thereafter by W. Ketterle at Harvard. Discoveries in this field continue to enlighten our understanding of many body quantum physics, quantum information and non linear quantum field theory. We hardly touch on this subject in this book, which is already well covered in a number of recent excellent texts, choosing instead to highlight some aspects of the emerging field of quantum atom optics.