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基于小波域隐马科夫模型的 图像去噪

Image Denoising Using Wavelet Domain
Hidden Markov Models

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Abstract

Most recently, wavelet domain hidden Markov models (WD-HMMs) are proposed for image processing. The basic idea of WD-HMM is that the dependencies among wavelet coefficients can be efficiently captured by hidden Markov models (HMMs) since the dependency between two wavelet coefficients dies down quickly as their distance becomes big. Besides this, WD-HMM also allows us to approximate the distribution of the wavelet coefficient with a Gaussian Mixture model (GMM). Since WD-HMMs construct the HMMs on the wavelet domain, they lead the model simpler than the traditional statistical models in the spatial domain.

However, most of the improvements of the WD-HMM focus on how to add some additional structures on the original WD-HMM. Generally speaking, the new structures will improve the performance of image processing, however, even some simple additional structures will lead to increasing computational complexity significantly. Image denoising without blurring the edges is a difficult problem. Constructing adaptable denoising algorithms is an important technique to solve this problem. However, most of spatial denoising techniques blur the edges and singularities of the images since they consider the details of the images are the same as the noise. In this book, we firstly propose a new adaptive denoising framework on wavelet domain and then extend it to WD-HMMs with some new local structures.

The new adaptive denoising techniques based on the fact that the

images are non - stationary with singularities and some smooth areas, which can be considered as stationary. Firstly, the singularities are separated from the smooth areas. Thus we can handle the different coefficients separately. The local squares is defined on the context, that is, the variance of the singularity will be estimated in a relatively large square while the variance of the smooth coefficients will be estimated in a smaller square. This new framework is different from traditional local context methods, which estimate the variance of the signal using the pixels with the same context in a moving windows. In order to reduce the artifacts in the denoising images, we construct a template in LL subband and then used it to all subbands. This new technique combing with the block model will be extended to WD - HMMs.

Our new frameworks consider each subband of the wavelet coefficients to be a Gaussian Mixture field (GMF), that is, each wavelet coefficient is a random variable with GMM, and allows the dependency links among the hidden states of the wavelet coefficients. Therefore, the joint distribution of each subband can be easily decided by the new frameworks. Then the standard parameter estimation of the new models can be obtained from the EM algorithm and the estimated parameters are used for signal and image denoising.

In order to obtain the adaptable image denoising results, we must obtain the local estimated parameters firstly. Based on carefully designed local structure on wavelet domain, we can use the same local squares and further consider the block structure which coincides the non - stationary of images. That is, in the local squares, we will consider not only the number of coefficients with different context but also consider the block labels of these coefficients. This help us to correct the local structure of the local denoising technique. Thus the estimation will be in the

same adaptive squares and blocks. We know that the template can be used to reduce the artifacts in the denoising images. In fact, the template also can help us properly construct the adaptive structure in noisy. This will be discussed in our book.

After obtaining the estimated parameters on the wavelet domain, we can use these parameters for image denoising on the wavelet domain. Finally, the denoised image can be obtained from an inverse wavelet transform.

We give some examples on signal and image denoising using the block HMM and the template HMM relatively to show the power and potentiality of the new frameworks. The experimental results show that the block HMM and the template HMM can efficiently improve the spatial adaptability in a simple way. They also show that signals with relatively stable nature and images with a proper structure of the texture have better denoising results. Finally, we give the summary of our works and discuss the future work.

CONTENTS

Chapter 1	Introduction	(1)
1.1	Motivation	(2)
1.2	Outline of Book	(5)
1.3	Original Contributions	(7)
Chapter 2	Wavelet and Hidden Markov Model (HMM)	(10)
2.1	Wavelet Analysis	(11)
2.1.1	Integral Wavelet Transform	(11)
2.1.2	Discrete Wavelet Transform	(13)
2.1.3	Multiresolution Analysis (MRA) of $L^2(\mathbf{R})$	(14)
2.1.4	Wavelet Packet	(22)
2.1.5	Two Dimensional Wavelets	(23)
2.2	Hidden Markov Model	(28)
2.2.1	From Markov Process to Markov Model	(28)
2.2.2	Elements of an Hidden Markov Model	(32)
2.2.3	The Three Basic Problems for HMM	(34)
2.2.4	Solution to Problem 1: Probability Evaluation	(36)
2.2.5	Solution to Problem 2: "Optimal" State Sequence	(38)
2.2.6	Solution to Problem 3: Parameter Estimation	(40)
2.2.7	Continuous Observation Densities in HMMs	(45)

Chapter 3	Image Denoising on Wavelet Domain	(48)
3.1	Introduction	(48)
3.2	Important Statistical Preparations	(51)
3.2.1	Prior and Posterior Distribution	(52)
3.2.2	Markov Random Field	(54)
3.2.3	Maximum Likelihood Estimate	(56)
3.2.4	Expectation – Maximization (EM)	(58)
3.3	Description of Image Denoising	(60)
3.3.1	Gaussian White Noise	(61)
3.3.2	Signal and Noise Ratios	(73)
3.3.3	Criteria	(77)
3.4	Wavelet Thresholding	(78)
3.4.1	Hard and Soft Thresholding	(79)
3.4.2	Improvements of Wavelet Thresholding	(80)
3.5	Least Square Estimation	(85)
3.6	Spatial Image Denoising	(90)
3.6.1	New Frameworks	(93)
3.6.2	Denoising Results	(96)
3.7	Summary	(110)
Chapter 4	Spatial Wavelet Domain Hidden Markov Model	(111)
4.1	Preparations	(112)
4.1.1	Statistical Models and Wavelet	(112)
4.1.2	Gaussian Mixture Model	(115)
4.1.3	EM Algorithm	(120)
4.1.4	Least Square Estimation on Wavelet Domain	(125)
4.2	Wavelet Domain – Hidden Markov Models	(126)
4.2.1	Independent Mixture (IM) Model	(126)
4.2.2	Hidden Markov Tree (HMT) Model	(127)

4.2.3	Contextual Hidden Markov Model (CHMM)	(133)
4.3	Spatial Wavelet Domain – Hidden Markov Models	(137)
4.3.1	Gaussian Markov Field	(138)
4.3.2	Block	(139)
4.3.3	Template	(144)
4.3.4	EM Algorithm of the Block HMM	(148)
4.3.5	EM Algorithm of the Template HMM	(150)
4.3.6	Some Views about Improved WD – HMM	(151)
Chapter 5	Signal Denoising Using the Block HMM	(154)
5.1	Signal Denoising	(156)
5.2	The Signal Denoising Algorithm Using the Block HMM	(159)
5.3	Experimental Results and Discussion	(165)
5.4	Conclusions	(171)
Chapter 6	Image Denoising Using the Template HMM	(173)
6.1	Image Denoising	(175)
6.2	The Image Denoising Algorithm Using the Template HMM	(180)
6.3	Experimental Results and Discussion	(186)
6.4	Conclusions	(193)
Chapter 7	Conclusions and Perspectives	(195)
Bibliography		(199)

Chapter 1

Introduction

Over the last decade, there has been abundant interest for noise removal in signals and images. In many hundreds of papers published in journals throughout the scientific and engineering disciplines, a wide range of tools and ideas have been proposed and studied. However, the searching for efficient image denoising methods still is a valid challenge, at the crossing of functional statistics and other important techniques.

Most of the denoising techniques consider the image is stationary with Gaussian distributions. However, these two assumptions are not real in most of images. In fact, most of images are nonstationary with some smooth areas and singularities. In order to match this nature, one important technique, named the spatial denoising techniques is discussed in some works [26] – [28]. However, all these spatial denoising techniques is based on the context and a set of moving windows, which lead to blur the edges of denoising image since they consider the noise is the same as the details of the images.

In our book, we firstly construct a new adaptive denoising framework on wavelet domain, and then extend it to the WD – HMMs with some new structures. Since our new method can efficiently separate the singularities and smooth areas in the images and estimates these different coefficients in different squares, it can obtain better denoising results especially in heavy noise.

The contents in this Chapter is as follows: we will discuss the moti-

vation about our book, and then gives the outline of this book. Finally, we will introduce our original contributions.

1.1 Motivation

An image model attempts to capture the key characteristics of an image based on which image processing problems can be formulated and solved mathematically and systematically. The solution of the problem depends on the developing mathematical tools.

One important tool is wavelet transform that obtains the powerful image processing results based on simpler scale transform. While its origins go back many decades, the subject of wavelets has become a popular tool in image processing only within the last two decades or so partly as a result of Ingrid Daubechies's celebrated work on the construction of compactly supported, orthonormal wavelets. However, most of wavelet techniques regard each wavelet coefficient independent each other. For the images, it is obvious that there are some resident dependencies among wavelet coefficients from the similarity for the same position of all subbands and from the similarity for neighboring wavelet coefficients. Therefore, some new frameworks capturing the dependencies among the wavelet coefficients maybe have better processing results.

The statistical model is a suitable tool for describe the dependencies among some random variables. It considers an image as a realization of a random field, that is, it considers each position in the image as a random variable with a distribution and the value of the position is a observation for the random variable. Based on the distribution of each position, the statistical techniques allow some dependency structures to capture the key dependencies among the image or the signal. Then the sig-

nal and image processing can be obtained from the statistical information about the image. However, the biggest weakness of the statistical techniques is its complexity increases quickly as the sample points of the image increases.

Wavelet transform is an efficient tool for reducing the complexity of image processing since the image can be represented as a few wavelet coefficients by wavelets. The natural idea to reduce the complexity of the statistical techniques is to process the wavelet coefficients using statistical techniques since wavelets “simplify” the original image. Fortunately, statistical techniques also help us to handle the problem of how to capture the dependencies among wavelet coefficients using some statistical models and how to describe the distributions of the wavelet coefficients. A model, called hidden Markov model (HMM), is a famous one. The basic idea for the HMM is that the dependency between two pixels in an image reduces quickly as the distance of these two pixels increases. In summary, we can use HMM to capture the dependencies between a pixel and its neighbors on the wavelet domain.

Most recently, wavelet domain hidden Markov models (WD-HMM) are used to image processing and obtain powerful processing results. The basic idea of WD-HMM is that the dependencies among wavelet coefficients can be efficiently captured by hidden Markov model that coincides the nature of the dependency between two positions dies down quickly when the two positions are far away. Besides this, WD-HMM also allows us to approximate the distribution of the wavelet coefficients using Gaussian Mixture model (GMM). Since WD-HMMs construct a HMM on the wavelet domain, it leads the model simpler than the traditional statistical models in spatial domain. Therefore, WD-HMM is a simple and powerful model that allows us properly modelling the non-

Gaussian of wavelet coefficients and dependencies among wavelet coefficients.

However, most of improvements of the WD – HMM focus on how to add an additional structure on the original WD – HMM. Generally speaking, the new structure must improve the performance of image processing, however, even a simple additional structure will lead to increasing the computation complexity of image processing quickly.

Most of the denoising techniques consider the image is stationary. However, it is not real in images. In fact, most of images are nonstationary with some smooth areas and singularities. In order to match this nature, one important technique, named the spatial denoising techniques is discussed in some works [26] – [28]. However, all these spatial denoising techniques is based on the context and a set of moving windows, which lead to blurring edges of the denoising image since they consider the noise is the same as the details of the images.

We firstly construct a new adaptive denoising framework on wavelet domain, and then extend it to the WD – HMMs with some new structures. Since our new method can efficiently separate the singularities and smooth areas in the images and estimates these different coefficients in different squares, it can obtain better denoising results especially in heavy noise.

In this book, we study WD – HMMs regarding both statistical image modelling, and the applications to various image denoising problems. The aim of the book is that finds a more reasonable way to construct spatial image denoising frameworks. The outline and original contributions of this work are given as follows.

1.2 Outline of Book

This book has seven Chapters and their contents are shown on Figure 1.1.

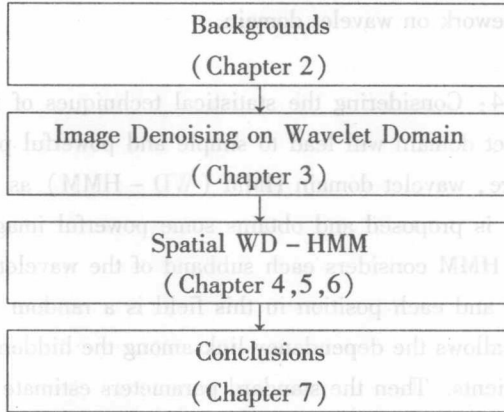


Figure 1.1 Outline of the book

Chapter 2: We firstly review the theory of wavelet transforms, including to several 1 - D wavelet transforms, wavelet packet, 2 - D wavelet transforms. We also introduce the multiresolution analysis (MRA), the decomposition and the reconstruction algorithms of the wavelet transform, that is, an image can be decomposed into some wavelet coefficients, and also can be recovered using an inverse wavelet transform. The wavelet transform is a kind of MRA that localized both on the time and spatial domain. After that, we review the basic concept of the HMM, that is, HMM is composed by two Markov processes and one can be observed; the other is hidden. In order to process the image using HMM, we have to face its three element problems, the solutions of these

problems are given in final part of this Chapter.

Chapter3: We firstly introduce relative statistical image processing techniques in this Chapter. Then the nature of noise is discussed, after that, wavelet shrinkage is reviewed. Finally, we propose a new spatial denoising framework on wavelet domain.

Chapter 4: Considering the statistical techniques of image processing on wavelet domain will lead to simple and powerful processing results. Therefore, wavelet domain HMM (WD - HMM) as a new image processing tool is proposed and obtains some powerful image processing results. WD - HMM considers each subband of the wavelet transform is a random field and each position in this field is a random variable with GMM. It also allows the dependency link among the hidden states of the wavelet coefficients. Then the standard parameters estimate problem can be achieved by the EM algorithm. Since WD - HMMs lack the spatial adaptability, we focus on how to obtain local and reliable estimated parameters in the EM algorithm. However, it is a tradeoff for obtaining both local and reliable estimated parameters. Fortunately, the block is a proper balance of this tradeoff. In order to reduce the effect of the noise for the block HMMs, the template is constructed in the subband of the scaling coefficients and then it is used to the all subbands of the wavelet coefficients. In this Chapter, we discuss the constructing techniques of the block and template, also discuss the EM algorithms about these two models.

In order to demonstrate the power and potentiality of the improved WD - HMMs, we give two examples of signal and image denoising using the block HMM and the template HMM relatively.

Chapter 5: To obtain the spatial adaptability and the low computation complexity, we develop a new WD – HMM, called the block HMM for signal denoising. It is a highly localized model with rich local statistics. Therefore, the new model can obtain state – of – art denoising performance at the low computation complexity.

Chapter 6: The image denoising not blurring the edges is a difficult problem. We propose a new framework, called the template HMM, to reduce the noise effects, and obtain both reliable and local estimated parameters in a simple way. The new framework can improve the image denoising results largely at a low computation complexity and also avoids denoising artifacts in denoised images.

Chapter 7: Finally, conclusions and perspectives are given. Based on the results from Chapter 6 to Chapter 5, we conclude that the new WD – HMMs are proven efficient for statistical image denoising problems, and suggest that they can be used for specific applications with distinct considerations and requirements. Meanwhile, the image denoising algorithms are also found essential to the applications of wavelet domain HMMs. We predict the future developments of this work.

1.3 Original Contributions

Corresponding to the outline above, the original contributions of this book consist of three parts as follows:

Statistical Modeling: Firstly, we propose a new spatial denoising framework on wavelet domain. This model can efficiently separate the singularities from the smooth areas and estimate the variance of different coefficients in local squares with different sizes. The size of a local square is decided by the number of coefficients with the same context. Based on the new denoising framework proposed by us, we proposed two new frameworks, called the block HMM and the template HMM, and give EM algorithms about these two frameworks. We also give a system discussion about how to modify the blocks and the templates in a proper way. These two frameworks improve the spatial adaptability of the WD - HMMs in a simple way and can be extended to all the WD - HMMs easily. In this book, we use the context combined with the local character grouping and classification techniques to improve the performance of statistical modelling in the wavelet domain. These schemes improve WD - HMMs in terms of training efficiency and modelling accuracy, and provide some new ideas to improve the performance of the WD - HMMs.

Signal Denoising: Keeping the abrupt changes of signals is difficult in signal denoising. The most important technique for obtaining this target is to improve the spatial adaptability of the statistical modelling. However, it is a tradeoff between obtaining a local and reliable estimated parameters that will be used to the signal denoising late. Fortunately, a statistical modelling, called the block HMM, proposed by us in this book, is a proper balance for this tradeoff. The experimental results show that the new framework can efficiently improve the spatial adaptability of the WD - HMM in a simple way and obtain powerful denoising results.