牛 津

学科英语基础丛书

GCSE

数 学

through diagrams

MATHEMATICS

牛津图解中学数学

Andrew Edmondson

英汉双语

上海教育出版社

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（英汉双语）

Andrew Edmondson 著

韩 希 塘 译

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CONTENTS 目 录

Number 数

Algebra 代数

注意　高水平的材料标有 H

说明：本书采用一页对一页的排法，左边一页是英语，右边一页是汉语，两页的页码一致。原书前10页的内容与正文无关，因此本书从第11页开始。

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CONTENTS 目 录

Whole Numbers

1. Basic Arithmetic

Make sure you can answer these questions *without* using a calculator.

1. *There are 857 boys and 946 girls in a school. How many children are there altogether?* $\begin{array}{r} 857 \\ +\ \underline{946} \end{array}$
2. *The battle of Waterloo took place in 1815. The battle of Hastings took place in 1066. How many years apart are these events?* $\begin{array}{r} 1815 \\ -\ \underline{1066} \end{array}$
3. *Boxes contain 17 tiles each. How many tiles are there in 12 boxes?* $\begin{array}{r} 17 \\ \times\ \underline{12} \end{array}$
4. *Lollipops cost 13p each. How many can be bought for 234p?* $13\,\overline{)\,234}$

2. Sum, Difference, Product, Quotient

The **sum** of 6 and 2 is written $6+2$ or $2+6$. Both calculations give the same answer, 8.

The **difference** of 2 and 6 is written $6-2$ or $2-6$. The **positive difference** is $6-2 = 4$. The **negative difference** is $2-6 = -4$.

The **product** of 2 and 6 is written 2×6 or 6×2. Both calculations give the same answer, 12.

The **quotient** of 2 and 6 is written $6\div2$ or $2\div6$. These calculations give different answers.

NOTE Another way of writing $6\div2$ is $\frac{6}{2}$.

3. Brackets

A bracket contains a *single* number. For example, the bracket $(6+2)$ contains the single number 8.

When a bracket is multiplied by a number, the × sign is usually omitted. For example:

$3\times(4+1)$ is usually written $3(4+1)$

$(2+3)\times(5-2)$ is usually written $(2+3)(5-2)$

4. BoDMAS

Always use the following order of working when performing a calculation.

① Work out **B**rackets
② **D**ivide and **M**ultiply
③ **A**dd and **S**ubtract

Remember this using the word BoDMAS

Examples

$3(4+1) = 3\times(4+1) = 3\times(5) = 3\times5 = 15$
brackets first........... drop brackets......then multiply

$4+3\times2 = 4+6 = 10$
multiply first.........then add

$3+4(3-1)+6\div3-5 = 3+4\times2+6\div3-5 = 3+8+2-5 = 8$
brackets first.................................multiply and divide.............add and subtract

$4\times3\times2 = 12\times2 = 24$
$4\times3\times2 = 4\times6 = 24$
Multiply any two numbers first; you will always get the same answer

$12\div2\div3 = 6\div3 = 2$
$6\div3\times4 = 2\times4 = 8$
Work from left to right with combinations of × and ÷ signs

5. Natural Numbers

Whole positive numbers are sometimes called **natural** numbers:

1, 2, 3, 4, 5, 6, ...

6. Multiplying and Dividing by Zero

Multiplying numbers by 0 always gives 0.

For example, $5\times0 = 0$, $0\times5 = 0$, $3\times5\times0 = 0$

Dividing 0 by any number (other than 0) always gives 0.

For example, $0\div5 = 0$ or $\frac{0}{5} = 0$

Dividing by 0 is impossible.

For example, $5\div0$ or $\frac{5}{0}$ is impossible

7. Powers

4^3 is shorthand for $4\times4\times4$ and is called the **3rd power** of 4. It is commonly called the **cube** of 4. We also say that 4 has been **raised to the 3rd power**. So, $4^3 = 4\times4\times4 = 64$.

NOTE 4^3 does ***not*** mean the same as 4×3. Here is the difference:
$4^3 = 4\times4\times4$ whereas $4\times3 = 4+4+4$

Similarly, 4^2 is shorthand for 4×4 and is called the **2nd power** of 4, or more commonly the **square** of 4 (or 4 squared). So, $4^2 = 4\times4 = 16$.

4^1 is another way of writing 4

Question *Calculate the value of* 2^4+3^2.
$2^4+3^2 = 2\times2\times2\times2 + 3\times3 = 16+9 = 25$

Base $\longrightarrow$ 4^3 $\longleftarrow$ Index (plural: indices)

8. Power of a Bracket

$(\ )^2$ means $(\ )\times(\ )$ $\qquad$ $(\ )^3$ means $(\ )\times(\ )\times(\ )$

So, $(5+3)^2 = (5+3)\times(5+3) = 8\times8 = 64$

And $(6-1)^3 = (6-1)\times(6-1)\times(6-1) = 5\times5\times5 = 125$

Also $(5\times3)^2 = (5\times3)\times(5\times3) = 15\times15 = 225$.

You get the same answer by squaring each number in the bracket and multiplying the results together:
$(5\times3)^2 = 5^2\times3^2 = 5\times5 \times 3\times3 = 25\times9 = 225$

So we have the result: $(5\times3)^2 = 5^2\times3^2$

Similarly: $(6\div3)^2 = 6^2\div3^2 = \left(\frac{6}{3}\right)^2$

NOTE $(5+3)^2$ is ***not*** equal to 5^2+3^2. Here's why:
$(5+3)^2 = 8^2 = 64$ whereas $5^2+3^2 = 25+9 = 34$

整数

1. 基本算术

确信你能不用计算器回答这些问题。

1. 某校有857个男孩，946个女孩，一共有多少孩子？

$$\begin{array}{r} 857 \\ +\ 946 \\ \hline \end{array}$$

2. 滑铁卢战役发生于1815年，哈斯丁战役发生于1066年，这两个事件相隔多少年？

$$\begin{array}{r} 1815 \\ -\ 1066 \\ \hline \end{array}$$

3. 每个盒子装17条领带，12个盒子共装多少条领带？

$$\begin{array}{r} 17 \\ \times\ 12 \\ \hline \end{array}$$

4. 棒糖每根13便士，234便士能买多少棒糖？

$$13\,\overline{)\,234}$$

2. 和，差，积，商

6与2的**和**写作6+2或2+6．两者计算得到同一个答案：8.

2与6的**差**写作6−2或2−6．

正的差是6−2=4．**负的差**是2−6=−4．

2与6的**积**写作2 × 6或6 × 2．两者计算得到同一个答案：12．

2与6的**商**写作6 ÷ 2或2 ÷ 6．它们的计算结果不同．

注意　6 ÷ 2的另一种写法是 $\frac{6}{2}$．

3. 括号

括号包含了一个单一的数．例如，括号(6+2)包含了单一的数8．

当一个括号乘以某一个数时，乘号“×”通常被省略，例如：

3 × (4+1)通常写作3(4+1)

(2+3) × (5−2)通常写作(2+3)(5−2)

4. 运算顺序

进行计算时总是按下列顺序：

①去括号
②除或乘
③加或减

用去括号，先乘除后加减记忆

例

$3(4+1) = 3\times(4+1) = 3\times(5) = 3\times5 = 15$

先算括号内……去括号…………再乘

$4+3\times2 = 4+6 = 10$

先乘…….再加

$3+4(3-1)+6\div3-5 = 3+4\times2+6\div3-5 = 3+8+2-5 = 8$

先算括号内…………………………乘和除……………………加和减

$4\times3\times2 = 12\times2 = 24$　任意两个数先乘，

$4\times3\times2 = 4\times6 = 24$　你总是得到相同的答案

$12\div2\div3 = 6\div3 = 2$

$6\div3\times4 = 2\times4 = 8$

乘除号组合时从左到右运算

5. 自然数

正整数有时又叫**自然数**：

1, 2, 3, 4, 5, 6, ...

6. 被零乘和除

任意数乘以0都得0．

例如，5 × 0=0，0 × 5=0，3 × 5 × 0=0

0除以任意数(0除外)都得0．

例如，0 ÷ 5=0　或 $\frac{0}{5}$=0

除以0无意义．

例如，5 ÷ 0　或 $\frac{5}{0}$ 无意义

7. 幂

4^3是4 × 4 × 4的简略写法，称为4的**3次幂**．通常称作4的**立方**．我们也说4**自乘到3次幂**．于是，4^3=4 × 4 × 4=64．

注意　4^3与4 × 3的意义不一样．其差别在于：

4^3=4 × 4 × 4而4 × 3=4+4+4

类似地，4^2是4 × 4的简略写法，称为4的**2次幂**，或更常用的4的**平方**(或4自乘)．于是，4^2=4 × 4=16．

4^1是4的另一种写法

问题　计算2^4+3^2的值．

2^4+3^2=2 × 2 × 2 × 2+3 × 3=16+9=25

底 ⟶ 4^3 ⟵ 指数(index 的复数形式是 indices)

8. 括号的幂

$(\ )^2$意思是(　) × (　)　　$(\ )^3$意思是(　) × (　) × (　)

因此，$(5+3)^2$=(5+3) × (5+3)=8 × 8=64

$(6-1)^3$=(6−1) × (6−1) × (6−1)=5 × 5 × 5=125

同样　$(5 \times 3)^2$=(5+3) × (5+3)=15 × 15=225．

如果通过自乘括号中的每个数，然后把结果相乘也能得得同样的答案：

$(5 \times 3)^2=5^2 \times 3^2$=5 × 5 × 3 × 3=25 × 9=225

于是我们有结论：$(5\times3)^2 = 5^2\times3^2$

类似地：$(6\div3)^2 = 6^2\div3^2 = \left(\frac{6}{3}\right)^2$

注意　$(5+3)^2$不等于5^2+3^2．这是因为：

$(5 \times 3)^2=8^2$=64　而5^2+3^2=25+9=34

Whole Numbers (Contd)

9. Roots

The opposite process of calculating the power of a number is finding its **root**.

The 3rd power (cube) of 2 is written 2^3 and is equal to 8. The **3rd root (cube root)** of 8 is written $\sqrt[3]{8}$ and equals 2.

$$\begin{tikzcd} 2 \arrow[r, bend left, "\text{Cube (3rd power)}"] & 8 \arrow[l, bend left, "\text{Cube root (3rd root)}"] \end{tikzcd}$$

The 2nd power (square) of 5 is written 5^2 and equals 25. The **2nd root (square root)** of 25 is written $\sqrt[2]{25}$, or simply $\sqrt{25}$, and is equal to 5.

$$\begin{tikzcd} 5 \arrow[r, bend left, "\text{Square (2nd power)}"] & 25 \arrow[l, bend left, "\text{Square root (2nd root)}"] \end{tikzcd}$$

Question *Calculate $\sqrt[4]{16}$ without using a calculator.*

$\sqrt[4]{16}$ is the 4th root of 16.

The answer must be 2, because:

$$\begin{tikzcd} 2 \arrow[r, bend left, "\text{4th power}"] & 16 \arrow[l, bend left, "\text{4th root}"] \end{tikzcd}$$

4^{th} power of 2 = $2^4 = 2\times2\times2\times2 = 16$

So, $\sqrt[4]{16} = 2$ because $2\times2\times2\times2 = 16$

NOTE $\sqrt[4]{16}$ does ***not*** mean $16\div4$ (see above).

$\sqrt{16+9}$ is ***not*** equal to $\sqrt{16} + \sqrt{9} = 4+3 = 7$.

$\sqrt{16+9} = \sqrt{25} = 5$ is correct.

10. Powers and Roots

Powers and roots cancel each other out.

For example:

$$6 \xrightarrow{\text{Square}} 6^2 = 36 \xrightarrow{\text{Square root}} \sqrt{36} = 6$$

Another way of writing this is $\sqrt{6^2} = 6$

Similarly, $(\sqrt{6})^2 = 6$ or $\sqrt{6}\times\sqrt{6} = 6$

11. Multiples

$$\left.\begin{array}{l} 1\times3 = 3\\ 2\times3 = 6\\ 3\times3 = 9\\ 4\times3 = 12\\ 5\times3 = 15\\ 6\times3 = 18\\ 7\times3 = 21\\ 8\times3 = 24\\ 9\times3 = 27 \end{array}\right\} \text{These numbers are called } \textbf{multiples} \text{ of 3.} \qquad \left.\begin{array}{l} 1\times4 = 4\\ 2\times4 = 8\\ 3\times4 = 12\\ 4\times4 = 16\\ 5\times4 = 20\\ 6\times4 = 24\\ 7\times4 = 28\\ 8\times4 = 32\\ 9\times4 = 36 \end{array}\right\} \text{These numbers are called } \textbf{multiples} \text{ of 4.}$$

Some of the multiples of 3 and 4 are the same. They are called **common multiples**.

3, 6, 9, 12, 15, 18, 21, 24, 27, ... Multiples of 3

4, 8, 12, 16, 20, 24, 28, 32, 36, ... Multiples of 4

12 and 24 are common multiples of 3 and 4.

12 is the **lowest common multiple (LCM)** of 3 and 4. It is the smallest number that both 3 and 4 divide into exactly.

12. Factors

The **factors** of 12 are those numbers that divide exactly into 12: 1, 2, 3, 4, 6, 12

(Don't forget to include 1 and 12.)

The factors of 18 are: 1, 2, 3, 6, 9, 18

Some of the factors of 12 and 18 are the same; they are called **common factors**.

1, 2, 3, 4, 6, 12 Factors of 12

1, 2, 3, 6, 9, 18 Factors of 18

The common factors of 12 and 18 are 1, 2, 3, and 6.

The **highest common factor (HCF)** of 12 and 18 is 6.

13. Prime Numbers

A **prime number** is a number that can be divided exactly only by itself and 1. For example, 11 is prime. Here are the first few prime numbers:

2, 3, 5, 7, 11, 13, 17, 19, 23, 29

1 is not considered to be a prime number.

14. Prime Factors

The factors of 12 are: 1, 2, 3, 4, 6, 12

Of these, 2 and 3 are prime numbers and so are called **prime factors**.

Question *What are the prime factors of 2^5?*

$2^5 = 2\times2\times2\times2\times2 = 32$

The factors of 32 are: 1, 2, 4, 8, 16, 32

So, 2 is the only prime factor of 2^5.

15. Product of Prime Factors

Any number can be written as a product of its prime factors, i.e. can be broken down into its prime factors.

Question *Express 36 as the product of its prime factors*

① Write down the first few prime numbers:
2, 3, 5, 7, 11, 13, ...

② Divide 36 by the first prime number, 2, as many times as possible (see opposite).

③ Then divide by the next prime number, 3, as many times as possible, and so on until you get to 1.

$$\begin{array}{r|l} 2 & 36\\ 2 & 18\\ 3 & 9\\ 3 & 3\\ & 1\ \text{stop} \end{array}$$

④ Write down the product of all the prime numbers you divided by:

$36 = 2\times2\times3\times3$

⑤ Write any repeated prime numbers as powers (i.e. using index form):

$36 = 2^2\times3^2$

This is called **expressing a number as a product of primes using index form.**

整数(续)

9. 根

计算一个数的幂的相反过程是求它的**根**.

2 的 3 次幂(立方)写作 2^3，等于8.

8 的 **3 次根(立方根)**写作 $\sqrt[3]{8}$，等于2.

$$\begin{tikzcd} 2 \arrow[r, bend left, "\text{立方(3 次幂)}"] & 8 \arrow[l, bend left, "\text{立方根(3 次根)}"] \end{tikzcd}$$

5 的 2 次幂(平方)写作 5^2，等于25.

25 的 **2 次根(平方根)**写作 $\sqrt[2]{25}$ 或简写作 $\sqrt{25}$，等于5.

$$\begin{tikzcd} 5 \arrow[r, bend left, "\text{平方(2 次幂)}"] & 25 \arrow[l, bend left, "\text{平方根(2 次根)}"] \end{tikzcd}$$

问题　不用计算器计算 $\sqrt[4]{16}$.

$\sqrt[4]{16}$ 是16的4次根．答案必须是2, 因为：

$$\begin{tikzcd} 2 \arrow[r, bend left, "\text{4 次幂}"] & 16 \arrow[l, bend left, "\text{4 次根}"] \end{tikzcd}$$

2 的 4 次幂 $=2^4=2 \times 2 \times 2 \times 2=16$

所以, $\sqrt[4]{16}=2$，因为 $2 \times 2 \times 2 \times 2=16$

注意　$\sqrt[4]{16}$ 不是 $16 \div 4$(见上).

$\sqrt{16+9}$ 不等于 $\sqrt{16}+\sqrt{9}=4+3=7$.

$\sqrt{16+9}=\sqrt{25}=5$ 才是正确的.

10. 幂和根

幂和根互相抵消.

例如：

$$6 \xrightarrow{\text{平方}} 6^2 = 36 \xrightarrow{\text{平方根}} \sqrt{36} = 6$$

另一种写法是 $\sqrt{6^2} = 6$

类似地, $(\sqrt{6})^2 = 6$ 或 $\sqrt{6}\times\sqrt{6} = 6$

11. 倍数

$1\times3 = 3$
$2\times3 = 6$
$3\times3 = 9$
$4\times3 = 12$
$5\times3 = 15$
$6\times3 = 18$
$7\times3 = 21$
$8\times3 = 24$
$9\times3 = 27$

这些数叫做 3 的倍数.

$1\times4 = 4$
$2\times4 = 8$
$3\times4 = 12$
$4\times4 = 16$
$5\times4 = 20$
$6\times4 = 24$
$7\times4 = 28$
$8\times4 = 32$
$9\times4 = 36$

这些数叫做 4 的倍数.

3 的倍数和 4 的倍数中有一些是相同的，它们叫做**公倍数**.

3, 6, 9, 12, 15, 18, 21, 24, 27, ….　3 的倍数
4, 8, 12, 16, 20, 24, 28, 32, 36, ….　4 的倍数

12 和 24 是 3 和 4 的公倍数．

12是 3 和 4 的**最小公倍数(LCM)**．它是能同时被 3、4 整除的最小的数．

12. 因数

12的**因数**是那些能整除12的数：1, 2, 3, 4, 6, 12 (别忘了包括1和12.)

18的因数是：1, 2, 3, 6, 9, 18

12 和 18 的因数中有一些是相同的，它们叫做**公因数**.

1	2	3	4	6	12	12 的因数
1	2	3	6	9	18	18 的因数

12和18的公因数是1, 2, 3和6.

12 和 18 的**最大公因数(HCF)**是 6.

13. 素数

只能被1和它本身整除的数称为**素数**. 例如, 11是素数. 下面是前几个素数：

2, 3, 5, 7, 11, 13, 17, 19, 23, 29

1不是一个素数.

14. 素因数

12的因数是：1, 2, 3, 4, 6, 12

其中 2 和 3 是素数，因此称为**素因数**.

问题　2^5 的素因数是什么?

$2^5=2 \times 2 \times 2 \times 2 \times 2=32$

32的因数是：1, 2, 4, 8, 16, 32

因此，2 是 2^5 的唯一的素因数.

15. 素因数的积

任何数都可以写成它的素因数的积，也就是说，可以分解成它的素因数的积.

问题　将36表示成它的素因数的积.

① 写出前几个素数：
2, 3, 5, 7, 11, 13, …

② 36 被第一个素数 2 除，尽可能多除几次(见右).

③ 然后用下一个素数3除，尽可能多除几次，如此等等，直至得到 1.

2) 36
2) 18
3) 9
3) 3
1
停止

④ 把你除过的所有素数的积写下来：
$36=2 \times 2 \times 3 \times 3$

⑤ 把重复的素数用幂表示(即用指数形式)：
$36=2^2 \times 3^2$

这就叫**用指数形式把一个数表示为素数的积**.

Number Patterns (Sequences)

1. Number Patterns (Sequences)

A **number pattern** or **sequence** is an ordered list of numbers connected by a rule.

The numbers in the pattern below start with 3 and are connected by the rule 'add 2', i.e. adding 2 to one number gives the next number.

$$3 \xrightarrow{\text{Add 2}} 5 \xrightarrow{\text{Add 2}} 7 \xrightarrow{\text{Add 2}} 9 \xrightarrow{\text{Add 2}} 11$$

Some rules involve several steps. For example, starting with 1 and using the rule 'double then add 3' gives the number pattern:

$$1 \xrightarrow{\text{Double, add 3}} 5 \xrightarrow{\text{Double, add 3}} 13 \xrightarrow{\text{Double, add 3}} 29$$

Some rules involve several previous numbers in the pattern. For example, starting with the numbers 2, 3 and using the rule 'add the two previous numbers' gives the number pattern:

$$2 \overbrace{\qquad}^{\text{Add}} 3 \underbrace{\qquad}_{\text{Add}} 5 \overbrace{\qquad}^{\text{Add}} 8 \underbrace{\qquad}_{\text{Add}} 13$$

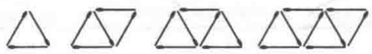
Some number patterns involve negative numbers. For example, starting with 6 and using the rule 'subtract 2' gives the number pattern:

6, 4, 2, 0, −2, −4, ...

Other sequences involve fractions. For example, starting with $\frac{1}{2}$ and using the rule "increase the numerator by 1, increase the denominator by 1" gives the number pattern below:

$$\frac{1}{2} \quad \frac{2}{3} \quad \frac{3}{4} \quad \frac{4}{5} \quad \frac{5}{6} \quad \dots$$

Number patterns often arise in coursework investigations. For example, the numbers of matches in these triangles form a number pattern:



Number of matchsticks 3 5 7 9 .....

2. Number Patterns You Should Know

Even numbers	2, 4, 6, 8, 10, ...
Odd numbers	1, 3, 5, 7, 9, 11, ...
Prime numbers	2, 3, 5, 7, 11, 13, 15, 17, 19, 23, ...
Natural numbers	1, 2, 3, 4, 5, 6, 7, 8, 9, ...
Square numbers	$1^2, 2^2, 3^2, 4^2, 5^2, 6^2$, ... ↓ ↓ ↓ ↓ ↓ ↓ 1, 4, 9, 16, 25, 36, ...
Cube numbers	$1^3, 2^3, 3^3, 4^3, 5^3$, ... ↓ ↓ ↓ ↓ ↓ 1, 8, 27, 64, 125, ...
Powers of 2	$2^0, 2^1, 2^2, 2^3, 2^4, 2^5, 2^6$, ... ↓ ↓ ↓ ↓ ↓ ↓ ↓ 1, 2, 4, 8, 16, 32, 64, ...

3. Finding the Rule for a Number Pattern

Follow these steps to find the rule for your number pattern.

① **Is the pattern one you know?**

9, 16, 25, 36, ... Square numbers, starting with 3^2

② **Is the pattern a modification of one you know?**

2, 5, 10, 17, 26, ... Square numbers + 1

③ **Try adding consecutive numbers:**

$$2 \overbrace{\qquad}^{\text{Add}} 4 \underbrace{\qquad}_{\text{Add}} 6 \overbrace{\qquad}^{\text{Add}} 10 \underbrace{\qquad}_{\text{Add}} 16 \ \dots$$

RULE Start with 2 and 4; add two consecutive numbers to get the next

④ **Look at the differences between consecutive numbers:**

$$\begin{array}{ccccccccc} 7 & & 10 & & 13 & & 16 & & 19\ \dots \\ & 3 & & 3 & & 3 & & 3 & \end{array}$$

Same difference

RULE Start with 7; add 3

$$\begin{array}{ccccccccc} 1 & & 2 & & 4 & & 7 & & 11\ \dots \\ & 1 & & 2 & & 3 & & 4 & \end{array}$$

Increasing differences

RULE Start with 1; add 1, then add 2, then add 3, etc.

⑤ **Try multiplying consecutive numbers:**

$$1 \overbrace{\qquad}^{\text{Multiply}} 2 \underbrace{\qquad}_{\text{Multiply}} 2 \overbrace{\qquad}^{\text{Multiply}} 4 \underbrace{\qquad}_{\text{Multiply}} 8 \ \dots$$

RULE Start with 1 and 2; multiply two consecutive numbers to get the next

⑥ **Try dividing consecutive numbers:**

$$\begin{array}{ccccccc} 2 & & 6 & & 18 & & 54\ \dots \\ & 3 & & 3 & & 3 & \\ & (6 \div 2) & & (18 \div 6) & & (54 \div 18) & \end{array}$$

RULE Start with 2; multiply by 3

$$\begin{array}{ccccccc} 48 & & 24 & & 12 & & 6\ \dots \\ & 2 & & 2 & & 2 & \\ & (48 \div 24) & & (24 \div 12) & & (12 \div 6) & \end{array}$$

RULE Start with 48; divide by 2

If none of these steps worked, try the methods in Box 4

4. Terms of a Sequence

Each number in a sequence (number pattern) is called a **term** and occupies a certain **position** in the sequence.

Position	1st,	2nd,	3rd,	4th,	5th, ...
Term	3,	6,	9,	12,	15, ...

You can often find the value of a term by knowing its position. In the above sequence, each term can be found by multiplying its position by 3:

$$\begin{array}{lcccccc} \text{Position} & 1 & 2 & 3 & 4 & 5 & \dots \\ & \downarrow{\times 3} & \downarrow{\times 3} & \downarrow{\times 3} & \downarrow{\times 3} & \downarrow{\times 3} & \\ \text{Term} & 3 & 6 & 9 & 12 & 15 & \dots \end{array}$$

So we can now also find the 20th and 100th terms:

20th term = 20×3 = 60 100th term = 100×3 = 300

more

数的模型(序列)

1. 数的模型(序列)

数的模型或序列，是用某一规则联系起来的一列有序的数．

下面模型中的数从3开始，用规则“加2”联系起来，即一个数加2得到下一个数．

$$3 \xrightarrow{\text{加}2} 5 \xrightarrow{\text{加}2} 7 \xrightarrow{\text{加}2} 9 \xrightarrow{\text{加}2} 11$$

有些规则包含几步．例如，从1开始，用规则“加倍再加3”得出的数的模型：

$$1 \xrightarrow{\text{加倍再加}3} 5 \xrightarrow{\text{加倍再加}3} 13 \xrightarrow{\text{加倍再加}3} 29$$

有些规则含有模型中前几个数．例如，从2和3开始，用规则“前二个数相加”得出的数的模型：

$$\begin{tikzcd}
2 \arrow[r, bend left, dash, "\text{加}"] & 3 \arrow[r, bend right, dash, "\text{加}"'] & 5 \arrow[r, bend left, dash, "\text{加}"] & 8 \arrow[r, bend right, dash, "\text{加}"'] & 13
\end{tikzcd}$$

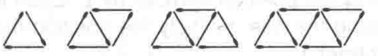
有些数的模型含有负数．例如，从6开始，用规则“减2”得出的数的模型：

6, 4, 2, 0, −2, −4, ……

另一些序列含有分数．例如，从$\frac{1}{2}$开始，用规则“分子增加1，分母增加1”得出下面的数的模型：

$$\frac{1}{2} \quad \frac{2}{3} \quad \frac{3}{4} \quad \frac{4}{5} \quad \frac{5}{6} \quad \ldots.$$

数的模型常产生于研究过程．例如，在这些三角形中的火柴数形成一个数的模型：



火柴数　3　5　7　9　....

2. 你应该知道的数的模型

偶　数	2, 4, 6, 8, 10, …
奇　数	1, 3, 5, 7, 9, 11, …
素　数	2, 3, 5, 7, 11, 13, 15, 17, 19, 23, …
自然数	1, 2, 3, 4, 5, 6, 7, 8, 9, …

平方数
$$\begin{array}{cccccc} 1^2, & 2^2, & 3^2, & 4^2, & 5^2, & 6^2, \ \ldots \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 1, & 4, & 9, & 16, & 25, & 36, \ \ldots \end{array}$$

立方数
$$\begin{array}{ccccc} 1^3, & 2^3, & 3^3, & 4^3, & 5^3, \ \ldots \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 1, & 8, & 27, & 64, & 125, \ \ldots \end{array}$$

2的幂
$$\begin{array}{ccccccc} 2^0, & 2^1, & 2^2, & 2^3, & 2^4, & 2^5, & 2^6, \ \ldots \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 1, & 2, & 4, & 8, & 16, & 32, & 64, \ \ldots \end{array}$$

3. 寻找一个数的模型的规则

下面这些步骤用于寻找一个数的模型的规则．

① **这个模型你是否知道?**

9, 16, 25, 36, …　平方数，从3^2开始．

② **这个模型是否是你知道的某个模型的变形?**

2, 5, 10, 17, 26, …　平方数 +1

③ **尝试添加接下去的几个数：**

$$\begin{tikzcd}
2 \arrow[r, bend left, dash, "\text{加}"] & 4 \arrow[r, bend right, dash, "\text{加}"'] & 6 \arrow[r, bend left, dash, "\text{加}"] & 10 \arrow[r, bend right, dash, "\text{加}"'] & 16 \ldots.
\end{tikzcd}$$

规则 从2和4开始；相邻两个数相加得到下一个数．

④ **观察相邻数的差：**

$$\begin{tikzcd}[column sep=tiny]
7 \arrow[dr, dash] & & 10 \arrow[dl, dash] \arrow[dr, dash] & & 13 \arrow[dl, dash] \arrow[dr, dash] & & 16 \arrow[dl, dash] \arrow[dr, dash] & & 19 \ldots. \arrow[dl, dash] \\
& 3 & & 3 & & 3 & & 3 &
\end{tikzcd}$$

差相同

规则 从7开始；加3.

$$\begin{tikzcd}[column sep=tiny]
1 \arrow[dr, dash] & & 2 \arrow[dl, dash] \arrow[dr, dash] & & 4 \arrow[dl, dash] \arrow[dr, dash] & & 7 \arrow[dl, dash] \arrow[dr, dash] & & 11 \ldots. \arrow[dl, dash] \\
& 1 & & 2 & & 3 & & 4 &
\end{tikzcd}$$

差增加

规则 从1开始：加1，然后加2，然后加3，等等．

⑤ **尝试乘相邻的数：**

$$\begin{tikzcd}
1 \arrow[r, bend left, dash, "\text{乘}"] & 2 \arrow[r, bend right, dash, "\text{乘}"'] & 2 \arrow[r, bend left, dash, "\text{乘}"] & 4 \arrow[r, bend right, dash, "\text{乘}"'] & 8 \ldots.
\end{tikzcd}$$

规则 从1和2开始；相邻两数相乘得到下一个数．

⑥**尝试除相邻的数：**

$$\begin{tikzcd}[column sep=tiny]
2 \arrow[dr, dash] & & 6 \arrow[dl, dash] \arrow[dr, dash] & & 18 \arrow[dl, dash] \arrow[dr, dash] & & 54 \ldots. \arrow[dl, dash] \\
& \underset{(6\div2)}{3} & & \underset{(18\div6)}{3} & & \underset{(54\div18)}{3} &
\end{tikzcd}$$

规则 从2开始；乘3.

$$\begin{tikzcd}[column sep=tiny]
48 \arrow[dr, dash] & & 24 \arrow[dl, dash] \arrow[dr, dash] & & 12 \arrow[dl, dash] \arrow[dr, dash] & & 6 \ldots. \arrow[dl, dash] \\
& \underset{(48\div24)}{2} & & \underset{(24\div12)}{2} & & \underset{(12\div6)}{2} &
\end{tikzcd}$$

规则 从48开始；除以2.

如果这些方法均不见效，尝试用方框4中的方法．

4. 序列的项

序列(数的模型)中的每一个数叫做项，它在序列中占一个位置．

位置　第1，第2，第3，第4，第5，…

项　　3，　6，　9，　12，　15，…

知道某一项的位置，你就可以找到这一项的值．在上面的序列中，每一项都可以通过将它的位置乘以3得到：

$$\begin{tikzcd}
\text{位置} & 1 \arrow[d, bend right, "\times3"'] & 2 \arrow[d, bend right, "\times3"'] & 3 \arrow[d, bend right, "\times3"'] & 4 \arrow[d, bend right, "\times3"'] & 5 \arrow[d, bend right, "\times3"'] & \ldots. \\
\text{项} & 3 & 6 & 9 & 12 & 15 & \ldots.
\end{tikzcd}$$

因此我们现在可以找到第20项和第100项：

第20项 =20 × 3 =60　　第100项 =100 × 3 =300

未完

Number Patterns (Contd)

4. Terms of a Sequence (Contd)

Question *Find the 7th, 20th and 100th terms of the sequence: 3, 7, 11, 15, 19, 23, ...*

First, write down the positions of the terms:

Position 1, 2, 3, 4, 5, 6, ...
Term 3, 7, 11, 15, 19, 23, ...

Then find the rule connecting each term and its position number:

Position	1	2	3	4	5 ....
Term	3	7	11	15	19 ....

$$3 \underbrace{\;\;}_{4} 7 \underbrace{\;\;}_{4} 11 \underbrace{\;\;}_{4} 15 \underbrace{\;\;}_{4} 19$$

A difference of 4 means that the rule involves multiplying the position number by 4:

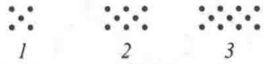
Position 1, 2, 3, 4, 5, 6, ...
4× Position 4, 8, 12, 16, 20, 24, ...
Term 3, 7, 11, 15, 19, 23, ...

You can see that each term is calculated by multiplying each position by 4 and then subtracting 1.

So, 7th term $= 4\times7-1 = 28-1 = 27$
20th term $= 4\times20-1 = 80-1 = 79$
100th term $= 4\times100-1 = 400-1 = 399$

Question *As part of his coursework, Philip drew the following sequence of patterns:*

Pattern Number



(a) Complete the table below.

Pattern number	*1*	*2*	*3*	*4*	*5*
Number of dots	*5*	*8*	*11*	*14*	

(b) What is the number of the pattern with 137 dots?

(a) The number of dots can be found using the rule 'multiply the pattern number by 3 and add 2'.

Number of dots in pattern 5 is: $3\times5+2 = 17$

(b) *Reverse the rule* to find the number of the pattern with 137 dots, i.e. 'subtract 2, then divide by 3':

$137-2 = 135$ then $135\div3 = 45$

So, the 45th pattern has 137 dots.

Question *Find the next term in the sequence below.*

Position 1st, 2nd, 3rd, 4th, 5th, ...
Term 2, 5, 10, 17, 26, ...

One of the terms of this sequence is 226. What is the position of this term?

Try squaring the position numbers.

Position number 1, 2, 3, 4, 5, ...
Square of position number 1, 4, 9, 16, 25, ...
Term 2, 5, 10, 17, 26, ...

You can see that each term is found using the rule 'square the position number and add 1'.

So, the next term = 6th term $= 6^2+1 = 36+1 = 37$

To find the position number of the term 226, *reverse the rule,* i.e. 'subtract 1 and then square root'.

Subtract 1 $226-1 = 225$
Square root $\sqrt{225} = 15$

So, the term 226 has position number 15.

5. Describing Sequences Using Algebra

Sequences can be briefly described using algebra.

n represents the position of any term
u_1 represents the 1st term
u_2 represents the 2nd term, etc.
u_n represents the nth term (the **general term**)

For the sequence 3, 6, 9, 12, 15, ... we have:

Position 1, 2, 3, 4, 5, 6, ..., n, ...
Term 3, 6, 9, 12, 15, 18, ..., nth term, ...
u_1, u_2, u_3, u_4, u_5, u_6, ..., u_n, ...

Each term in this sequence is found using the rule 'multiply the position number by 3'. So,

1st term $= u_1 = 3\times1 = 3$
2nd term $= u_2 = 3\times2 = 6$
3rd term $= u_3 = 3\times3 = 9$, etc.
nth term $= u_n = 3\times n = 3n$

Question *Write down the first three terms of the sequence where $u_n = n^2+1$.*

1st term $= u_1 = 1^2+1 = 1+1 = 2$
2nd term $= u_2 = 2^2+1 = 4+1 = 5$
3rd term $= u_3 = 3^2+1 = 9+1 = 10$

Question *Write down the nth term, u_n, for the sequence 1, 3, 5, 7, 9, ...*

First, write down the positions of the terms:

Position 1, 2, 3, 4, 5, 6, ..., n, ...
Term 1, 3, 5, 7, 9, 11, ..., u_n, ...

Find the rule connecting the terms and their positions.

Position	1	2	3	4	5	6 .... u_n ....
Term	1	3	5	7	9	11 .... n ....

$$1 \underbrace{\;\;}_{2} 3 \underbrace{\;\;}_{2} 5 \underbrace{\;\;}_{2} 7 \underbrace{\;\;}_{2} 9 \underbrace{\;\;}_{2} 11$$

Same difference

A difference of 2 means that the rule involves multiplying the position number by 2. Each term is found using the rule 'multiply the position number by 2, then subtract 1':

Term = 2×(Position number)−1

So, u_n = nth term $= 2\times n-1 = 2n-1$

Question *For the sequence 2, 6, 12, 20, 30, ...*
(a) Find the next term.
(b) Write down an expression for u_n.
(c) Find the value of n when $u_n = 4970$.

(a) Write down the positions of the terms:

Position	1	2	3	4	5	6	7 ....
Term	2	6	12	20	30	?	....

Each term is found by multiplying its position number by the next position number:

$$\text{Position}\quad 1 \overset{\text{Multiply}}{\frown} 2 \overset{\text{Multiply}}{\frown} 3 \overset{\text{Multiply}}{\frown} 4 \;....\; n \overset{\text{Multiply}}{\frown} n+1 \;....$$

Term 2 6 12 20 .... u_n u_{n+1} ....

So, the next term is $6\times7 = 42$

(b) The position of u_n is n and the next position is $n+1$. So, $u_n = n\times(n+1) = n(n+1)$

(c) We must find n such that $n(n+1) = 4970$. Since $n+1$ is close to n, then $n(n+1)$ is close to $n\times n$ or n^2. So, $n^2 \approx 4970$, giving $n \approx \sqrt{4970} = 70.5$. Try $n = 70$: $n(n+1) = 70\times71 = 4970$. Correct

数的模型(续)

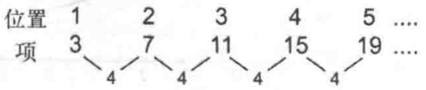
4. 序列的项(续)

问题 *找出序列：3，7，11，15，19，23，…的第7、第20和第100项．*

首先，写下项的位置：

位置　1，2，3，4，5，6，…
项　3，7，11，15，19，23，…

然后找出每一项和它的位置数的联系规则：



差4意味着规则包含位置数乘以4：

位置　1，2，3，4，5，6，…
4 × 位置　4，8，12，16，20，24，…
项　3，7，11，15，19，23，…

你会发现每一项都是该位置乘以4再减1算得的．

于是，第7项 $= 4 \times 7-1 = 28-1 = 27$
第20项 $= 4 \times 20-1 = 80-1 = 79$
第100项 $= 4 \times 100-1 = 400-1 = 399$

问题 *菲力浦在他的作业中画了如下模型的序列：*

模型数　*1*　*2*　*3*

(a) 完成下表：

模型数	*1*	*2*	*3*	*4*	*5*
点　数	*5*	*8*	*11*	*14*	

(b) 137点的模型数是多少？

(a) 点数能用规则“模型数乘以3再加2”求得．
模型数5对应的点数是：$3 \times 5+2=17$

(b) 逆用规则可求出137点的模型数，那就是“减2，再除以3”：
$137-2 = 135$，再 $135 \div 3 = 45$
因此，第45个模型数有137点．

问题 *求下列序列的下一项：*

位置　第1项，第2项，第3项，第4项，第5项，…
项　2，5，10，17，26，…

这个序列的某一项是226．该项在什么位置？

尝试将位置数平方．

位置数　1，2，3，4，5，…
位置数的平方　1，4，9，16，25，…
项　2，5，10，17，26，…

你会发现每一项都是用规则“位置数平方再加1”求得的．

因此，下一项＝第6项 $= 6^2+1 = 36+1 = 37$

为了求出项226的位置数，*逆用规则*，即“减1，再开方”．

减1　$226-1 = 225$

开方　$\sqrt{225} = 15$

所以项226的位置数为15．

5. 用代数描述序列

序列可以用代数简单地描述．

n　表示任意项的位置
u_1　表示第1项
u_2　表示第2项，等等．
u_n　表示第n项(通项)

对于序列3，6，9，12，15，…我们有：

位置　1，2，3，4，5，6，…，n，…
项　3，6，9，12，15，18，…，第n项，…
u_1，u_2，u_3，u_4，u_5，u_6，…，u_n，…

这个序列中的每一项都可以用规则“位置数乘以3”求得．因此，

第1项 $= u_1 = 3 \times 1 = 3$
第2项 $= u_2 = 3 \times 2 = 6$
第3项 $= u_3 = 3 \times 3 = 9$，等等．
第n项 $= u_n = 3 \times n = 3n$

问题 *写出序列 $u_n = n^2+1$ 的前三项．*

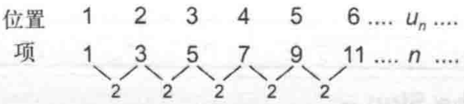
第1项 $= u_1 = 1^2+1 = 1+1 = 2$
第2项 $= u_2 = 2^2+1 = 4+1 = 5$
第3项 $= u_3 = 3^2+1 = 9+1 = 10$

问题 *已知序列1，3，5，7，9，…，写出第n项 u_n．*

首先，写出各项的位置：

位置　1，2，3，4，5，6，…n，…
项　1，3，5，7，9，11，…u_n，…

找出项和它们的位置间的联系．



差相同

差2意味着规则含有位置数乘以2．每一项都是用规则“位置数乘以2，再减1”求出的：

项 = 2 × 位置数 −1

因此，u_n = 第n项 $=2 \times n-1 = 2n-1$

问题 *对于序列2，6，12，20，30，…*

(a) 求下一项．

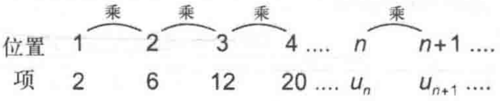
(b) 写出 u_n 的表达式．

(c) 当 $u_n = 4970$ 时，求n的值．

(a) 写出各项的位置：

位置　1　2　3　4　5　6　7 …
项　2　6　12　20　30　? …

每一项是用它的位置数乘以下一个位置数求得的：



因此，下一项是 $6 \times 7 = 42$

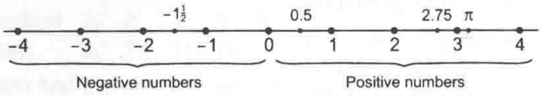
(b) u_n的位置数是n，而下一项的位置数是$n+1$．
因此，$u_n = n \times (n+1) = n(n+1)$

(c) 我们要求出满足 $n(n+1) = 4970$ 的n．由于$n+1$接近n，而$n(n+1)$接近$n \times n$或n^2．因此，$n^2 \approx 4970$，$n \approx \sqrt{4970} = 70.5$．尝试$n$=70：$n(n+1) = 70 \times 71 =4970$．正确

Negative Numbers

1. Positive and Negative Numbers

Positive and negative numbers can be represented as points on a straight line called the **number line**.



There are several ways of writing positive and negative numbers. For example,

2 can be written +2, (+2) or $^{+}2$

−2 can be written (−2) or $^{-}2$ and is called *negative 2*

NOTE $^{+}2$ and $^{-}2$ are called **directed numbers**

2. Integers

Whole numbers are sometimes called **integers**. They include the positive and negative numbers and zero.

.... −3, −2, −1, 0, 1, 2, 3, ...

3. Plus and Minus

Minus is the name of the sign −

Plus is the name of the sign +

So, 3+4−2 expressed in words is 'three plus four minus 2'

4. The Sign −

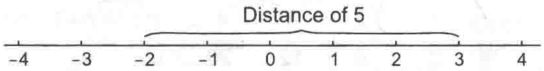
In calculations, the sign − can mean subtract or take away, or it can mean negative. For example,

5−2 means '5 subtract 2' or '5 take away 2'

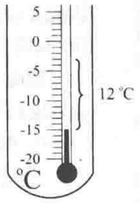
−2+5 means 'negative 2 add 5' (the negative number −2 added to 5)

5. Distance Between Numbers

The distance between two numbers on the number line is always positive. For example, the distance between the numbers −2 and 3 is 5, as shown on the number line below.



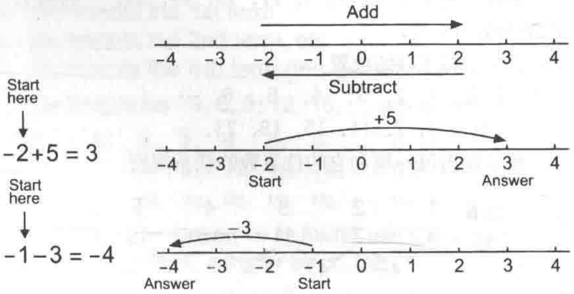
Question *The temperature in a thawing freezer increased from −15 °C to −3 °C. What was the rise in temperature?*



The diagram shows that the temperature rose by 12 °C (the distance between −15 and −3 is 12).

6. Adding and Subtracting

Use the number line to add/subtract two numbers.



Sometimes you will see two + or − signs next to each other, e.g. 2+−3. Use the rules on the right to replace them with a single sign.

++	gives	+
−−	gives	+
+−	gives	−
−+	gives	−

Examples

2+−3 = 2−3 = −1

−(−5)−2 = −−5−2 = +5−2 = 3

7. Rearranging Numbers

−2+5 and +5−2 both give the same answer of 3.

So, −2+5 can be rearranged to give +5−2. Notice how **the signs stay with the numbers**.

Similarly, −2+5−1+4 = +5+4−2−1 = 9−3 = 6

8. Adding/Subtracting Several Numbers

When adding/subtracting more than two numbers, first combine the + numbers into a single number and then the − numbers into a single number:

−3+2+4−1−6+3+5	
= +2+4+3+5−3−1−6	Rearrange numbers first
= +14−10	Combine numbers
= 4	

9. Multiplying and Dividing

Use these rules for multiplying numbers.

+×+ = +	−×− = +
+×− = −	−×+ = −

Examples

−2×−5 = +10 = 10 $(-3)^2 = (-3)\times(-3) = +9 = 9$

−2+−3×4 = −2+−12 = −2−12 = −14 or alternatively

−2+−3×4 = −2−3×4 = −2−12 = −14

Use the same rules for dividing numbers. For example:

−6÷−2 = +3 = 3 or equivalently $\frac{-6}{-2} = +3 = 3$

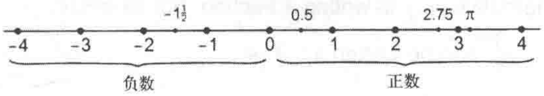
$\frac{-8}{2} = \frac{-8}{+2} = -4$ and $\frac{8}{-2} = \frac{+8}{-2} = -4$ and $-\frac{8}{2} = -4$

The last example shows that $\frac{-8}{2} = \frac{8}{-2} = -\frac{8}{2} = -4$

负数

1．正数和负数

正数和负数可以表示为称作**实数直线**的直线上的点．



正负数有几种写法．例如

2 可以写成 +2，(+2) 或 $^{+}2$

−2 可以写成 (−2) 或 $^{-}2$，读作‘负 2’

注意 $^{+}2$ 和 $^{-}2$ 称作**有向数**

2．整数

整数包括正整数、负整数和零．

…−3，−2，−1，0，1，2，3，…

3．正号和负号

负号是符号 − 的名称

正号是符号 + 的名称

因此，3+4−2 用话表示就是“3 正号 4 负号 2”

4．符号−

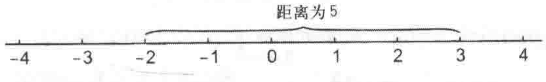
在计算中，符号−表示减或去掉，也可以表示负．例如，

5−2 意思是“5 减 2”或“5 去掉 2”

−2+5 意思是“负 2 加 5”(负数 −2 加上 5)

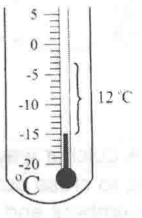
5．两数间的距离

实数直线上两数间的距离总是正的．例如，−2 和 3 之间的距离是 5，如下面的实数直线所示．



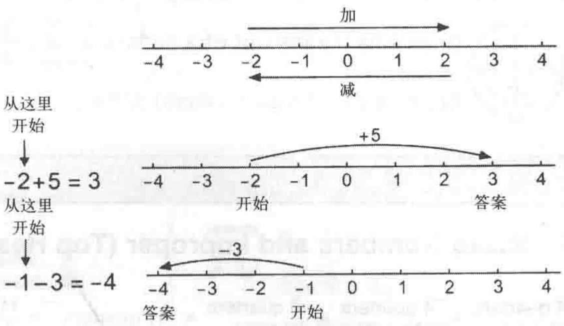
问题　温度计在解冻时从−15℃增加到−3℃．温度上升了多少度？

图中显示温度上升了 12℃(−15 和−3 间的距离为12)．



6．加和减

用实数直线进行两数的加／减．



有时你会看到两个＋或－符号连在一起，例如2+−3．按右面的规则用一个符号代替它们．

++	得	+
−−	得	+
+−	得	−
−+	得	−

例

2+−3=2−3=−1

−(−5)−2=−−5−2=+5−2=3

7．重新排列数

−2+5 和 +5−2 都有相同的答数3．

因此，−2+5可以重新排列成+5−2．注意符号是怎样跟着数走的．

类似地，−2+5−1+4=+5+4−2−1=9−3=6

8．加／减几个数

当加／减两个以上的数时，首先把正数合并成一个数，把负数合并成一个数：

−3+2+4−1−6+3+5

=+2+4+3+5−3−1−6　首先重新排列数

=+14−10　合并数

=4

9．乘和除

数相乘用这些规则．

$+\times+=+$　　$-\times-=+$

$+\times-=-$　　$-\times+=-$

例

$-2\times-5=+10=10$　　$(-3)^2=(-3)\times(-3)=+9=9$

$-2+-3\times4=-2+-12=-2-12=-14$　亦可

$-2+-3\times4=-2-3\times4=-2-12=-14$

数相除用相同的规则．例如：

$-6\div-2=+3=3$　　或相等于　$\dfrac{-6}{-2}=+3=3$

$\dfrac{-8}{2}=\dfrac{-8}{+2}=-4$　及　$\dfrac{8}{-2}=\dfrac{+8}{-2}=-4$　及　$-\dfrac{8}{2}=-4$

最后一个例子表示　$\dfrac{-8}{2}=\dfrac{8}{-2}=-\dfrac{8}{2}=-4$