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# 从资产组合决策推断风险规避

INFERRING RISK AVERSION  
FROM THE PORTFOLIO DECISION

刘德溯 著



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# Chapter One

## Introduction

This dissertation examines how to infer the Arrow-Pratt measures of risk aversion for an expected utility maximizing decision maker, based on her observed portfolio choice(s). It contains successful attempts from which three different papers have been extracted, as well as some current thinking that may later help in developing other papers. A literature review is provided in Chapter 2. I discuss the intuition and summarize each of the three main chapters of this dissertation in the following.

Chapter 3 mainly studies the role of nonfinancial wealth components when using wealth allocation decisions to infer the slope of relative risk aversion for *consumption*. The literature gives different definitions of wealth but the wealth measures that are frequently used exclude important nonfinancial elements as other sources of consumption. In particular, I focus on the effect of uncaptialized future income, which provides another source of consumption and thus may imply a different slope of relative risk aversion for consumption. This analytical finding is based on the

same response of the risky asset share to changes in wealth. The finding has an implication for applied economists who are interested in the current debate on whether constant relative risk aversion (CRRA) power utility or decreasing relative risk aversion (DRRA) habit formation utility is a more appropriate functional form of utility for consumption.

It is often assumed in multi-period models that all future income can be capitalized into current wealth for the portfolio allocation decision; that is, current wealth equals lifetime wealth. As a result, consumption can only come from the return on wealth. This assumption on the wealth measure, however, does not match what is observed from the real world, where some sources of future income are not a component of current wealth. Examples of uncanceled future income include labor income, social security benefits, pensions, government transfer, appreciation of housing equity, and other forms. This can happen as long as an agent has sufficient current wealth for consumption and for investment. For instance, a tenured professor has very stable future labor income but he may choose not to capitalize every penny into current wealth. It is also possible that for some reasons, a decision maker “fails” to integrate these sources of income into current wealth. Finally, due to some imperfection, financial frictions or legal restrictions, market does not allow one to fully capitalize various forms of

future income.

In a recent study to test the existence of time-varying risk aversion that results from external habit formation utility, Brunnermeier and Nagel (2008) (B-N) find that the share of wealth allocated to the risky assets is essentially not affected by wealth changes across time periods. This may imply that relative risk aversion for certain measure of wealth is constant for a representative of households from the Panel Study of Income Dynamics (PSID). It seems that empirical evidence cannot reconcile the positive contemporaneous relationship shown in their testable equation which is derived from the theory. B-N interpret the finding as evidence against the presence of DRRA habit formation utility for consumption at the micro level and suggest that CRRA power utility for consumption may prevail.

With the correction for uncapitalized future income, it is demonstrated in my theoretical analysis that the sign of the slope of relative risk aversion for consumption can be totally different from without this correction. First, the study of a two-period model shows that if the comparative static change in the initial wealth has no effect on the risky asset proportion, utility function must exhibit DRRA for consumption. An infinite horizon model assuming habit formation utility and an exogenous inflow of future income is then examined. If the present value of future income is relatively close to that of future habits, the risky

asset share may not respond to changes in wealth over time. These two analytical findings concerning DRRA utility functions for consumption can be used to reinterpret recent empirical micro-level findings, including the one by B-N that there is an absence of wealth effect on households' asset allocation over time.

A workable future direction is to test the existence of habit formation using housing data at the micro level. Housing is the largest wealth component for many households, and is also an illiquid asset with risk properties being unclear. For one thing, housing is a durable good and provides constant consumption flow which may be treated as a constant habit. Second, home mortgage helps capitalize one's future income to certain extent, since mortgage loan is usually earmarked and is different from a consumer loan which does not require a specific use. These two features may enable housing to be incorporated into multi-period models in which habit formation utility and a future income stream are assumed.

Chapter 4 examines how to infer the magnitude of the Pratt-Arrow measures of risk aversion for *wealth* using one or more observations on the portfolio allocation decision. While this magnitude is very useful in asset pricing models and in the determination of insurance premium, the literature presents little direct empirical evidence, as Meyer and Meyer (2006) point out.

The endeavor in this chapter is in part driven to provide more of such information. More importantly, it is also because the main existing approach to connecting the portfolio decision to risk aversion for wealth infers risk aversion in the small (for small risks), rather than risk aversion in the large (for large risks). This does not make much sense given the fact that portfolio risk is definitely a large risk, often measured in terms of the standard deviation of its returns. For example, during the period of 1890 to 1979, investing \$1 in the Standard & Poor 500 Index had an annualized mean return of \$1.07 and a standard deviation of \$0.17; this is compared to the investment of \$1 in the short-term U.S. treasury bills with an annualized return and a standard deviation of \$1.01 and \$0.05 respectively in the same period.

Friend and Blume (F-B) (1975) provide a formula that can be used to infer the measure of risk aversion for wealth in the small at the point of the initial wealth, based on a single observation on the portfolio allocation. This formula is reached using a specific approximation procedure by assuming that time interval is very small. As a consequence, the portfolio risk being evaluated only leads to small wealth variations, and risk aversion for small risks can be inferred at the point of the initial wealth. The same formula is recently utilized by Chiappori and Paiella (2011) (C-P) in the study of Italian household wealth allocation

across time periods. My main concern about the F-B's methodology is whether it can be applied to infer or estimate the magnitude of risk aversion for risks whose sizes cannot be assumed to be zero or close to zero.

I study a standard one-period two-asset portfolio allocation model, in which time interval is one year and hence the risks from investing in the risky asset are substantial. Two different methods to infer risk aversion in the large are proposed, assessed and compared with the one used by F-B to infer risk aversion in the small. The first method quadratically approximates the utility function for wealth, and then maximizes the expectation of the approximated utility. This gives rise to an estimate of risk aversion in the large, which only depends on the mean and variance of the risky asset return. The second method directly employs functional forms of utility or risk aversion to infer risk aversion in the large. The procedure involves specifying one or more portfolio choices to identify the same number of unknown parameters in an assumed functional form of utility for wealth. The second method requires complete prior information on the probability distribution function for the risky asset return.

Three functional forms of utility or marginal utility belonging to the family of isoelastic risk preferences recently proposed by Meyer (2010) are considered. These include two commonly used utilities: Power (CRRA) and exponential

(CARA), and one marginal utility chosen to display DRRA. In addition, historical market data of annualized returns on the Standard & Poor 500 Index and on the U. S. treasury bills are borrowed. Using one observed portfolio decision, computed solutions show that picking one of the three functional forms and then inferring relative risk aversion performs much better than assuming a quadratic utility or using the F-B in the small procedure, if the true utility is from the isoelastic risk preferences group. It seems that when the goal is to estimate risk aversion level under regular conditions, choosing a functional form of utility that possesses the property of isoelastic risk preferences (even if it is wrong) to infer risk aversion in the large prevails over the F-B's methodology of inferring risk aversion in the small without restricting functional forms of utility.

Chapter 5 provides a detailed discussion of three published papers: F-B, C-P and B-N. The methodologies used and the empirical evidence presented in these papers have led to the writing of Chapters 3 and 4. The theoretical findings in these two chapters are utilized to reinterpret the empirical findings concerning the magnitudes and the slopes of relative risk aversion. There are three tentative conclusions. First, relative risk aversion for liquid financial wealth is probably constant. Second, relative risk aversion for consumption can be decreasing, if uncaptialized future income, an often ignored part of wealth, is

assumed to provide another source of consumption. Third, the opinions on the magnitude of relative risk aversion for Arrow-Pratt wealth are still divergent but at the mean return it usually does not exceed 10 unless for extremely impoverished investors. Meanwhile, two major econometric issues that may confound the identification of the effect of wealth changes over time on the risky asset share are indicated.



## Chapter Two

### Literature Review

This chapter consists of two parts: theoretical analysis and recent empirical evidence. In the first part, I review a portion of literature that studies the demand for risky assets in one-period models by making assumptions on the magnitude and/or the slope of risk aversion for wealth, some literature which focuses on tradeoff between consumption and savings in two-period consumption models by assuming that risk aversion for consumption satisfies certain properties, and several papers that provide analytical solutions for consumption or the risky asset share using multiperiod models in which the functional form of utility for consumption is assumed. Major papers in macroeconomics that use DRRA habit formation utility for consumption to address the equity premium puzzle are also reviewed. In the second part, I review recent literature that uses data on portfolio choice and/or consumption to either deduce or estimate relative risk aversion for wealth and relative risk aversion for consumption. A detailed discussion of three papers