11 Martin Braun

TEXTS IN APPLIED MATHEMATICS

Differential Equations and Their Applications

Fourth Edition

微分方程及其应用 第4版

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Martin Braun

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To four beautiful people:

Zelda Lee

Adeena Rachelle, I. Nasaŋayl, and Shulamit

Series Preface

Mathematics is playing an ever more important role in the physical and biological sciences, provoking a blurring of boundaries between scientific disciplines and a resurgence of interest in the modern as well as the classical techniques of applied mathematics. This renewal of interest, both in research and teaching, has led to the establishment of the series: *Texts in Applied Mathematics (TAM)*.

The development of new courses is a natural consequence of a high level of excitement on the research frontier as newer techniques, such as numerical and symbolic computer systems, dynamical systems, and chaos, mix with and reinforce the traditional methods of applied mathematics. Thus, the purpose of this textbook series is to meet the current and future needs of these advances and encourage the teaching of new courses.

TAM will publish textbooks suitable for use in advanced undergraduate and beginning graduate courses, and will complement the *Applied Mathematical Sciences (AMS)* series, which will focus on advanced textbooks and research level monographs.

Preface to the Fourth Edition

There are two major changes in the Fourth Edition of *Differential Equations and Their Applications.* The first concerns the computer programs in this text. In keeping with recent trends in computer science, we have replaced all the APL programs with Pascal and C programs. The Pascal programs appear in the text in place of the APL programs, where they are followed by the Fortran programs, while the C programs appear in Appendix C.

The second change, in response to many readers' suggestions, is the inclusion of a new chapter (Chapter 6) on Sturm-Liouville boundary value problems. Our goal in this chapter is not to present a whole lot of technical material. Rather it is to show that the theory of Fourier series presented in Chapter 5 is not an isolated theory but is part of a much more general and beautiful theory which encompasses many of the key ideas of linear algebra.

To accomplish this goal we have included some additional material from linear algebra. In particular, we have introduced the notions of inner product spaces and self-adjoint matrices, proven that the eigenvalues of a self-adjoint matrix are real, and shown that all self-adjoint matrices possess an orthonormal basis of eigenvectors. These results are at the heart of Sturm-Liouville theory.

I wish to thank Robert Giresi for writing the Pascal and C programs.

New York City
May, 1992

Martin Braun

Preface to the Third Edition

There are three major changes in the Third Edition of *Differential Equations and Their Applications*. First, we have completely rewritten the section on singular solutions of differential equations. A new section, 2.8.1, dealing with Euler equations has been added, and this section is used to motivate a greatly expanded treatment of singular equations in sections 2.8.2 and 2.8.3.

Our second major change is the addition of a new section, 4.9, dealing with bifurcation theory, a subject of much current interest. We felt it desirable to give the reader a brief but nontrivial introduction to this important topic.

Our third major change is in Section 2.6, where we have switched to the metric system of units. This change was requested by many of our readers.

In addition to the above changes, we have updated the material on population models, and have revised the exercises in this section. Minor editorial changes have also been made throughout the text.

New York City
November, 1982

Martin Braun

Preface to the First Edition

This textbook is a unique blend of the theory of differential equations and their exciting application to "real world" problems. First, and foremost, it is a rigorous study of ordinary differential equations and can be fully understood by anyone who has completed one year of calculus. However, in addition to the traditional applications, it also contains many exciting "real life" problems. These applications are completely self contained. First, the problem to be solved is outlined clearly, and one or more differential equations are derived as a model for this problem. These equations are then solved, and the results are compared with real world data. The following applications are covered in this text.

1. In Section 1.3 we prove that the beautiful painting "Disciples of Emmaus" which was bought by the Rembrandt Society of Belgium for $170,000 was a modern forgery.

2. In Section 1.5 we derive differential equations which govern the population growth of various species, and compare the results predicted by our models with the known values of the populations.

3. In Section 1.6 we derive differential equations which govern the rate at which farmers adopt new innovations. Surprisingly, these same differential equations govern the rate at which technological innovations are adopted in such diverse industries as coal, iron and steel, brewing, and railroads.

4. In Section 1.7 we try to determine whether tightly sealed drums filled with concentrated waste material will crack upon impact with the ocean floor. In this section we also describe several tricks for obtaining information about solutions of a differential equation that cannot be solved explicitly.

5. In Section 2.7 we derive a very simple model of the blood glucose regulatory system and obtain a fairly reliable criterion for the diagnosis of diabetes.

6. Section 4.5 describes two applications of differential equations to arms races and actual combat. In Section 4.5.1 we discuss L. F. Richardson's theory of the escalation of arms races and fit his model to the arms race which led eventually to World War I. This section also provides the reader with a concrete feeling for the concept of stability. In Section 4.5.2 we derive two Lanchestrian combat models, and fit one of these models, with astonishing accuracy, to the battle of Iwo Jima in World War II.

7. In Section 4.10 we show why the predator portion (sharks, skates, rays, etc.) of all fish caught in the port of Fiume, Italy rose dramatically during the years of World War I. The theory we develop here also has a spectacular application to the spraying of insecticides.

8. In Section 4.11 we derive the "principle of competitive exclusion," which states, essentially, that no two species can earn their living in an identical manner.

9. In Section 4.12 we study a system of differential equations which govern the spread of epidemics in a population. This model enables us to prove the famous "threshold theorem of epidemiology," which states that an epidemic will occur only if the number of people susceptible to the disease exceeds a certain threshold value. We also compare the predictions of our model with data from an actual plague in Bombay.

10. In Section 4.13 we derive a model for the spread of gonorrhea and prove that either this disease dies out, or else the number of people who have gonorrhea will ultimately approach a fixed value.

This textbook also contains the following important, and often unique features.

1. In Section 1.10 we give a complete proof of the existence–uniqueness theorem for solutions of first-order equations. Our proof is based on the method of Picard iterates, and can be fully understood by anyone who has completed one year of calculus.

2. In Section 1.11 we show how to solve equations by iteration. This section has the added advantage of reinforcing the reader's understanding of the proof of the existence–uniqueness theorem.

3. Complete Fortran and APL programs are given for every computer example in the text. Computer problems appear in Sections 1.13–1.17, which deal with numerical approximations of solutions of differential equations; in Section 1.11, which deals with solving the equations $x=f(x)$ and $g(x)=0$; and in Section 2.8, where we show how to obtain a power-series solution of a differential equation even though we cannot explicitly solve the recurrence formula for the coefficients.

4. A self-contained introduction to the computing language APL is presented in Appendix C. Using this appendix we have been able to teach our students APL in just two lectures.

5. Modesty aside, Section 2.12 contains an absolutely super and unique treatment of the Dirac delta function. We are very proud of this section because it eliminates all the ambiguities which are inherent in the traditional exposition of this topic.

6. All the linear algebra pertinent to the study of systems of equations is presented in Sections 3.1–3.7. One advantage of our approach is that the reader gets a concrete feeling for the very important but extremely abstract properties of linear independence, spanning, and dimension. Indeed, many linear algebra students sit in on our course to find out what's really going on in their course.

Differential Equations and Their Applications can be used for a one- or two-semester course in ordinary differential equations. It is geared to the student who has completed two semesters of calculus. Traditionally, most authors present a "suggested syllabus" for their textbook. We will not do so here, though, since there are already more than twenty different syllabi in use. Suffice it to say that this text can be used for a wide variety of courses in ordinary differential equations.

I greatly appreciate the help of the following people in the preparation of this manuscript: Douglas Reber who wrote the Fortran programs, Eleanor Addison who drew the original figures, and Kate MacDougall, Sandra Spinacci, and Miriam Green who typed portions of this manuscript.

I am grateful to Walter Kaufmann-Bühler, the mathematics editor at Springer-Verlag, and Elizabeth Kaplan, the production editor, for their extensive assistance and courtesy during the preparation of this manuscript. It is a pleasure to work with these true professionals.

Finally, I am especially grateful to Joseph P. LaSalle for the encouragement and help he gave me. Thanks again, Joe.

New York City
July, 1976

Martin Braun

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