Solving Problems in:

Fluid Mechanics

Volume 2

J. F. DOUGLAS & R. D. MATTHEWS

3rd Edition

流体力学题解

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Solving Problems in Fluid Mechanics

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Volume 2

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Preface

The treatment adopted in this second volume is exactly the same as that employed so successfully in the first volume, the subject matter of each section being presented in the form of question and answer. The reader will find all the definitions and theory required, together with selected problems which are fully worked out, and plenty of exercise questions with numerical answers on which to practice and develop skill and understanding.

The material included in this volume covers more advanced work in Fluid Mechanics for engineering students in Universities, Polytechnics and Colleges of Higher Education. The fullness of the treatment has in some places had to be restricted owing to the limited space available. The reader seeking further information in any particular field will find it helpful to refer to "Fluid Mechanics" by Douglas, Gasiorek and Swaffield (Pitman 2nd. Edn 1985).

I would again like to express my appreciation of the assistance which I have received from my former colleagues in the teaching profession. I am particularly indebted to Dr. R.D. Matthews for his advice on the preparation of this new text and for the provision of examples and exercises with particular reference to Chapter 9.

I hope that my readers will not hesistate to let me know of any difficulties that they may experience with this text and I will be glad to receive any constructive criticism.

John Douglas September 1985

Preface to third edition

I am delighted to be associated with Dr J.F. Douglas in the production of this latest edition of Solving Problems in Fluid Mechanics. The tried and trusted format has been retained although some updating and correction, as well as some additions and a little deletion have taken place. The books as ever provide a wealth of basic fluid mechanics theory developed through worked solutions. In addition, the chapters open with some brief competency statements and conclude with a chapter summary of outcomes. In many chapters there are applications examples which will involve students in main project work in the library, laboratory or at home.

In volume 2 there has been some amalgamation of material and an additional chapter added introducing Computational Fluid Dynamics. While this chapter contains worked examples the authors feel that students should follow this with hands-on work using software packages as available to them.

The authors are indebted to Mr Ewan Bennett for helpful contributions in volumes 1 and 2, particularly on computational fluid dynamics.

We hope that these books will continue to be of help to students and academics studying the many fascinating branches of fluid mechanics.

Richard Matthews
Twickenham

September 1995

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1

Dimensional analysis

Introduction

Dimensional analysis is a mathematical method which is of considerable value in problems which occur in fluid mechanics. As explained in Volume One all quantities can be expressed in terms of certain primary quantities which in mechanics are Length (L), Mass (M) and Time (T). For example

$$\text{Force} = \text{mass} \times \text{acceleration}$$
$$= \text{mass} \times \text{length/time}^2$$

Thus the dimensions of Force will be MLT^{-2}.

In any equation representing a real physical event every term must contain the same powers of the primary quantities (L, M and T). In other words, like must be compared with like or else the equation is meaningless, although it may balance numerically. Even if an equation does balance in its dimensions, it may still be meaningless. Pure numbers (the coefficient of each term in an equation) have no dimensions and are not accounted for by dimensional analysis.

This principle of homogeneity of dimensions can be used, (1) to check whether an equation has been correctly formed, (2) to establish the form of an equation relating a number of variables, and (3) to assist in the analysis of experimental results.

Learning outcomes

After working through the examples and problems in this chapter, the student should be able to

1. Determine whether a given equation is a mathematically plausible relationship by means of dimensional analysis and state any problems associated with such a conclusion.
2. Equate indices of fundamental quantities to determine the influence of physical quantities such as viscosity, density, velocity, etc., on a particular physical quantity of interest.

1.1 Checking equations

Show by dimensional analysis that the equation

$$p + \tfrac{1}{2}\rho v^2 + \rho g z = H$$

is a possible relationship between the pressure p, velocity v and height above datum z for frictionless flow along a streamline of a fluid of mass density ρ, and determine the dimensions of the constant H.

Solution. If the equation represents a physically possible relationship each term must have the same dimensions and therefore contain the same powers of the primary quantities L, M and T.

The procedure to be adopted is first to determine the dimensions of each of the variables in terms of L, M and T, and then to examine the dimensions of each term in the equation.

The dimensions of the variables are

$$\text{Pressure } p = \frac{\text{force}}{\text{area}} = \frac{\text{mass} \times \text{acceleration}}{\text{area}}$$

$$= ML^{-1}T^{-2}$$

$$\text{Mass density } \rho = \frac{\text{mass}}{\text{volume}} = ML^{-3}$$

$$\text{Velocity } v = \frac{\text{length}}{\text{time}} = LT^{-1}$$

$$\text{Gravitational acceleration } g = LT^{-2}$$

$$\text{Height above datum } z = L$$

The dimensions of each term on the left-hand side are

$$p = ML^{-1}T^{-2}, \quad \tfrac{1}{2}\rho v^2 = ML^{-3} \times L^2T^{-2} = ML^{-1}T^{-2}$$

$$\rho g z = ML^{-3} \times LT^{-2} \times L = ML^{-1}T^{-2}$$

Thus all terms have the same dimensions and the equation is physically possible if the constant H also has the dimensions $ML^{-1}T^{-2}$.

The constant of $\frac{1}{2}$ in the second term is a pure number and thus not amenable to dimensional analysis. Had this been any other value, the process would have been equally valid. Something in addition to dimensional analysis is required to settle the fact that this constant is $\frac{1}{2}$.

1.2 Velocity of a pressure wave

The velocity of propagation a of a pressure wave through a liquid could be expected to depend upon the elasticity of the liquid represented by the bulk modulus K and its mass density ρ. Establish by dimensional analysis the form of a possible relationship.

Solution. Assume a simple exponential equation

$$a = CK^a\rho^b \tag{1}$$

where C is a numerical constant and a and b are unknown powers.

The dimensions of the variables are: velocity $a = LT^{-1}$, bulk modulus $K = ML^{-1}T^{-2}$, mass density $\rho = ML^{-3}$. If equation (1) is to be correct the powers a and b must be such that both sides of the equation contain the same powers of M, L and T. Rewrite equation (1) replacing each quantity by its dimensions, remembering that the constant C is a pure number.

$$LT^{-1} = M^a L^{-a} T^{-2a} \times M^b L^{-3b}$$

Equating powers of M, L and T,

$$0 = a + b$$

$$1 = -a - 3b$$

$$-1 = -2a$$

from which

$$a = +\tfrac{1}{2} \quad \text{and} \quad b = -\tfrac{1}{2}$$

Thus a possible equation is $a = C\sqrt{\left(\dfrac{K}{\rho}\right)}$.

Compare this result with example **10.5.**

Dimensional analysis gives the form of a possible equation but the value of the constant C would have to be determined experimentally.

1.3 Pipe flow

Show that a rational formula for the loss of pressure when a fluid flows through geometrically similar pipes is

$$p = \frac{\rho l v^2}{d}\phi\left(\frac{vd\rho}{\mu}\right)$$

where d is the diameter of the pipe, l is the length of the pipe, ρ is the mass density and μ the dynamic viscosity of the fluid, v is the mean velocity of flow through the pipe and ϕ means "a function of".

Solution. Assume $p = C\rho^a l^b v^c d^e \mu^f$ where C is a numerical constant and a, b, c, e, f are unknown powers.

The dimensions of the quantities are: $p = ML^{-1}T^{-2}$, $\rho = ML^{-3}$, $l = L$, $v = LT^{-1}$, $d = L$ and $\mu = ML^{-1}T^{-1}$.

Substituting these dimensions for the quantities,

$$ML^{-1}T^{-2} = M^a L^{-3a} \times L^b \times L^c T^{-c} \times L^e \times M^f L^{-f} T^{-f}$$

Equating powers of M, L and T,

$$1 = a + f \qquad (1)$$

$$-1 = -3a + b + c + e - f \qquad (2)$$

$$-2 = -c - f \qquad (3)$$

There are five unknown powers and only three equations, so that it must be decided to solve for three of the unknown powers in terms of the others. In practice this decision is made from experience; in examination problems some indication is usually given in the question as to the form of the final result, which depends on the choice of unknowns to be solved. In this case solve for the powers of ρ, v and d, namely a, c and e.

From equation (1) $a = 1 - f$

From equation (3) $c = 2 - f$

From equation (2) $e = -1 + 3a - c - b + f$

$= -f - b$

Substituting these values in the original equation

$$p = C\rho^{1-f} l^b v^{2-f} d^{-f-b} \mu^f$$
$$= C\rho v^2 \left(\frac{l}{d}\right)^b \left(\frac{\rho v d}{\mu}\right)^{-f}$$
$$= \frac{\rho v^2 l}{d} C \left(\frac{l}{d}\right)^{b-1} \left(\frac{\rho v d}{\mu}\right)^{-f}$$

For geometrically similar pipes (l/d) is a constant and $(l/d)^{b-1}$ can be combined with C. Putting $K = C(l/d)^{b-1}$

$$p = \frac{\rho l v^2}{d} K \left(\frac{\rho v d}{\mu}\right)^{-f}$$

Since neither K nor f are known this is written simply as

$$p = \frac{\rho l v^2}{d} \phi \left(\frac{\rho v d}{\mu}\right) \quad (4)$$

It is interesting to compare this result with the Darcy formula

$$h_f = 4f \frac{l}{d} \frac{v^2}{2g}$$

From equation (4) $h_f = \dfrac{p}{\rho g} = \dfrac{l v^2}{dg} \phi \left(\dfrac{\rho v d}{\mu}\right)$

which indicates that the Darcy coefficient f must be a function of the pipe Reynolds number $\rho vd/\mu$. This has already been shown by more orthodox methods (*see* volume 1).

1.4 Pipe flow

A rational formula for loss of pressure when fluid flows through geometrically similar pipes is

$$p = \frac{\rho l v^2}{d} \phi \left(\frac{\rho v d}{\mu}\right)$$

The measured loss of head in a 50 mm diam pipe conveying water at 0.6 m/s is 800 mm of water per 100 m length. Calculate the loss of head in millimetres of water per 400 m length when air flows through a 200 mm diam pipe at the *corresponding speed.* Assume that the pipes have geometrically similar roughness and take the densities of air and water as 1.23 and 1000 kg/m^3 and the absolute viscosities as 1.8×10^{-5} and 0.12 Pa/s respectively.

Solution. The formula $p = (\rho l v^2/d)\phi(\rho v d/\mu)$, derived by dimensional analysis in example **1.3**, might appear to be of little use since the nature of the function $\phi(\rho v d/\mu)$ is unknown, but it can be used for comparison of the pressure drops in two geometrically similar pipes provided that the value of the Reynolds number $\rho v d/\mu$ is the same in both cases. Then

$$\phi\left(\frac{\rho_1 v_1 d_1}{\mu_1}\right) = \phi\left(\frac{\rho_2 v_2 d_2}{\mu_2}\right)$$

and the ratio of pressure drops simplifies to

$$\frac{p_1}{p_2} = \frac{\rho_1}{\rho_2} \cdot \frac{l_1}{l_2} \cdot \frac{v_1^2}{v_2^2} \cdot \frac{d_2}{d_1}$$

The velocity of flow in the second pipe required to make the Reynolds number the same in both is known as the *corresponding speed.* Using the suffix w for the pipe containing water and a for that containing air, for equality of Reynolds numbers,

$$\frac{\rho_w v_w d_w}{\mu_w} = \frac{\rho_a v_a d_a}{\mu_a}$$

Corresponding speed for air

$$= v_a = v_w \frac{\rho_w}{\rho_a} \frac{d_w}{d_a} \frac{\mu_a}{\mu_w}$$

$$= 0.6 \frac{1000}{1.23} \times \frac{50}{200} \times \frac{1.8 \times 10^{-5}}{0.12} = 1.83 \text{ m/s}$$

Ratio of pressure drops

$$\frac{p_a}{p_w} = \frac{\rho_a}{\rho_w} \frac{l_a}{l_w} \frac{d_w}{d_a} \frac{v_a^2}{v_w^2}$$

$$= \frac{1.23}{1000} \times \frac{400}{100} \times \frac{50}{200} \times \frac{1.83^2}{0.6^2} = 0.001\,44$$

If loss of head per 100 m in 50 mm pipe is 800 mm of water

Loss of head per 400 m in 200 mm pipe $= 0.001\,44 \times 800$

$= \mathbf{9.15}$ **mm of water**

1.5 Resistance to a partially-submerged body

Find by dimensional analysis a rational formula for the resistance to motion R of geometrically similar bodies moving partially submerged through a viscous, compressible fluid of density ρ and coefficient of dynamic viscosity μ with a uniform velocity V.

Solution. The resistance R will be due to skin friction, wave resistance and compressibility of the fluid and will depend on the size of the body denoted by a characteristic length l, the velocity V, the density ρ, viscosity μ and bulk modulus K of the fluid and the gravitational acceleration g (for wave resistance). Thus R is a function of l, V, ρ, μ, K and g. The form of this function may be simple as was assumed in example **1.4** or may consist of a series of terms made up of the product of the variables each raised to suitable powers

$$R = Al^x V^y \rho^z \mu^p K^q g^r + A_1 l^{x_1} V^{y_1} \rho^{z_1} \mu^{p_1} K^{q_1} g^{r_1} + \cdots \quad (1)$$

where A, A_1, ... *are numerical constants,* x, x_1, ..., y, y_1, ... etc. are unknown indices. Thus

$$\frac{R}{Al^x V^y \rho^z \mu^p K^q g^r} = 1 + \frac{A_1}{A} l^{x_1 - x} V^{y_1 - y} \rho^{z_1 - z} \mu^{p_1 - p} K^{q_1 - 1} g^{r_1 - r}$$

Since the first term on the right-hand side is a pure number, the equation will only be correct if dimensionally

$$R = Al^x V^y \rho^z \mu^p K^q g^r$$

The dimensions of the quantities are: $R = MLT^{-2}$, $l = L$, $V = LT^{-1}$, $\rho = ML^{-3}$, $\mu = ML^{-1}T^{-1}$, $K = ML^{-1}T^{-2}$, $g = LT^{-2}$. Substituting in equation (1)

$$MLT^{-2} = L^x \times L^y T^{-y} \times M^z L^{-3z} \times M^p L^{-p} T^{-p} \times M^q L^{-q} T^{-2q} \times L^r T^{-2r}$$

Equating powers of M, L and T

$$1 = z + p + q \quad (2)$$

$$1 = x + y - 3z - p - q + r \quad (3)$$

$$-2 = -y - p - 2q - 2r \quad (4)$$

Equations (2), (3) and (4) allow of three solutions only. A useful result is obtained by solving for x, y and z giving

$$z = 1 - p - q, \quad y = 2 - p - 2q - 2r, \quad x = 2 - p + r.$$

All the other terms on the right-hand side of equation (1) are similar to the first so that by the same dimensional reasoning

$$x_1 = 2 - p_1 + r_1, \quad y_1 = 2 - p_1 - 2q_1 - 2r_1, \quad z_1 = 1 - p_1 - q_1$$

and so on. Substituting in equation (1)

$$R = \rho V^2 l^2 \left\{ A \left(\frac{\rho V l}{\mu}\right)^{-p} \left(\frac{V}{\sqrt{(K/\rho)}}\right)^{-2q} \left(\frac{V}{\sqrt{(lg)}}\right)^{-2r} + A_1 \left(\frac{\rho V l}{\mu}\right)^{-p_1} \left(\frac{V}{\sqrt{(K/\rho)}}\right)^{2q_1} \left(\frac{V}{\sqrt{(lg)}}\right)^{-2r_1} + \cdots \right\}$$

The series in brackets is an unknown function of $(\rho Vl/\mu)$, $(V/\sqrt{(K/\rho)})$ and $(V/\sqrt{(lg)})$ and can be written

$$R = \rho V^2 l^2 \phi \left\{ \frac{\rho V l}{\mu}, \frac{V}{\sqrt{(K/\rho)}}, \frac{V}{\sqrt{(lg)}} \right\} \quad (5)$$

The terms in the function are all dimensionless groups,

$\frac{\rho V l}{\mu}$ is the Reynolds number,

$\frac{V}{\sqrt{(K/\rho)}}$ is the Mach number and

$\frac{V}{\sqrt{(lg)}}$ is the Froude number.

Equation (5) may also be written

$$\frac{R}{\rho V^2 l^2} = \phi \left\{ \frac{\rho V l}{\mu}, \frac{V}{\sqrt{(K/\rho)}}, \frac{V}{\sqrt{(lg)}} \right\}$$

in which case $R/\rho V^2 l^2$ will also be found to be dimensionless.

1.6 Thrust of screw propeller

Assuming that the thrust F of a screw propeller is dependent upon the diameter d, speed of advance v, fluid density ρ, revolutions per second n and coefficient of viscosity μ, show that it can be expressed by the equation

$$F = \rho d^2 v^2 \phi \left\{ \frac{\mu}{\rho d v}, \frac{dn}{v} \right\}$$

Solution. F will be a function of d, v, ρ, n and μ. Instead of expanding this function fully as in example **1.5**, since all the terms are similar we can write

$$F = \sum A d^m v^p \rho^q n^r \mu^s \quad (1)$$

where A is a numerical constant and m, p, q, r and s are unknown powers.

The dimensions of the variables are $F = MLT^{-2}$, $d = L$, $v = LT^{-1}$, $\rho = ML^{-3}$, $n = T^{-1}$, $\mu = ML^{-1}T^{-1}$.

Substituting the dimensions for the variables, equation (1) will be true if

$$MLT^{-2} = L^m \times L^p T^{-p} \times M^q L^{-3q} \times T^{-r} \times M^s L^{-6} T^{-3}$$

Equating powers of

$$\begin{array}{ll} M & 1 = q + s \\ L & 1 = m + p - 3q - s \\ T & -2 = -p - r - s \end{array}$$

The equation given in the problem indicates that it is desirable to solve for m, p and q in terms of r and s.

$$q = 1 - s, \quad p = 2 - r - s$$
$$m = 1 - p + 3q + s = 2 + r - s$$

Substituting in equation (1)

$$F = \sum A d^{2+r-s} v^{2-r-s} \rho^{1-s} n^r \mu^s$$

Regrouping by powers

$$F = \sum A \rho d^2 v^2 \left(\frac{\mu}{\rho d v}\right)^s \left(\frac{dn}{v}\right)^r$$

which can be written

$$F = \rho d^2 v^2 \phi\left\{\frac{\mu}{\rho d v}, \frac{dn}{v}\right\}$$

where ϕ means "a function of".

1.7 Buckingham's Pi theorem

> State Buckingham's Π theorem and apply it to the problem of example **1.6.**

Solution. Buckingham's Π theorem states that if there are n variables in a problem and these variables contain m primary dimensions (for example M, L and T) the equation relating the variables will contain $n - m$ dimensionless groups. Buckingham referred to these dimensionless groups as Π_1, Π_2, etc., and the final equation obtained is

$$\Pi_1 = \phi(\Pi_2, \Pi_3, \ldots \Pi_{n-m})$$

Thus in example **1.5** there are seven variables with three primary dimensions so that the final equation

$$\frac{R}{\rho V^2 l^2} = \phi\left\{\frac{\rho V l}{\mu}, \frac{V}{\sqrt{(K/\rho)}}, \frac{V}{\sqrt{(lg)}}\right\}$$

is formed of four dimensionless groups.

In the problem of example **1.6** there are six variables, F, ρ, d, v, μ and n and three primary dimensions. The equation relating the variables will therefore be formed of 6−3=3 dimensionless groups and will be

$$\Pi_1 = \phi(\Pi_2, \Pi_3)$$

The dimensionless groups can be formed as follows:

(1) Choose a number of variables equal to the number of primary dimensions and including all these dimensions, in this case F, ρ and v.

(2) Form dimensionless groups by combining the variables selected in (1) with each of the others in turn.

Combining F, ρ and v with d to form a dimensionless group:

$$\Pi_1 = \frac{F}{\rho v^2 d^2}$$

Combining F, ρ and v with n,

$$\Pi_2 = \frac{F n^2}{\rho v^4}$$

Combining F, ρ and v with μ,